

**EECS20n, Quiz 5, 4/27/05**

The quiz will take 15 minutes. Write your response on the sheet.

Please print your name and lab time here:

Last Name \_\_\_\_\_ First \_\_\_\_\_ Lab time \_\_\_\_\_

Recall:

The (complex) Fourier series representation of a discrete-time periodic signal  $x$  with period  $p$  is given by:

$$x(n) = \sum_{k=0}^{p-1} X_k e^{\frac{i2\pi kn}{p}}$$

where  $X_k$ 's are the Fourier coefficients, given by:

$$X_k = \frac{1}{p} \sum_{n=0}^{p-1} x(n) e^{-\frac{i2\pi kn}{p}}.$$

The discrete-time Fourier transform (DTFT)  $X$  of a discrete-time signal  $x$  is related to  $x$  by:

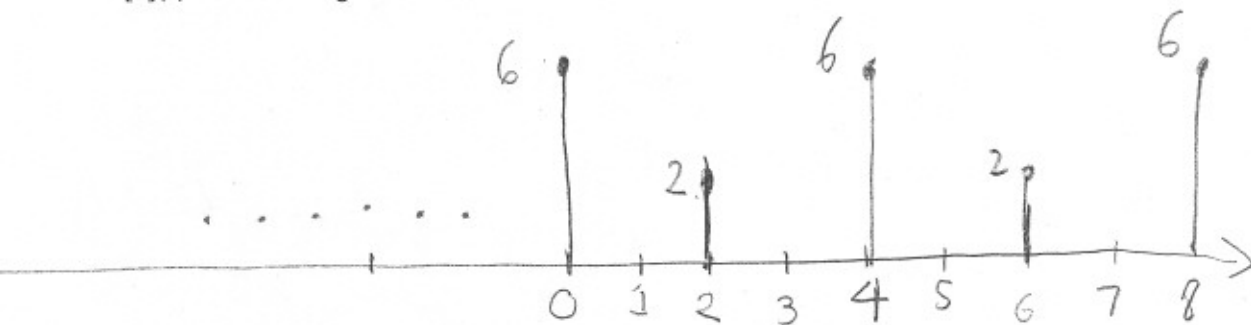
$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-i\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{i\omega n} d\omega$$

1. [20 points total] Consider the discrete-time signal  $x$ , given by

$$x(n) = 2 \sum_{m=-\infty}^{\infty} \delta(n-2m) + 4 \sum_{m=-\infty}^{\infty} \delta(n-4m), \quad \text{for all integers } n$$

[5](a) Sketch the signal.



[8] (b) Is  $x$  periodic? If so, determine its period  $p$  and the Fourier coefficients in its (complex) Fourier series representation.

yes.

$$p=4.$$

$$X_0 = 2, \quad X_1 = \frac{1}{4}(6 + 2e^{-i\pi}) = 1.$$

$$X_2 = \frac{1}{4}(6 + 2e^{-i2\pi}) = 2.$$

$$X_3 = \frac{1}{4}(6 + 2e^{-i3\pi}) = 1.$$

[7] (c) Determine  $X$ , the DTFT of  $x$ . Is  $X$  periodic? If so, what is the period?

$$X(\omega) = 2\pi \left( 2\delta(\omega) + \delta\left(\omega - \frac{\pi}{2}\right) + 2\delta(\omega - \pi) + \delta\left(\omega - \frac{3\pi}{2}\right) \right) \quad \text{for } \omega \in [0, 2\pi).$$

and is periodic with

period  $2\pi$ .

Fundamental frequency  $\omega_0 = \frac{2\pi}{p} = \frac{\pi}{2}$

2. [15 points total] Consider a system where the discrete-time input  $x$  is related to the output  $y$  by:

$$y(n) = (-1)^n x(n) \quad \text{for all integers } n.$$

- [3] (a) Is the system linear?

Yes.

- [3] (b) Is the system time-invariant?

No.

- [9] (c) Determine  $Y$ , the DTFT of the signal  $y$  in terms of  $X$ , the DTFT of the signal  $x$ .

$$\begin{aligned} Y(\omega) &= \sum_{n=-\infty}^{\infty} y(n) e^{-i\omega n} \\ &= \sum_{n=-\infty}^{\infty} (-1)^n x(n) e^{-i\omega n} \\ &= \sum_{n=-\infty}^{\infty} x(n) \left(-e^{-i\omega}\right)^n \\ &= \sum_{n=-\infty}^{\infty} x(n) e^{-i(\omega + \pi)n} \\ &= X(\omega + \pi) \end{aligned}$$