EECS 20N: Structure and Interpretation of Signals and Systems QUIZ 1
Department of Electrical Engineering and Computer Sciences 8 February 2007
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name Philter FIRST Name Notch
Lab Time Sunday, Noon–3pm

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes, up to a maximum of 30 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- We will provide you with scratch paper. Do not use your own.
- **The quiz printout consists of pages numbered 1 through 4.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the four numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	5
1	20	20
2	10	10
3	10	10
Total	**45**	45

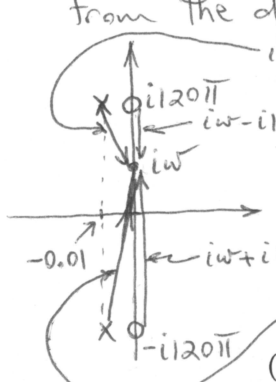
For a more detailed analysis (beyond what this problem asked for in the Quiz), see p. 4.

Q1.1 (20 Points) The frequency response function $F : \mathbb{R} \to \mathbb{C}$ of an analog filter is characterized as follows:

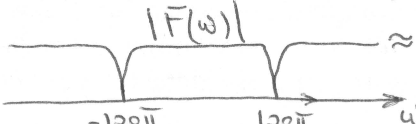
$$\forall \omega \in \mathbb{R}, \quad F(\omega) = \frac{(i\omega - i\,120\pi)(i\omega + i\,120\pi)}{[i\omega - (-0.01 + i\,120\pi)][i\omega - (-0.01 - i\,120\pi)]}.$$

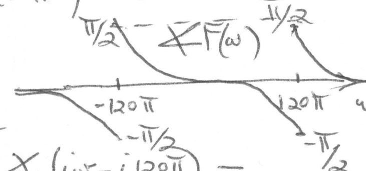
(a) (15 Points) Determine an expression for—and sketch and label the salient features of the graphs of—the magnitude $|F(\omega)|$ and angle $\angle F(\omega)$ of function F, where $F(\omega) = |F(\omega)|\, e^{i\angle F(\omega)}$ is the polar representation of F.

Note that for ω sufficiently distant from -120π, $i\omega + i120\pi \approx i\omega - (-0.01 - i120\pi)$. Similarly, for ω sufficiently distant from $+120\pi$, $i\omega - i120\pi \approx i\omega - (-0.01 + i120\pi)$. For ω sufficiently distant from both $+i120\pi$ and $-i120\pi$, $F(\omega) \approx 1 \implies |F(\omega)| \approx 1$, $\angle F(\omega) \approx 0$. You can see this from the diagram below:



Also note that $F(120\pi) = F(-120\pi) = 0$. Hence,





For $\angle F(\omega)$, note that as $\omega \nearrow 120\pi$ from below, $\angle F(\omega) \approx \angle(i\omega - i120\pi) - \angle(i\omega - (-0.01 + i120\pi))$, $\angle F(\omega) \to -\pi/2$ and as $\omega \searrow 120\pi$ from above, $\angle F(\omega) \to \pi/2$. Similarly for the vicinity of $-i120\pi$.

(b) (5 Points) It is known that if a complex exponential input signal $x : \mathbb{R} \to \mathbb{C}$ of frequency $\omega_0 \in \mathbb{R}$ (in radians/second) is applied to the linear filter, i.e.,

$$\forall t \in \mathbb{R}, \quad x(t) = e^{i\omega_0 t},$$

then the corresponding output $y : \mathbb{R} \to \mathbb{C}$ of the filter is

$$\forall t \in \mathbb{R}, \quad y(t) = F(\omega_0)\, e^{i\omega_0 t}.$$

Determine the filter's output in response to the signal $\cos(2\pi \cdot 60t)$.

$\cos 120\pi t = \frac{1}{2} e^{i120\pi t} + \frac{1}{2} e^{-i120\pi t}$. We are told that the system is linear $\implies$ The response to the cosine is

$$y(t) = \frac{1}{2} \underbrace{F(120\pi)}_{=0} e^{i120\pi t} + \frac{1}{2} \underbrace{F(-120\pi)}_{=0} e^{-i120\pi t} \implies y(t) = 0$$

Q1.2 (10 Points) The frequency response function $G : \mathbb{R} \to \mathbb{C}$ of a discrete-time filter is characterized as follows:

$$\forall \omega \in \mathbb{R}, \quad G(\omega) = \frac{\alpha + e^{-i\omega}}{1 + \alpha\, e^{-i\omega}},$$

where $-1 < \alpha < +1$ (note $\alpha \in \mathbb{R}$).

Determine a simple expression for the magnitude response $|G(\omega)|$, $-\pi < \omega \leq +\pi$.

$$G(\omega) = \frac{\alpha + e^{-i\omega}}{1+\alpha e^{-i\omega}} = e^{-i\omega} \frac{1+\alpha e^{i\omega}}{1+\alpha e^{-i\omega}}$$

Note that $1+\alpha e^{i\omega} = (1+\alpha e^{-i\omega})^* \Rightarrow |1+\alpha e^{i\omega}| = |1+\alpha e^{-i\omega}|$

$$|G(\omega)| = \left| e^{-i\omega} \frac{1+\alpha e^{i\omega}}{1+\alpha e^{-i\omega}} \right| = |e^{-i\omega}| \left| \frac{1+\alpha e^{i\omega}}{1+\alpha e^{-i\omega}} \right| = \underbrace{\cancel{|e^{-i\omega}|}}_{1} \underbrace{\cancel{\frac{|1+\alpha e^{i\omega}|}{|1+\alpha e^{-i\omega}|}}}_{1} = 1$$

$$\underline{|G(\omega)| = 1}$$

Q1.3 (10 Points) Let A and B denote two sets having *finite* cardinalities $|A|$ and $|B|$, respectively. Their intersection $A \cap B$ has cardinality $|A \cap B|$.

Which of the following three expressions correctly depicts $|P(A \cup B)|$, the cardinality of the power set of the union $A \cup B$? Circle your choice of correct expression. Be sure to explain your decision succinctly, but clearly and convincingly.

$$\boxed{\text{(I)} \quad |P(A \cup B)| = \frac{2^{|A|}\, 2^{|B|}}{2^{|A \cap B|}}.}$$

(II) $|P(A \cup B)| = \dfrac{2^{|A \cap B|}}{2^{|A|}\, 2^{|B|}}$.

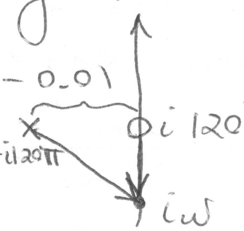
(III) $|P(A \cup B)| = 2^{|A|}\, 2^{|B|}\, 2^{|A \cap B|}$.

$$|P(A \cup B)| = 2^{|A \cup B|} = 2^{|A| + |B| - |A \cap B|} = \frac{2^{|A|}\, 2^{|B|}}{2^{|A \cap B|}}$$

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

Q.1.1: What happens in the vicinity of $\omega = 120\pi$?

$$F(\omega) \approx \frac{i\omega - i120\pi}{i\omega - (-0.01 + i120\pi)}$$

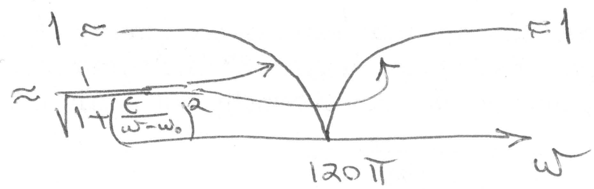


Let's use symbols to declutter the expressions: $\omega_0 = 120\pi$, $\epsilon = 0.01$

$$F(\omega) \approx \frac{i(\omega-\omega_0)}{i(\omega-\omega_0)+\epsilon} \implies |F(\omega)|^2 \approx \frac{(\omega-\omega_0)^2}{\epsilon^2+(\omega-\omega_0)^2} = \frac{1}{1+\left(\frac{\epsilon}{\omega-\omega_0}\right)^2}$$

$$\implies |F(\omega)| \approx \frac{1}{\sqrt{1+\left(\frac{\epsilon}{\omega-\omega_0}\right)^2}}$$

Where $\omega \approx \omega_0 \pm \epsilon$, we have $|F(\omega)| \approx \frac{1}{\sqrt{2}}$



A similar analysis applies to the vicinity of $\omega = -120\pi$.

$$F(\omega) \approx \frac{i\omega + i120\pi}{i\omega - (-0.01 - i120\pi)} = \frac{i(\omega+\omega_0)}{i(\omega+\omega_0)+\epsilon} \implies$$

$$|F(\omega)| \approx \frac{1}{\sqrt{1+\left(\frac{\epsilon}{\omega+\omega_0}\right)^2}}$$

ω in a small neighborhood of -120π.

Phase: (In the vicinity of 120π)

$$F(\omega) \approx \frac{i(\omega-\omega_0)}{\epsilon + i(\omega-\omega_0)} = \frac{1}{1+\frac{\epsilon}{i(\omega-\omega_0)}} = \frac{1}{1-i\frac{\epsilon}{\omega-\omega_0}} \implies \measuredangle F(\omega) \approx -\tan^{-1}\left(\frac{-\epsilon}{\omega-\omega_0}\right)$$

$$\implies \measuredangle F(\omega) \approx \tan^{-1}\left(\frac{\epsilon}{\omega-\omega_0}\right)$$

Similarly, for $\omega \approx -120\pi \implies \measuredangle F(\omega) \approx \tan^{-1}\left(\frac{\epsilon}{\omega+\omega_0}\right)$