EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 2
Department of Electrical Engineering and Computer Sciences 20 March 2007
UNIVERSITY OF CALIFORNIA BERKELEY

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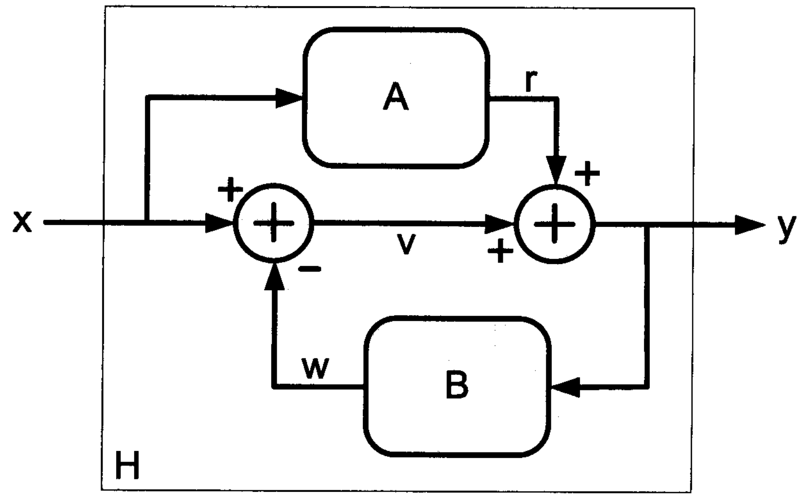
Lab Time Secret

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except two double-sided 8.5" × 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT2.1 (15 Points) Consider a well-structured interconnection H of discrete-time LTI systems A and B, as shown in the figure below. Each of the individual systems is a function defined on $[\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$.



Let the respective frequency responses be A and B, defined on $\mathbb{R} \to \mathbb{C}$.

Determine the composite system's frequency response $H : \mathbb{R} \to \mathbb{C}$ in terms of the frequency responses A and B of the individual components. Reduce your expression to the simplest form possible.

Note: The intermediate signals r, v, and w have been labeled on the diagram for your convenience. It is not necessary that you make use of them in your work.

$y = r + v \qquad r = A(x) \qquad v = x - w \quad , \quad w = B(y)$

Put them all together:

$y = A(x) + x - B(y) \implies y + B(y) = x + A(x)$

If $x(n) = e^{i\omega n}$, then $y(n) = H(\omega)e^{i\omega n}$, $r(n) = \underbrace{A(\omega)e^{i\omega n}}_{(A(x))(n)}$,

and $w(n) = \underbrace{B(\omega)H(\omega)e^{i\omega n}}_{(B(y))(n)}$

So, we have:

$H(\omega)e^{i\omega n} + B(\omega)H(\omega)e^{i\omega n} = e^{i\omega n} + A(\omega)e^{i\omega n} \implies H(\omega) = \frac{1+A(\omega)}{1+B(\omega)}$

MT2.2 (25 Points) The following discrete-time systems F, G, and H should be treated mutually independently; properties that hold for one system cannot be *assumed* to hold for the others.

For each part, explain your reasoning succinctly, but clearly and convincingly.

(a) (10 Points) A discrete-time system $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y,

$$y(n) = \cos\left(\frac{\pi}{4}n\right), \quad \forall n,$$

in response to the input signal x,

$$x(n) = e^{i\pi n/4}, \quad \forall n.$$

Select the strongest true assertion from the list below.

(i) The system must be LTI.

(ii) The system could be LTI, but does not have to be.

$\boxed{\text{(iii) The system cannot be LTI.}}$

The input contains only the frequency $\omega_0 = \frac{\pi}{4}$.
The output contains frequencies $\omega_0 = \frac{\pi}{4}$ and $-\omega_0 = -\frac{\pi}{4}$,
the latter not present in the input. LTI systems can't create new frequencies.

If your choice is (i) or (ii), please answer the following:

(I) Provide as much information about the frequency response of the (or an) LTI system consistent with the input-output pair of signals x and y. In particular, specify all inferrable values of the frequency response $F(\omega)$, $\omega \in \mathbb{R}$.

N/A

(II) Could the impulse response f of the system be real-valued? Explain your reasoning succinctly, but clearly and convincingly.

N/A

(b) (6 Points) A discrete-time system $G : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y,

$$y(n) = e^{i\pi n/4}, \quad \forall n,$$

in response to the input signal x,

$$x(n) = \cos\left(\frac{\pi}{4}n\right), \quad \forall n.$$

Select the strongest true assertion from the list below.

(i) The system must be LTI.

$\boxed{\text{(ii) The system could be LTI, but does not have to be.}}$

(iii) The system cannot be LTI.

The system $x \xrightarrow{X} \boxed{G} \xrightarrow{Y} y$ defined by $y(n) = e^{i\pi n/4}$, $\forall x \in X$ is neither linear nor time invariant, but is consistent w/ the input-output pair above.

If the system is LTI, then we know $G(\frac{\pi}{4}) = 2$, $G(-\frac{\pi}{4}) = 0$. $g(n)$ cannot be a real-valued impulse response. Why?

(c) (9 Points) A discrete-time system $H : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y,

$$y(n) = \cos\left(\frac{\pi}{4}n\right), \quad \forall n,$$

in response to the input signal x,

$$x(n) = \sin\left(\frac{\pi}{4}n\right), \quad \forall n.$$

Select the strongest true assertion from the list below.

(i) The system must be *memoryless*.

(ii) The system could be *memoryless*, but does not have to be.

$\boxed{\text{(iii) The system cannot be } \textit{memoryless}.}$

$x(0) = x(4) = 0$, but $y(0) = 1 \neq -1 = y(4)$

MT2.3 (35 Points) Consider a discrete-time LTI system $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ having input signal x and output signal y, as shown below:

$$x \longrightarrow \boxed{\mathsf{F}} \longrightarrow y$$

$\boxed{F(\omega) = 1 - \alpha e^{-i\omega}}$

If the input signal is the one-sided decaying exponential

$$x(n) = \alpha^n u(n), \quad \forall n,$$

where $0 < |\alpha| < 1$, the output signal is simply the Kronecker delta function, i.e.,

$$y(n) = \delta(n), \quad \forall n.$$

(a) (10 Points) Determine a simple expression for the frequency response values $F(\omega)$, $-\pi \leq \omega \leq +\pi$.

<u>Hint</u>: You may find the following helpful. If $|\beta| < 1$, then $\sum_{n=0}^{\infty} \beta^n = \frac{1}{1-\beta}$.

x can be thought of as being produced by an LTI system having impulse response x:

$$\delta \longrightarrow \boxed{X} \longrightarrow x$$

If system X is placed in cascade with F, we have $\delta \to \boxed{X} \xrightarrow{x} \boxed{F} \to \delta$

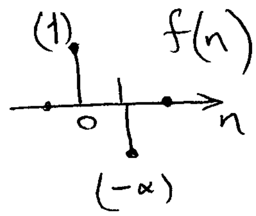
It must be, then, that $F(\omega) = \frac{1}{X(\omega)}$, where $X(\omega) = \frac{1}{1-\alpha e^{-i\omega}} \triangleq \sum_{n=-\infty}^{\infty} x(n) e^{-i\omega n}$

(b) (10 Points) Determine a simple expression for $f(n)$, $\forall n$, where f is the impulse response of the system F.

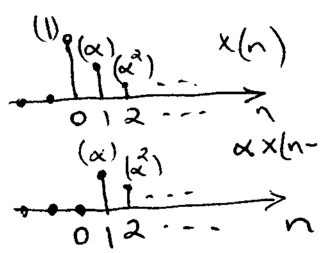
Note that it is possible to determine the impulse response f without knowing the frequency response F.

<u>Method 1</u>: Use $F(\omega) = 1 - \alpha e^{-i\omega}$

$$f(n) = \delta(n) - \alpha\,\delta(n-1)$$



<u>Method 2</u>:



$$\Longrightarrow \delta(n) = x(n) - \alpha\, x(n-1)$$

$$\Downarrow$$

$$f(n) = y(n) - \alpha\, y(n-1)$$

$$\boxed{f(n) = \delta(n) - \alpha\,\delta(n-1)}$$

→ Question: What should the impulse response of a memoryless LTI system look like?

(c) (7 Points) Select the strongest true assertion from the list below.

(i) The system must be *memoryless*.

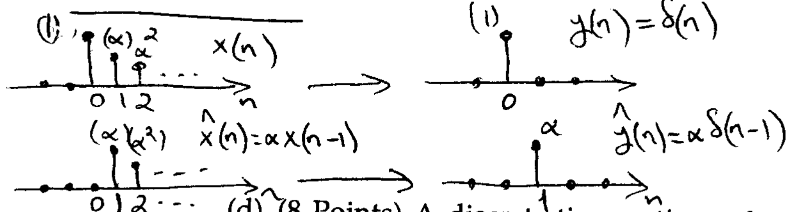
(ii) The system could be *memoryless*, but does not have to be.

$\boxed{\text{(iii) The system cannot be } \textit{memoryless}.}$

Method 1:
$$\left.\begin{array}{l} f(n) = \delta(n) - \alpha\,\delta(n-1) \\ y(n) = (x * f)(n) \end{array}\right\} \Rightarrow y(n) = x(n) - \underbrace{\alpha\, x(n-1)}_{\text{violates memorylessness}}$$

(Convolution w/ a shifted impulse shifts the signal to the location of the impulse)

Method 2:



Note $x(1) = \hat{x}(1)$

but $y(1) \neq \hat{y}(1)$

(d) (8 Points) A discrete-time *anti-causal* system is defined as a system whose instantaneous output does *not* depend on past values of its input.

That is, a system is said to be anti-causal if, and only if, the instantaneous output $y(n)$ depends *at most* on the input values $x(m), m \geq n$.

Select the strongest true assertion from the list below.

(i) The system must be *anti-causal*.

(ii) The system could be *anti-causal*, but does not have to be.

$\boxed{\text{(iii) The system cannot be } \textit{anti-causal}.}$

Anti-causal systems are counterparts of causal systems, as the name suggests. In particular, a system is anti-causal if, and only if, its responses y and $\hat{y}$ corresponding to any pair of inputs x, $\hat{x}$ such that

$$x(m) = \hat{x}(m) \quad \forall\, m \geq N$$

satisfy the equality $y(m) = \hat{y}(m) \quad \forall\, m \geq N$.

The signals x and $\hat{x}$ from part (c) satisfy the equality

$x(m) = \hat{x}(m) \quad \forall\, m \geq 1$ but $y(1) \neq \hat{y}(1)$.

MT2.4 (30 Points) Consider a discrete-time LTI system $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ having input signal x and output signal y, as shown below:

$$x \to \boxed{\mathsf{F}} \to y$$

The frequency response $F : \mathbb{R} \to \mathbb{C}$ is given by

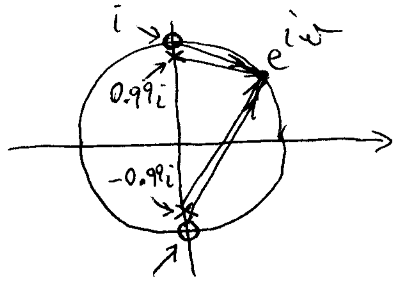
$$F(\omega) = \frac{1 + e^{-i2\omega}}{1 + (0.99)^2 e^{-i2\omega}}, \qquad -\pi \le \omega \le +\pi.$$

(a) (10 Points) Provide a well-labeled sketch of the magnitude response $|F(\omega)|$, $-\pi \le \omega \le +\pi$.

What type of filter is F: low-pass, band-pass, high-pass, all-pass, or notch?

Possibly helpful: $(0.99)^2 \approx 0.98$.

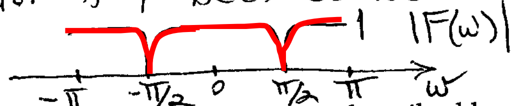
$$F(\omega) = \frac{e^{i2\omega} + 1}{e^{i2\omega} + (0.99)^2} = \frac{(e^{i\omega} - i)(e^{i\omega} + i)}{(e^{i\omega} - 0.99i)(e^{i\omega} + 0.99i)}$$



$|F(\omega)| \approx 1 \quad \forall\, \omega$ except $\pm\frac{\pi}{2}$.

At $\omega = \pm\pi/2$, the numerator of F becomes zero.

F is a notch filter:



In fact $\measuredangle F(\omega) \approx 0$ $\forall\, \omega$ except $\pm\pi/2$.
$\Downarrow$
$F(\omega) \approx 1$ except $\omega = \pm\pi/2$

(b) (10 Points) Suppose the input to the system is described by

$$x(n) = 1 + 2e^{i\pi n/4} + 3\cos\left(\frac{\pi}{2}n\right) + 4(-1)^n, \quad \forall n.$$

Determine the output values $y(n), \forall n$.

Every component passes through except $\cos(\frac{\pi}{2}n)$, which contains frequencies $\pi/2$ and $-\pi/2$.

$$y(n) \approx 1 + 2e^{i\pi n/4} + 4(-1)^n$$

(c) (10 Points) Determine the linear, constant-coefficient difference equation that governs the input-output behavior of the system.

$$\left[1 + (0.99)^2 e^{-i2\omega}\right] F(\omega) = 1 + e^{-i2\omega} \implies$$

$$y(n) + (0.99)^2\, y(n-2) = x(n) + x(n-2)$$

Causal Implementation: $y(n) = -(0.99)^2\, y(n-2) + x(n) + x(n-2)$

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Problem	Points	Your Score
Name	10	10
1	15	15
2	25	25
3	35	35
4	30	30
Total	**115**	115