EECS 20N: Structure and Interpretation of Signals and Systems QUIZ 1
Department of Electrical Engineering and Computer Sciences Solutions
UNIVERSITY OF CALIFORNIA BERKELEY 2 February 2006

LAST Name SET FIRST Name Power

Lab Time 365/24/7

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes—up to the end of today's lecture hour—to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- **The quiz printout consists of pages numbered 1 through 4.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the four numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	5
1	16	16
2	14	14
3	10	10
Total	**45**	45

Q1.1 (16 Points) Consider the following sets:

$$\mathsf{A} = \{a\}, \qquad \mathsf{B} = \{a, b\} \qquad \mathsf{C} = \{a, b, c\} \qquad \mathsf{T} = \{1, 2\}\,.$$

(a) Let $[\mathsf{B} \to \mathsf{T}]$ denote the set of all functions from B to T.

(i) List all the elements of $[\mathsf{B} \to \mathsf{T}]$.

There are $|\mathsf{T}|^{|\mathsf{B}|} = 2^2 = 4$ elements in $[\mathsf{B} \to \mathsf{T}]$, each of which is a function whose graph is an element of the following set:

$$\{\underbrace{\{(a,1),(b,1)\}}_{\mathsf{graph}(f_1)}, \underbrace{\{(a,1),(b,2)\}}_{\mathsf{graph}(f_2)}, \underbrace{\{(a,2),(b,1)\}}_{\mathsf{graph}(f_3)}, \underbrace{\{(a,2),(b,2)\}}_{\mathsf{graph}(f_4)}\}\,.$$

For example, one of the functions f_i, $i = 1, \ldots, 4$ is defined as follows: $f_1 : \mathsf{B} \to \mathsf{T}, f_1(a) = 1, f_1(b) = 1$. In effect, $[\mathsf{B} \to \mathsf{T}] = \{f_1, f_2, f_3, f_4\}$.

(ii) How many elements are in $\mathsf{P}([\mathsf{B} \to \mathsf{T}])$, the power set of $[\mathsf{B} \to \mathsf{T}]$? Specify one element of $\mathsf{P}([\mathsf{B} \to \mathsf{T}])$ other than the empty set ϕ.
There are $2^{|[\mathsf{B}\to\mathsf{T}]|} = 2^4$ elements in $\mathsf{P}([\mathsf{B} \to \mathsf{T}])$, namely,

$$\begin{aligned} \mathsf{P}([\mathsf{B} \to \mathsf{T}]) \;=\; & \{\phi, \{f_1\}, \{f_2\}, \{f_3\}, \{f_4\}, \{f_1, f_2\}, \{f_1, f_3\}, \\ & \{f_1, f_4\}, \{f_2, f_3\}, \{f_2, f_4\}, \{f_3, f_4\}, \{f_1, f_2, f_3\}, \\ & \{f_1, f_2, f_4\}, \{f_1, f_3, f_4\}, \{f_2, f_3, f_4\}, \{f_1, f_2, f_3, f_4\}\}\,. \end{aligned}$$

(b) Determine the truth or falsehood of each of the following assertions.
If you find an assertion to be false, provide a counterexample by finding an element in the purported subset which is not a member of the purported superset. If you find an assertion to be true, prove that every element in the purported subset is a member of the purported superset.

(i) $[\mathsf{A} \to \mathsf{C}] \subset [\mathsf{B} \to \mathsf{C}]$. FALSE! There are three functions f_1, f_2, and f_3 in $[\mathsf{A} \to \mathsf{C}]$ each having a graph consisting of a single ordered pair, i.e., $\mathsf{graph}(f_1) = \{(a,a)\}$, $\mathsf{graph}(f_2) = \{(a,b)\}$, and $\mathsf{graph}(f_3) = \{(a,c)\}$. In contrast, each of the nine functions $g_i, i = 1, \ldots, 9$ in $[\mathsf{B} \to \mathsf{C}]$ has a graph consisting of two ordered pairs, e.g., $\mathsf{graph}(g_1) = \{(a,a),(b,a)\}$. Therefore, *none* of the functions f_i in $[\mathsf{A} \to \mathsf{C}]$ appears as a function in $[\mathsf{B} \to \mathsf{C}]$.

(ii) $[\mathsf{A} \to \mathsf{B}] \subset [\mathsf{A} \to \mathsf{C}]$. TRUE! The set $[\mathsf{A} \to \mathsf{B}] = \{f_1, f_2, f_3\}$, where $\mathsf{graph}(f_1) = \{(a,a)\}$ and $\mathsf{graph}(f_2) = \{(a,b)\}$. The set $[\mathsf{A} \to \mathsf{C}] = \{f_1, f_2, f_3\}$, where f_1 and f_2 are as defined above, and f_3 is characterized by $\mathsf{graph}(f_3) = \{(a,c)\}$. Clearly, f_1 and f_2 are shared by the two sets, whereas $f_3 \in [\mathsf{A} \to \mathsf{C}]$ but $f_3 \notin [\mathsf{A} \to \mathsf{B}]$.

Q1.2 (14 points) (a) A function $H : \mathbb{R} \to \mathbb{C}$ is characterized as follows:

$$H(\omega) = \begin{cases} +i & \omega \leq 0 \\ -i & \omega > 0\,. \end{cases}$$

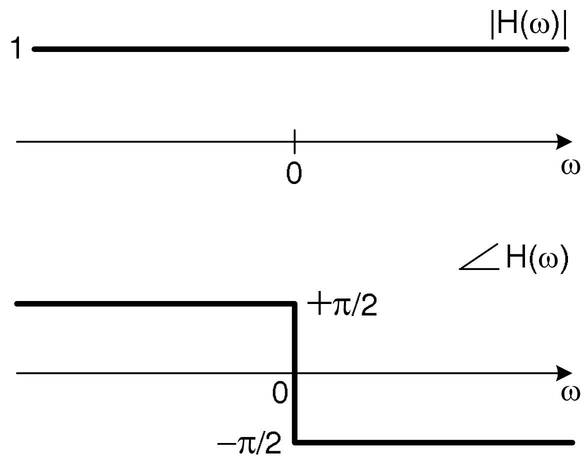
Determine and sketch the magnitude $|H(\omega)|$ and phase $\angle H(\omega)$ of the function H.

The magnitude is given as follows:

$$\begin{aligned} |H(\omega)| &= \begin{cases} |+i| = 1 & \omega \leq 0 \\ |-i| = 1 & \omega > 0\,. \end{cases} \\ &= 1, \quad \forall \omega \in \mathbb{R}\,. \end{aligned}$$

The phase is

$$\angle H(\omega) = \begin{cases} \angle + i = +\frac{\pi}{2} & \omega \leq 0 \\ \angle - i = -\frac{\pi}{2} & \omega > 0\,. \end{cases} = \begin{cases} +\frac{\pi}{2} & \omega \leq 0 \\ -\frac{\pi}{2} & \omega > 0\,. \end{cases}$$



(b) Numerically evaluate $\sin(i \ln i)$, where $\ln : \mathbb{C} - \{0\} \to \mathbb{C}$ denotes the natural logarithm function.

We simplify the argument of the $\sin$ function first. Noting that $i \ln i = \ln(i^i)$, we can evaluate i^i by noting that $i = e^{i\pi/2}$. Therefore, $i^i = e^{-\pi/2}$. We now find $i \ln i = \ln(i^i) = \ln(e^{-\pi/2}) = -\frac{\pi}{2} \underbrace{\ln(e)}_{=1} = -\frac{\pi}{2}$.

Therefore, $\sin(i \ln i) = \sin(-\pi/2) = -1$.

Q1.3 (10 points) Let the function $f : \mathbb{N}_0 \to \mathsf{V}$ be defined as follows:

$$\forall n \in \mathbb{N}_0\,, \quad f(n) = 2n\,,$$

where $\mathbb{N}_0 = \{0, 1, 2, 3, \ldots\}$ is the set of nonnegative integers, and V is a subset of the real numbers $\mathbb{R}$.

(a) Is the function f one-to-one? Explain your reasoning succinctly, but clearly and convincingly.

YES! To show this, we must prove that if $f(n_1) = f(n_2)$, then $n_1 = n_2$, where $n_1, n_2 \in \mathbb{N}_0$. Indeed, if $2n_1 = 2n_2$, then $n_1 = n_2$. Alternatively, we can show that if $n_1 \neq n_2$, then $f(n_1) = 2n_1 \neq 2n_2 = f(n_2)$. In other words, f maps no two distinct elements in $\mathbb{N}_0$ to the same element in V.

(b) If you are told that f is onto, determine the set V.

We systematically determine what each element in $\mathbb{N}_0$ maps to, and we collect all those values to define the set V. This leads to the set of nonnegative even integers:

$$\mathsf{V} = \{0, 2, 4, 6, \ldots\}\,.$$

You may use the blank space below for scratch work. Nothing written beyond this line will be considered in grading.
