DistOpt: A Ptolemy-based Tool to Model and Evaluate the Solutions of Optimization Problems in Distributed Environments

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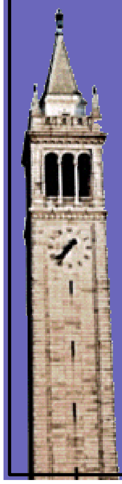
DistOpt and Distributed Optimization
Cassino, Italy

DistOpt and Distributed Optimization
Contents

DistOpt and Distributed Optimization
Credits

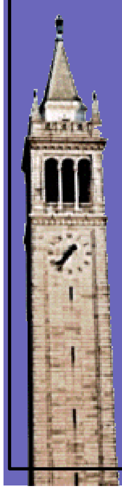
- Publications related to DistOpt
  - J. Contreras, A. Losi, M. Russo, and F.F. Wu, *DistOpt: A Software Framework for Modeling and Evaluating Optimization Problem Solutions in Distributed Environments*, Journal of Parallel and Distributed Computing, June 2000, pp. 741-763
  - J. Contreras, A. Losi, M. Russo, and F.F. Wu, *Simulation and Evaluation of Optimization Problem Solution in Distributed Energy Management Systems*, accepted for publication in IEEE Transactions on Power Systems.
  - A. Losi, and M. Russo, *A Note on the Application of the Auxiliary Problem Principle*, to be re-submitted (after the first review) to the Journal of Optimization Theory and Applications

DistOpt and Distributed Optimization
Introduction and motivation



- Coarse-grain algorithms, with focus on the structure of the optimization problem (not on the structure of the technique)
- Splitting of the optimization problem into subproblems, with grains corresponding to subproblems
- Decomposition/coordination approach
  - Suitable formulation of the problem
  - Decomposition of the optimization problem into smaller subproblems, iteratively solved
  - Coordination of the subproblems to drive their solutions to a solution of the original problem

DistOpt and Distributed Optimization
Introduction and motivation

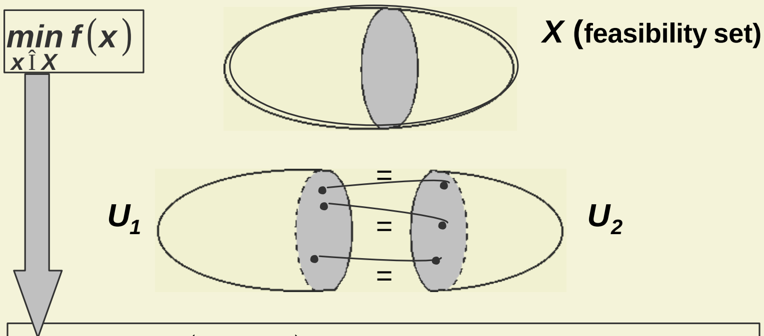


- Design variables
  - decomposition/coordination method
  - splitting of the problem into subproblems
  - technique for solving the subproblems
  - synchronization
  - computing capacity of the processors
  - characteristics of the communications (other packages)
- DistOpt helps modeling and evaluating coarse-grain distributed optimization algorithms, for wide classes of optimization problems and decomposition/coordination methods

DistOpt and Distributed Optimization

Optimization problem

$$\min_{x \in X} f(x)$$



$$\min_{u_1 \in U_1, \dots, u_n \in U_n} f(u_1, \dots, u_n), \qquad \text{with } \begin{cases} u_1 = \{x_1, y_{1,2}, \dots, y_{1,n}\}, \\ \vdots \\ u_n = \{x_n, y_{n,1}, \dots, y_{n,n-1}\} \end{cases}$$

$$\text{s.t. } \; y_{i,j} - y_{j,i} = 0, \quad i,j = 1, \dots, n \; (j > i)$$

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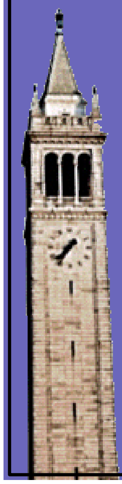
Auxiliary Problem Principle

- The Auxiliary Problem Principle (APP - by Guy Cohen, France) is a general decomposition/ coordination theory that:
  - does not require separability assumptions (such as additive cost or additive constraints)
  - is a generalization of many known methods
  - encompasses both one-level and two-level methods
  - for the two-level methods, the convergence conditions are quite general
- DistOpt is based on the APP's two-level methods



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<u>Auxiliary Problem Principle</u>

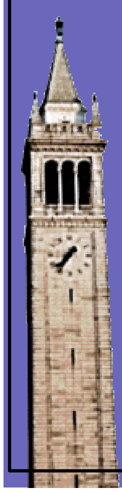
$$\min_{u_1 \in U_1, \ldots, u_n \in U_n} f(u_1, \ldots, u_n), \quad \text{s.t. } y_{i,j} - y_{j,i} = 0, \; i,j = 1, \ldots, n \; (j > i)$$

$$\text{with } \begin{cases} u_1 = \{x_1, y_{1,2}, \ldots, y_{1,n}\}, \ldots \\ \ldots, u_n = \{x_n, y_{n,1}, \ldots, y_{n,n-1}\} \end{cases}$$

iterative scheme (slightly modified)

$$\min_{u_i \in U_i} \left\{ \begin{array}{l} K_i(u_i) + \left\langle \varepsilon f'_{u_i}(u_1^k, \ldots, u_n^k) - K'_{u_i}(u_i^k), u_i \right\rangle + \\ + \varepsilon \sum_{j=1, j \neq i}^{n} \left\langle p_{i,j}^k + c\left(y_{i,j}^k - y_{j,i}^k\right), y_{i,j} \right\rangle \end{array} \right\} \Rightarrow u_i^{k+1} \quad i = 1, \ldots, n$$

$$p_{i,j}^{k+1} = p_{i,j}^k + \rho\left(y_{i,j}^{k+1} - y_{j,i}^{k+1}\right), \quad \text{for } j = 1, \ldots, n, \; j \neq i$$



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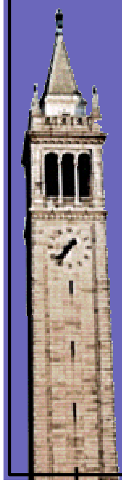
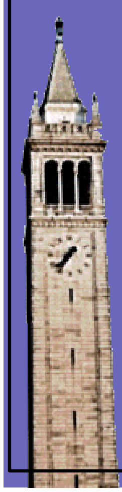
<u>Auxiliary Problem Principle</u>

- Global convergence conditions
  - Convex (and differentiable) objective function, f
  - closed convex feasibility sets, U_i (i=1,…,n)
  - convex coupling constraints (affine in the equality case)
  - strongly convex core function, K
  - defined bounds on the parameters c, ρ, ε
- In practical non-convex cases, with APP iterative scheme optimality necessary conditions are generally met in the limit, if convergence results.



DistOpt and Distributed Optimization
DistOpt structure

- Modular, flexible, extendible and conducive to cooperation
  - OOP
- Ptolemy (classic)
  - Discrete-Event (DE) domain
- Three levels of abstraction
  - definition of the problem and its splitting into subproblems
  - formulation of the transformed problem and of the subproblems, and set-up of the two-level algorithm
  - solution of the optimization subproblems



DistOpt and Distributed Optimization
DistOpt structure

- First level: Definition of the problem and its splitting
  - description of the optimization problem (objective function and constraints), and information on the splitting into subproblems - provided by the user
  - no constraints on the coding of the problem (data structure and functions internal to the application)
  - formal definition of the methods
  - defined data structure for the data exchange - through a file

DistOpt and Distributed Optimization
<u>DistOpt structure</u>

- **Second level: formulation of transformed problem, subproblems, two-level decomposition/ coordination algorithm**
  - problem transformation, with variable duplication
  - subproblem formulation and coordination
  - set-up of APP parameters (core function K, parameters c, e, r)
  - choice of synchronous/asynchronous execution of subproblems

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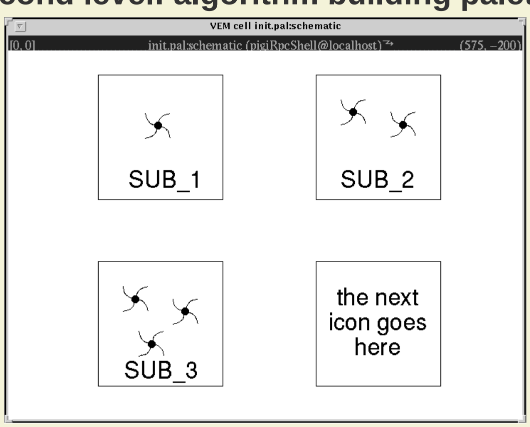
<u>DistOpt structure</u>

- **Second level: algorithm building palette**



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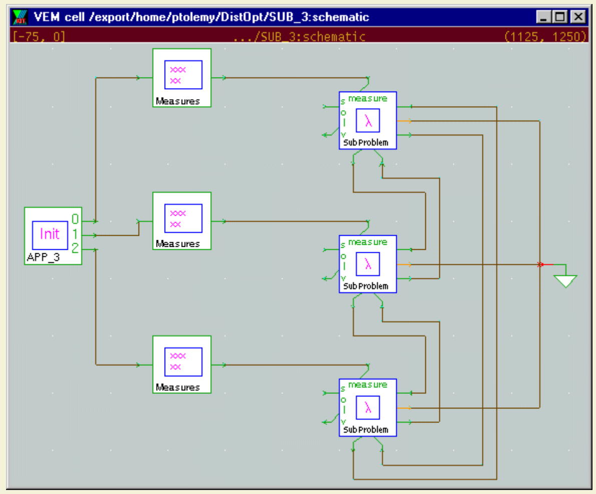
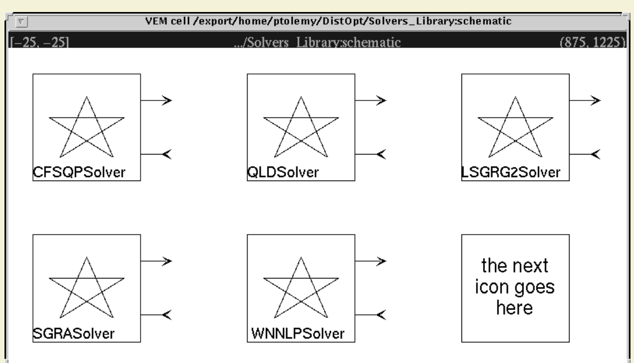
DistOpt structure

- Second level: block SUB_3



DistOpt and Distributed Optimization
DistOpt structure

- Third level: solution of the subproblems
  - choice of the solver for each subproblem



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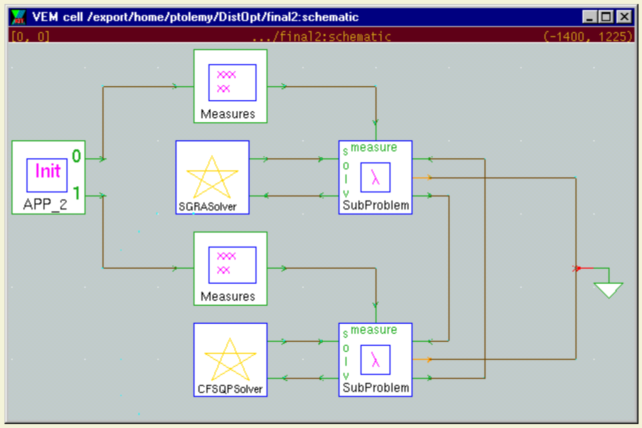
Using DistOpt

- A DistOpt universe



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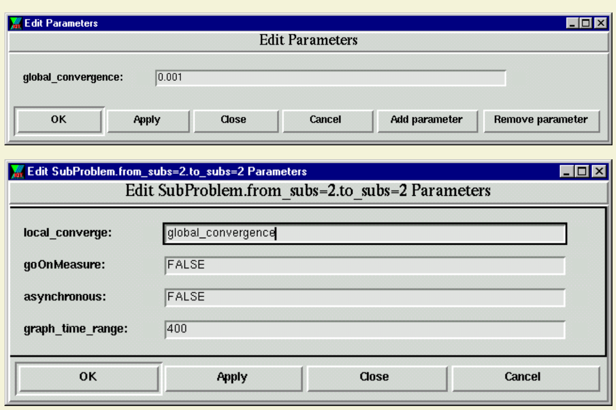
Using DistOpt

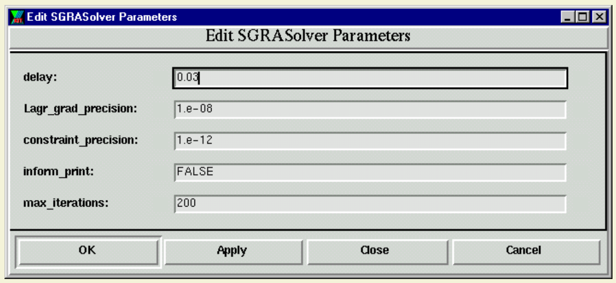
- Set-up of the computation



DistOpt and Distributed Optimization

Using DistOpt

- Set-up of the computation



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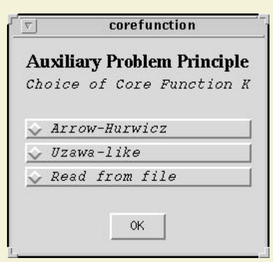
Using DistOpt

- Choice of the core function K



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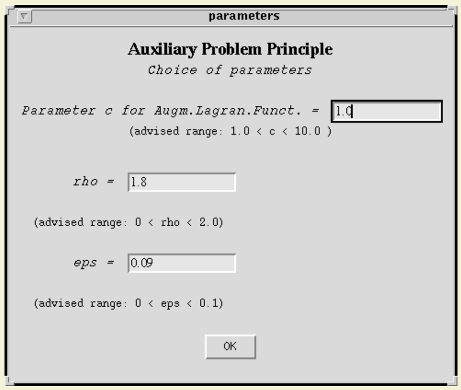
Using DistOpt

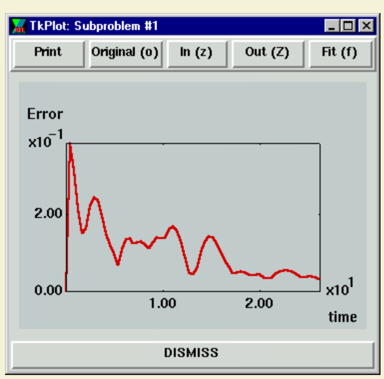
- Choice of convergence parameters



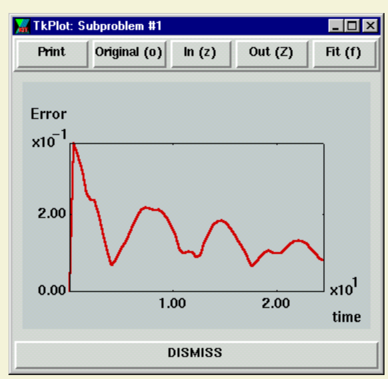
DistOpt and Distributed Optimization
Using DistOpt

- Some results

synchronous



asynchronous



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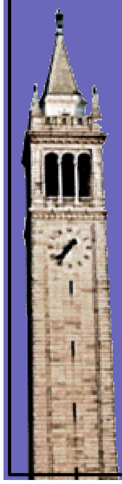

- ftp://sistelet.ing.unicas.it/DistOpt
  - readme.txt
  - userguide.pdf
  - DistOpt.tar.gz