



Figure 1: Block diagram for Problem 1

1 Sampling and reconstruction

1. Consider the block diagram of figure 1. The input to the system is a signal $y \in \text{DiscSignal}$. The block $\text{ImpulseGenerator}_T$ takes as input the signal y and produces as output the signal $w \in \text{ContSignal}$. w consists of a sequence of Dirac delta functions at times $kT, k \in \text{Ints}$ of magnitude $y(k)$. The reconstruction filter is an LTI system with impulse response h and frequency response is H . The output is the signal $z \in \text{ContSignals}$. Take $T = 1$ ms. Consider three signals y_1, y_2, y_3 given by:

$$\begin{aligned} y_1(n) &= 1, n = 0; = 0 \text{ elsewhere} \\ y_2(n) &= 1, n = 0, 1, 2; = 0 \text{ elsewhere} \\ y_3(n) &= -1, n = 3k - 1; = 0, n = 3k; = 1, n = 3k + 1 \end{aligned}$$

- (a) Plot y_i .
 - (b) Give an expression or a plot of w for each y_k .
 - (c) Also consider three different reconstruction filters as shown. For each of nine combinations of y_i and h_j calculate the corresponding output signal z_{ij} . You can give your answer either as an expression or as a plot.
2. Consider the following continuous time signals: $\forall t$

$$x_k(t) = \cos 2\pi kt, \quad k = 1, 2, 3.$$

This signal is sampled at T and passed through $\text{ImpulseGenerator}_T$ to produce the signal w , with CTFT W .

- (a) For $T = 0.1, 0.2, 0.4$, write an expression for w and sketch W .
- (b) Suppose that w is sent through an ideal low pass filter to produce output signal z . Obtain an expression for z in each case, and sketch its CTFT Z .

(c) Repeat the two parts above for the signal $x_1 + x_2 + x_3$.