EECS 20. Practice Problems No. 3.

You have the following sources of practice problems:

- Quizzes you have taken.
- Problem sets you have turned in.
- The exam archive on the web from prior semesters.
- The practice problems for the midterms.
- The labs you have done.
- The problem session problems.
- The following problems, mostly drawn from the problem sessions.

In the problems below, any references to figures and sections are references to your reader, except when the figure is included here.

8.4 **T** Consider a discrete-time LTI system with impulse response h given by

$$\forall\, n \in \textit{Ints}, \quad h(n) = \delta(n-1)/2 + \delta(n+1)/2$$

And consider the periodic discrete-time signal given by

$$\forall\, n \in \textit{Ints}, \quad x(n) = 2 + \sin(\pi n/2) + \cos(\pi n).$$

(a) Is the system causal?

(b) Find the frequency response of the system. Check that your answer is periodic with period 2π.

(c) For the given signal x, find the fundamental frequency ω_0 and the Fourier series coefficients X_k in the Fourier series expansion,

$$x(n) = \sum_{k=-\infty}^{\infty} X_k e^{i\omega_0 kn}.$$

Give the units of the fundamental frequency.

(d) Assuming the input to the system is x as given, find the output.

8.5 Consider the continuous-time moving average system S, whose impulse response is shown in figure 1. Find its frequency response. The following fact from calculus may be useful:

$$\int_a^b e^{c\omega} c d\omega = e^{cb} - e^{ca}$$

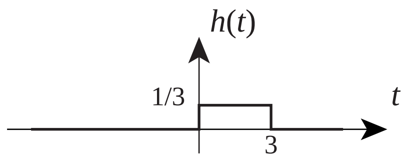

Figure 1: Signal example.

for real a and b and complex c. Use Matlab to plot the magnitude of this frequency response over the range -5 Hz to 5 Hz. Note the symmetry of the magnitude response, as required by the fact that

$$|H(\omega)| = |H(-\omega)|.$$

8.6 Consider a continuous-time LTI system with impulse response given by

$$\forall\, t \in \textit{Reals}, \quad h(t) = \delta(t-1) + \delta(t-2),$$

where δ is the Dirac delta function.

(a) Find a simple equation relating the input x and output y of this system.

(b) Find the frequency response of this system.

(c) Use Matlab to plot the magnitude frequency response of this system in the range -5 to 5 Hz.

9.8 Suppose a discrete-time signal x has DTFT given by

$$X(\omega) = i \sin(K\omega)$$

for some positive integer K. Note that $X(\omega)$ is periodic with period 2π, as it must be to be a DTFT.

(a) Determine from the symmetry properties of X whether the time-domain signal x is real.

(b) Find x. **Hint**: Use Euler's relation and the linearity of the DTFT.

9.9 Consider a periodic continuous-time signal x with period p and Fourier series $X \colon \textit{Ints} \to \textit{Comps}$. Let y be another signal given by

$$y(t) = x(t - \tau)$$

for some real constant τ. Find the Fourier series coefficients of y in terms of those of X.

9.10 Consider the continuous-time signal given by

$$x(t) = \frac{\sin(\pi t/T)}{(\pi t/T)}.$$

Show that its CTFT is given by

$$X(\omega) = \begin{cases} T, & \text{if } |\omega| \le \pi/T \\ 0, & \text{if } |\omega| > \pi/T \end{cases}$$

The following fact from calculus may be useful:

$$\int_a^b e^{c\omega} c d\omega = e^{cb} - e^{ca}$$

for real a and b and complex c.

9.11 If x is a continuous-time signal with CTFT X, then we can define a new time-domain function y such that

$$\forall\, t \in \textit{Reals}, \quad y(t) = X(t).$$

That is, the new time domain function has the same shape as the frequency domain function X. Then the CTFT Y of y is given by

$$\forall\, \omega \in \textit{Reals}, \quad Y(\omega) = 2\pi x(-\omega).$$

That is, the frequency domain of the new function has the shape of the time domain of the old, but reversed and scaled by 2π. This property is called **duality** because it shows that time and frequency are interchangeable. Show that the property is true.

9.12 Use the results of exercises and to show that a continuous time signal x given by

$$x(t) = \begin{cases} \pi/a, & \text{if } |t| \le a \\ 0, & \text{if } |t| > a \end{cases}$$

where a is a positive real number, has CTFT X given by

$$X(\omega) = 2\pi \frac{\sin(a\omega)}{(a\omega)}.$$

10.2 **E** A real-valued sinusoidal signal with a negative frequency is always exactly equal to another sinusoid with positive frequency. Consider a real-valued sinusoid with a negative frequency -440 Hz,

$$y(n) = \cos(-2\pi 440 nT + \phi).$$

Find a positive frequency f and phase θ such that

$$y(n) = \cos(2\pi f nT + \theta).$$

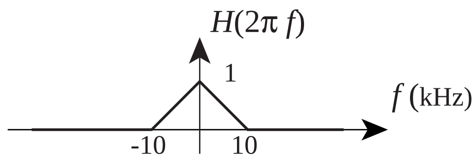

Figure 2: CTFT of an audio signal considered in exercise .

10.6 **T** Consider a continuous-time audio signal x with CTFT shown in figure 2. Note that it contains no frequencies beyond 10 KHz. Suppose it is sampled at 40 KHz to yield a signal that we will call x_{40}. Let X_{40} be the DTFT of x_{40}.

(a) Sketch $|X_{40}(\omega)|$ and carefully mark the magnitudes and frequencies.

(b) Suppose x is sampled at 20 KHz. Let x_{20} be the resulting sampled signal and X_{20} its DTFT. Sketch and compare x_{20} and x_{40}.

(c) Now suppose x is sampled at 15 KHz. Let x_{15} be the resulting sampled signal and X_{15} its DTFT. Sketch and compare X_{20} and X_{15}. Make sure that your sketch shows aliasing distortion.

1. **E** Consider a discrete-time LTI system with impulse response h given by

$$\forall\, n \in \textit{Ints}, \quad h(n) = \delta(n) + 2\delta(n-1)$$

(a) Plot the impulse response.

(b) Find and sketch the output when the input is the unit step,

$$u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(c) Find and sketch the output when the input is a ramp,

$$r(n) = \begin{cases} n & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(d) Find the frequency response.

(e) Show that the frequency response is periodic with period 2π.

(f) Show that the frequency response is conjugate symmetric.

(g) Give a simplified expression for the magnitude response.

(h) Give a simplified expression for the phase response.

(i) Suppose that the input x is given by

$$\forall\, n \in \textit{Ints}, \quad x(n) = \cos(\pi n/2 + \pi/6) + \sin(\pi n + \pi/3).$$

Find the output y.