# EECS20n, Quiz 1, 1/30/04, Solution

Indicate whether the following statements are **true** or **false**. There will be no partial credit, so please consider your answer carefully. Put a $\boxed{\text{box}}$ around your answer.

The solution below also gives a proof.

1. The sets $\{0, 1, 2, \cdots\}$ and $\{1, 2, 3, \cdots\}$ have the same cardinality. $\boxed{\text{True}}$
   The function $f$ given by $\forall n, f(n) = n + 1$ is one-to-one and onto.

2. The sets $\{0, 1, 2, \cdots\}$ and $[0, 1]$ have the same cardinality. $\boxed{\text{False}}$
   See p. 613 of text.

3. $\exists y \in$ *Reals* $\forall x \in$ *Reals* $\ y < x$. $\boxed{\text{False}}$
   Take $x = y + 1$.

4. $\forall x \in$ *Reals* $\exists y \in$ *Reals* $\ y < x$. $\boxed{\text{True}}$
   Take $y = x - 1$.

5. Consider the function $x$ where $\forall\ t \in$ *Reals*, $x(t) = 2$. Then $x \in [$*Reals* $\to$ *Reals*$]$. $\boxed{\text{True}}$
   $[$*Reals* $\to$ *Reals*$]$ is the space of ALL functions with domain *Reals* and range *Reals*. $x$ is such a function.

6. Let $f$: *Reals* $\to$ *Reals* and $g$: *Reals* $\to$ *Reals*. Define the functions $f + g$ by $\forall x \in$ *Reals*, $(f + g)(x) = f(x) + g(x)$, and $g \circ f$ by $\forall x \in$ *Reals*, $(g \circ f)(x) = f(g(x))$. Then

$$f + g = g + f \quad \boxed{\text{True}}$$
$$f \circ g = g \circ f \quad \boxed{\text{False}}$$

$$\forall x, (f + g)(x) = f(x) + g(x) = g(x) + f(x) = (g + f)(x).$$

   Define $f, g$ by $\forall x, f(x) = 1, \ \ g(x) = 2$. Then $\forall x, (f \circ g)(x) = 1, \ \ (g \circ f)(x) = 2$.

7. There is a function $f\colon \{1, 2\} \to \{a, b\}$ with *graph*$(f) = \{(1, a), (2, a)\}$. $\boxed{\text{True}}$
   The function is given by: $f(1) = f(2) = a$.

8. Let $f\colon X \to Y$. Then *graph*$(f) \subset X \times Y$. $\boxed{\text{True}}$
   *graph*$(f) = \{(x, y) \mid x \in X, y = f(x)\} \subset X \times Y$.

9. Let $G \subset X \times Y$. There exists a function $f$ such that *graph*$(f) = G$. $\boxed{\text{False}}$
   Take $X = \{1, 2\}, Y = \{a, b\}, G = \{(1, a)\}$.