EECS20n, Midterm 2, 11/17/00

Please print your name and your TA's name here:

Last Name ____________________ First ____________________ TA's name ____________________

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Problem 6:

Problem 7:

Problem 8:

Total:

Read the questions carefully before you answer. Good luck.

1. **10 points** Write the following in Cartesian coordinates (i.e. in the form $x + jy$)

   (a) $j^3 - j^2 + j + 1 =$

   (b) $(1 - j1)/(1 + j1) =$

   (c) $\sqrt{\cos \pi/4 + j \sin \pi/4} =$

   Write the following in polar coordinates (i.e. in the form $re^{j\theta}$)

   (a) $1 + j1 =$

   (b) $(1 + j1)/(1 - j1) =$

2. **10 points** Which of the following discrete-time or continuous-time signals is periodic. Answer yes or no. If the signal is periodic, give its fundamental period and state the units. Suppose that for a discrete-time signal, n denotes **seconds**, and for a continuous-time signal, t denotes **minutes**.

   (a) $\forall n \in \textit{Ints}, \quad x(n) = e^{\sqrt{2}\, n}$ Periodic (Y/N) Period =

   (b) $\forall t \in \textit{Reals}, \quad x(t) = e^{\sqrt{2}\, t}$ Periodic (Y/N) Period =

   (c) $\forall n \in \textit{Ints}, \quad x(n) = \cos 3\pi n + \sin(3\pi n + \pi/7)$ Periodic (Y/N) Period =

   Find A, θ, ω in the following expression:

$$A\cos(\omega t + \theta) = \cos(2\pi \times 10,000t + \frac{\pi}{4}) + \sin(2\pi \times 10,000t + \frac{\pi}{4}).$$

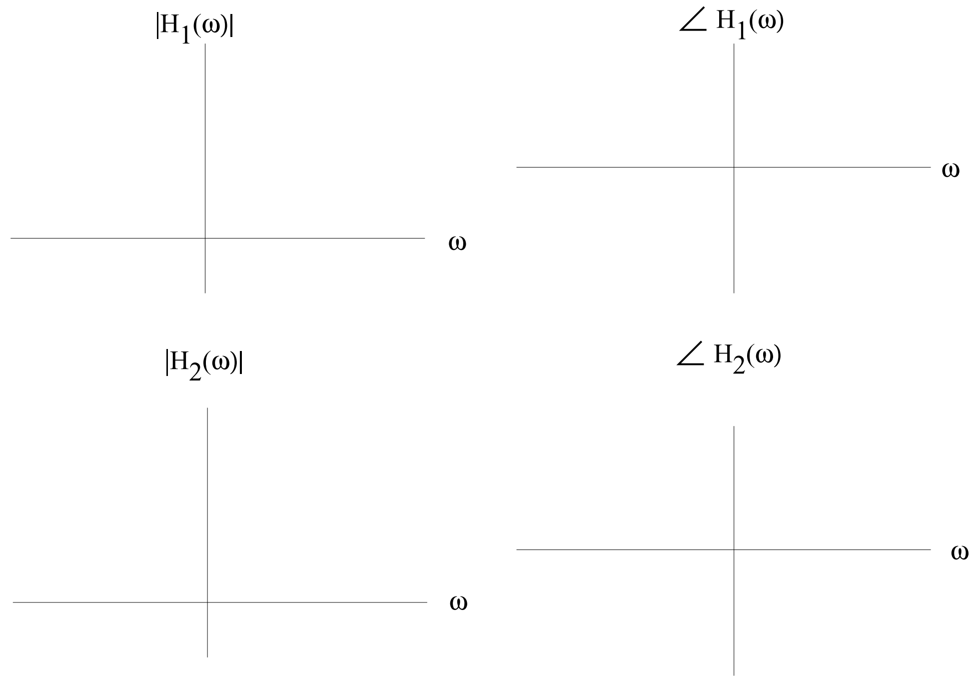

Figure 1: Plots for Problem 3

3. **10 points** On Figure 1 plot the amplitude and phase response of the following frequency responses. On your plots carefully mark the values for $\omega = 0$ and for one other non-zero value of ω.

   (a) $\forall \omega \in$ *Reals*, $\quad H_1(\omega) = 1 + j\omega$

   (b) $\forall \omega \in$ *Reals*, $\quad H_2(\omega) = 1 + \cos \omega$

   Which of H_1, H_2 can be the frequency response of a discrete-time system?

4. **10 points** A discrete-time system H has impulse response h : *Ints* $\to$ *Reals* given by

$$h(n) = \begin{cases} 1, & n = -1, 0, 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) What is the step response of H, i.e. the output signal when the input signal is *step*, where *step*$(n) = 1, n \geq 0$, and *step*$(n) = 0, n < 0$? You can give your answer as a plot or as an expression.

(b) What is the frequency response of H?

(c) What is the output signal of H for the following input signals?

i. $\forall n, \quad x(n) = \cos n$

ii. $\forall n, \quad x(n) = \cos(n + \pi/6)$

5. **10 points**

   (a) Find the frequency response for the LTI systems described by these differential equations (input is x, output is y)

      i. $\dot{y}(t) + 0.5y(t) = x(t)$

      ii. $\ddot{y}(t) + 0.5\dot{y}(t) + 0.25y(t) = \dot{x}(t) + x(t)$

   (b) What is the response of the first system above for the input $\forall t, \;\; x(t) = e^{j(100t+\pi/4)}$?

   (c) Find the frequency response for the LTI systems described by these difference equations (input is x, output is y)

      i. $y(n) + 0.5y(n-1) = x(n)$

      ii. $y(n) + y(n-1) + 0.25y(n-2) = x(n) + x(n-1)$

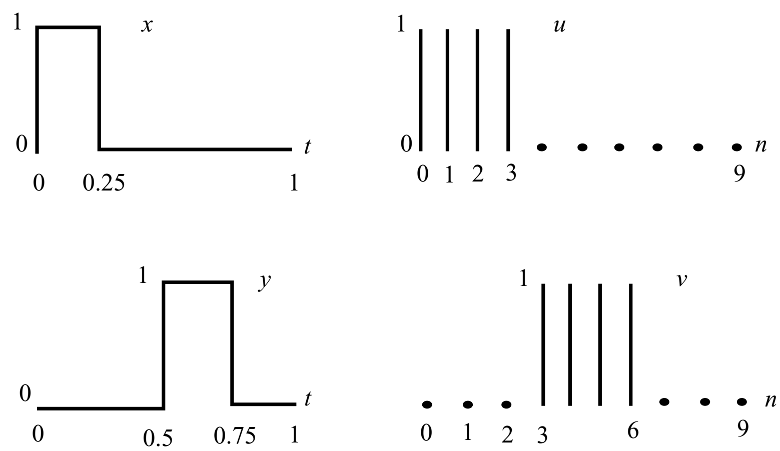

Figure 2: Periodic signals for Problem 6

6. **10 points** Figure 2 plots two continuous-time periodic signals x and y both with period 1 second, and two discrete-time signals u and v both with period 10 samples. The plots are given only for one period. Suppose the exponential Fouriers Series representations of these signals are given as:

$$\forall t \in \textit{Reals}, \quad x(t) = \; = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_x t}$$

$$\forall t \in \textit{Reals}, \quad y(t) = \; = \sum_{k=-\infty}^{\infty} Y_k e^{jk\omega_y t}$$

$$\forall n \in \textit{Ints}, \quad u(n) = \sum_{k=0}^{9} U_k e^{jk\omega_u n}$$

$$\forall n \in \textit{Ints}, \quad v(n) = \sum_{k=0}^{9} V_k e^{jk\omega_v n}$$

(a) Give the values of $\omega_x =$, $\omega_y =$, $\omega_u =$, $\omega_v =$. State the units of these frequencies.

(b) Calculate the values of the coefficients $X_0 =$, $Y_0 =$, $U_0 =$, $V_0 =$.

(c) Express y as a delayed version of x and v as a delayed version of u.

(d) Express the FS coefficients $\{Y_k\}$ in terms of $\{X_k\}$ and $\{V_k\}$ in terms of $\{U_k\}$.

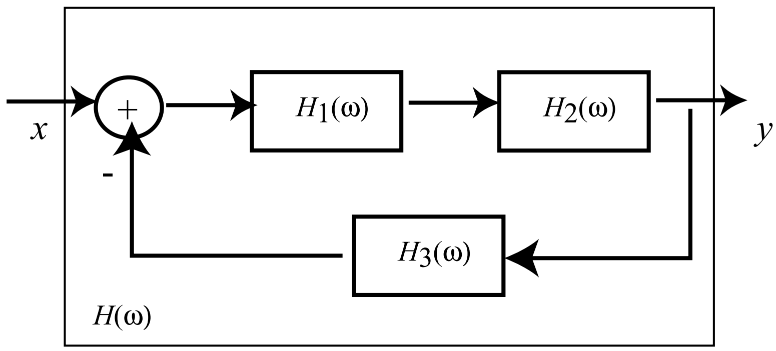

Figure 3: Feedback systems for Problem 7

7. **10 points** Figure 3 shows a feedback system obtained by composing three LTI systems. Note the negative feedback. In the figure, $H_k(\omega), k = 1, 2, 3$ is the frequency response of the three LTI systems.

   (a) Calculate the frequency response $H(\omega)$ of the feedback system in terms of the H_k.

   (b) Suppose $H_k(\omega) = 1/(1+j2\omega)$ for all $k = 1, 2, 3$. Calculate $H(0)$, $H(1)$ and $\lim_{\omega\to\infty} H(\omega)$.

8. **10 points** A continuous-time LTI system has the impulse response

$$\forall t \in \textit{Reals}, \quad h(t) = \begin{cases} 1, & |t| < 0.5 \\ 0, & \text{otherwise} \end{cases}$$

(a) Sketch the impulse response, and mark carefully the relevant points on your plot.

(b) Is this system causal? Answer yes or no.

(c) Sketch the step response of this system, i.e. the response to the input signal *step*$(t) = 1, t \geq 0$ and $= 0, t < 0$?

(d) Consider the input signal *impulsetrain*, where

$$\forall t \in \textit{Reals}, \quad \textit{impulsetrain}(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k).$$

Sketch *impulsetrain*.

(e) Sketch the response of the system to *impulsetrain*.