

EECS20n, Quiz 4 Solution, 12/1/00

1. **6 points** Suppose x has CTFT X . For $i = 1, 2, 3$ express the CTFT Y_i of y_i in terms of X , where $\forall t$

$$y_1(t) = x(t - 1), \quad y_2(t) = x(t) \cos(2\pi \times 100,000t), \quad y_3(t) = x(2t).$$

Write your answer below:

$$\begin{aligned} Y_1(\omega) &= X(\omega)e^{-j\omega} \\ Y_2(\omega) &= \frac{1}{2}[X(\omega - 200000\pi) + X(\omega + 200000\pi)] \\ Y_3(\omega) &= \frac{1}{2}X\left(\frac{\omega}{2}\right) \end{aligned}$$

2. **5 points** Consider the discrete time signal h , where $\forall n \in \text{Ints}, h(n) = 2^{-|n|}$. Calculate its DTFT H . What is $H(2\pi)$?

$$\begin{aligned} H(\omega) &= \sum_{n=-\infty}^{\infty} 2^{-|n|} e^{-j\omega n} \\ &= \sum_{n=-\infty}^0 2^n e^{-j\omega n} + \sum_{n=0}^{\infty} 2^{-n} e^{j\omega n} - 1 \\ &= \sum_{n=0}^{\infty} 2^{-n} e^{j\omega n} + \sum_{n=0}^{\infty} 2^{-n} e^{-j\omega n} - 1 \\ &= \frac{1}{1 - \frac{1}{2}e^{j\omega}} + \frac{1}{1 - \frac{1}{2}e^{-j\omega}} - 1 \\ &= \frac{2 - \cos \omega}{(1 - \frac{1}{2} \cos \omega)^2 + (\frac{1}{2} \sin \omega)^2} - 1 \end{aligned}$$

$$H(2\pi) = H(0) = 3.$$

3. **5 points** Suppose x has CTFT $X(\omega) = 2\delta(\omega) + \delta(\omega - 2) + \delta(\omega + 2)$. Then, $\forall t$,

$$x(t) = \frac{1}{2\pi}[2 + e^{j2t} + e^{-j2t}]$$