System-Level Modeling of Continuous and Discrete Dynamics

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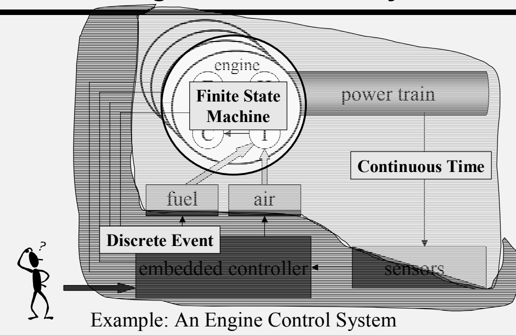
The heterogeneous view of systems



Example: An Engine Control System

Related Work

Mixing CT and DE (Mixed-Signal Model)

- SPICE → SPLICE
- Simulink 1.0 → Simulink 3.0
- VHDL-AMS ← VHDL
- Verilog-AMS ← Verilog

CT - - - - - - - - - - - - DE

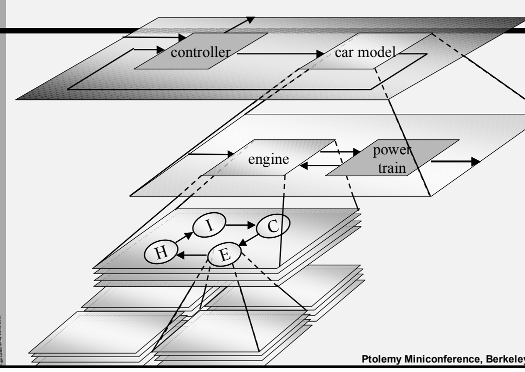
Mixing CT and FSM (Hybrid-System Model)

- Automatic Control → Hybrid automata ← Computer Science

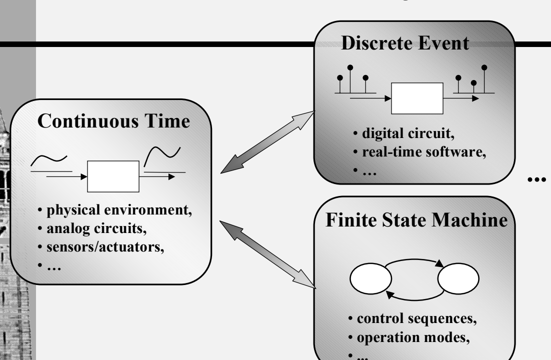
CT - - - - - - - - - - - - FSM

Omola
SHIFT
χ, ...

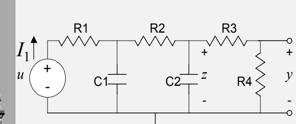
The Hierarchical View of Systems



Continuous and Discrete Dynamics



CT: Conservative Law Model



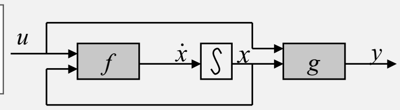
$$(R_1R_2C_1C_2)\ddot{z} + \dot{z} + (R_1C_1 + R_2C_2)z = u$$

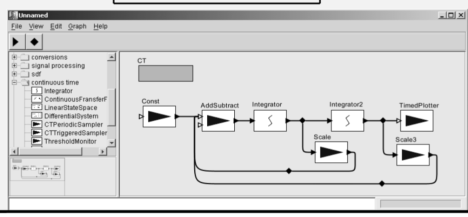
$$y = \frac{R_4}{(R_3+R_4)} z$$

$$\begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_2} & 0 & 0 & -1 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+C_1\frac{d}{dt} & -\frac{1}{R_2} & 0 & 0 \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3}+C_2\frac{d}{dt} & -\frac{1}{R_3} & 0 \\ 0 & 0 & -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ y \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ u \end{bmatrix}$$

CT: Signal-Flow Model

$$\dot{x} = f(x,u,t)$$
$$x(t_0) = x_0$$
$$y = g(x,u,t)$$



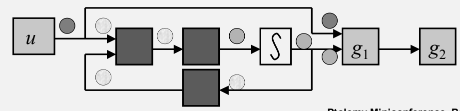


Solving ODEs by Token Flow

$$\dot{x} = f(x,u,t)$$
$$x(t_0) = x_0$$
$$y = g(x,u,t)$$

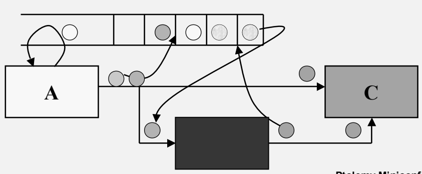
Example: Runge-Kutta 2

$$k_0 = f(x_k, t_k)$$
$$k_1 = f(x_k + \frac{h}{2} k_0, t_k + \frac{h}{2})$$
$$k_2 = f(x_k + \frac{3}{4} hk_1, t_k + \frac{3}{4} h)$$
$$x_{k+1} - x_k = h \cdot (\frac{2}{9} k_0 + \frac{1}{9} k_1 + \frac{4}{9} k_2)$$

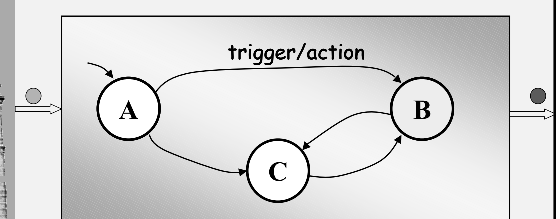


Discrete Event Model

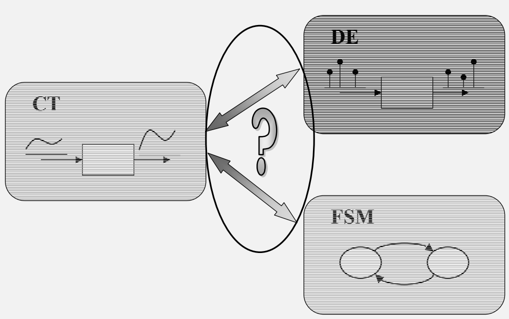
- Global notion of time
- event = (time_tag, data_token)
- Components are required to be causal



Finite State Machine

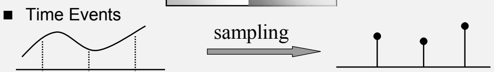


Signal Conversions

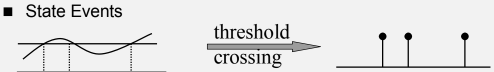


Event Generation

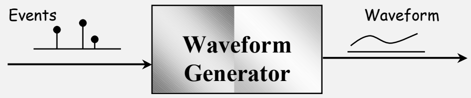
- Time Events



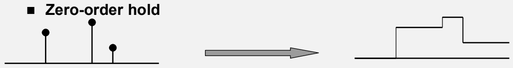
- State Events



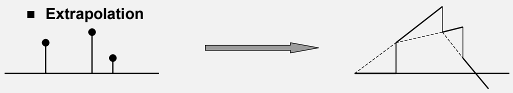
Waveform Generation



- Application Dependent
  - Zero-order hold



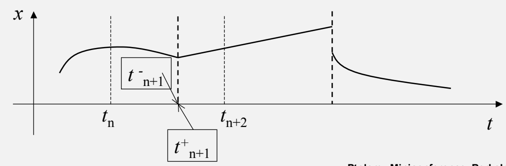
  - Extrapolation



Breakpoints

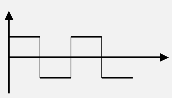
- Breakpoints are time instances where f is not sufficiently smooth or g is not continuous
  - u is not continuous
  - f discontinuous w.r.t. x, u, or t
  - g needs to generate events

$$\dot{x} = f(x,u,t)$$
$$x(t_0) = x_0$$
$$y = g(x,u,t)$$

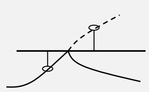


Two Classes of Breakpoints

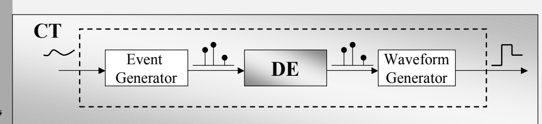
- predictable
  - known in advance
    - unsmoothness w.r.t. u
    - time events
  - queued in a table
  - lookup the table before each integration step.

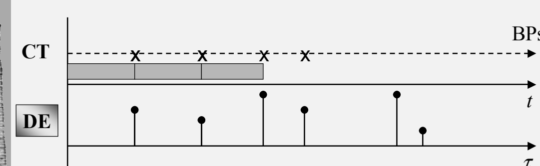


- unpredictable
  - not known in advance
    - unsmoothness w.r.t. x
    - state events
  - detect after each step
  - iteratively locate it with limited accuracy.

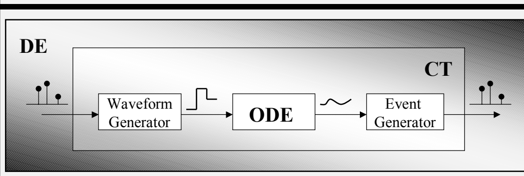


Hierarchical Composition — DE inside CT



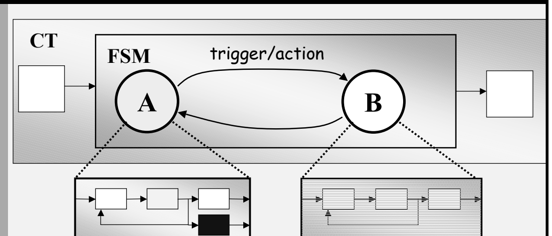


Hierarchical Composition — CT inside DE



- CT components must run ahead of the DE (global) time!
- They must be able to rollback if sees an input event in the past.
- They cannot emit detected events until the global time catches up
- Can show that: If there is only one CT component in a DE system, then no rollback is necessary.

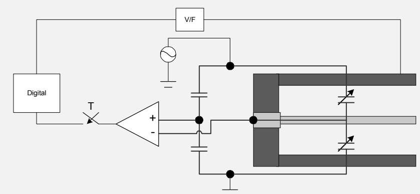
Hybrid Systems



Conclusion –
Implementation and Demos in Ptolemy II

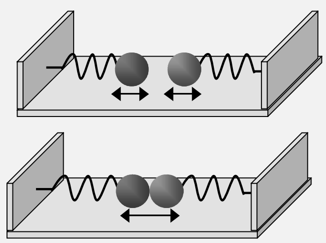
- CT Modeling and simulation
  - Higher order functions for transfer functions etc.
- Event Detection and step size control
- Mixed-Signal Modeling
  - Micro-accelerometer (sigma-delta) model
- Hybrid System Modeling
  - Sticky point masses model

Example: MEMS Accelerometer



M. A. Lemkin, "Micro Accelerometer Design with Digital Feedback Control",
Ph.D. dissertation, EECS, UC, Berkeley, Fall 1997

Example: Sticky Point Masses



The stickiness is exponentially decaying with respect to time.