EEC 20N: Structure and Interpretation of Signals and Systems — QUIZ 3
Department of Electrical Engineering and Computer Sciences — 19 April 2007
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name ~~Product~~ → Peace FIRST Name Inner

Lab Time Wee Hours of the Night

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 30 minutes to complete. You will be given at least 30 minutes, up to a maximum of 40 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- We will provide you with scratch paper. Do not use your own.
- **The quiz printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	5
1	15	15
2	15	15
3	10	10
Total	**45**	45

Basic Formulas:

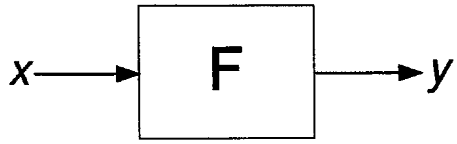
Discrete Fourier Series (DFS) Complex exponential Fourier series synthesis and analysis equations for a periodic discrete-time signal having period p:

$$x(n) = \sum_{k=\langle p\rangle} X_k\, e^{ik\omega_0 n} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p}\sum_{n=\langle p\rangle} x(n)\, e^{-ik\omega_0 n}\,,$$

where $\omega_0 = \dfrac{2\pi}{p}$ and $\langle p\rangle$ denotes a suitable discrete interval of length p (i.e., an interval containing p contiguous integers). For example, $\sum_{k=\langle p\rangle}$ may denote $\sum_{k=0}^{p-1}$ or $\sum_{k=1}^{p}$.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

Q3.1 (15 Points) Consider the discrete-time system $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ shown below:



The output signal y is related to the input signal x as follows:

$$\forall n \in \mathbb{Z}, \quad y(n) = |x(n)|^2.$$

Suppose the input signal x is periodic with fundamental period $p_x = 2$.

For some appropriate choice of fundamental frequency ω_x, the discrete Fourier series (DFS) expansion of x can be written as

$$\forall n \in \mathbb{Z}, \quad x(n) = X_0 + X_1 e^{i\omega_x n},$$

where $X_0 X_1^* = (X_0^* X_1)^* \neq 0$. $X_0^* X_1 \neq 0$ $\quad$ Assume these as instructed in class. $Re(X_0 X_1^*) \neq 0,\ Re(X_0^* X_1) \neq 0$

Select the strongest true assertion from the list below. To receive credit, you must explain your selection succinctly, but clearly and convincingly.

(I) The output signal y is periodic with fundamental period $p_1 = 3$. Accordingly, for some appropriate choice of fundamental frequency ω_1, it has the following DFS expansion:

$$\forall n \in \mathbb{Z}, \quad y(n) = Y_{-1} e^{-i\omega_1 n} + Y_0 + Y_1 e^{i\omega_1 n}.$$

If this is your selection, then determine w_1 numerically, and express the DFS expansion coefficients Y_{-1}, Y_0, and Y_1 in terms of X_0 and X_1.

(II) The output signal y is periodic with fundamental period $p_2 = 2$. Accordingly, for some appropriate choice of fundamental frequency ω_2, it has the following DFS expansion:

$$\forall n \in \mathbb{Z}, \quad y(n) = Y_0 + Y_1 e^{i\omega_2 n}.$$

If this is your selection, then determine w_2 numerically, and express the DFS expansion coefficients Y_0 and Y_1 in terms of X_0 and X_1.

(III) The output signal y cannot be periodic, because the system F is nonlinear.

Use the next page to show your work for this problem.

Show your work for Problem Q3.1 on this blank page:

$$y(n) = |x(n)|^2 = x(n)\,x^*(n) = \left(X_0 + X_1 e^{i\omega_x n}\right)\left(X_0^* + X_1^* e^{-i\omega_x n}\right)$$

$$= X_0 X_0^* + X_0 X_1^* e^{-i\omega_x n} + X_0^* X_1 e^{i\omega_x n} + X_1 X_1^*$$

$$= |X_0|^2 + |X_1|^2 + X_0 X_1^* e^{-i\omega_x n} + X_0^* X_1 e^{i\omega_x n}$$

But $P_x = 2 \Rightarrow \omega_x = \frac{2\pi}{P} = \pi \Rightarrow e^{i\omega_x n} = e^{i\pi n} = e^{-i\pi n} = e^{-i\omega_x n} = (-1)^n$

$$\Rightarrow y(n) = \underbrace{|X_0|^2 + |X_1|^2}_{Y_0} + \underbrace{2\,\mathrm{Re}\left(X_0 X_1^*\right) e^{i\pi n}}_{Y_1}$$

y is periodic with fundamental period $P_2 = 2 = P_x$

$\omega_2 = \pi$ is its fundamental frequency.

The proper choice is (II)

Note: The system is memoryless $\Rightarrow \exists f: \mathbb{C} \to \mathbb{C}$ such that $y(n) = f(x(n))\ \forall x, \forall n$

$\left.\right\}$ $f(\cdot) = |\cdot|$ in this case

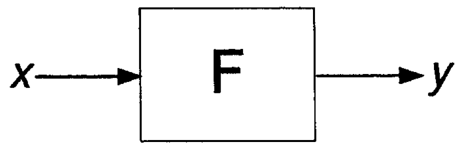
So if $x(n+P_x) = x(n)\ \forall n \Rightarrow$

$y(n+P_x) = f(x(n+P_x)) = f(x(n)) = y(n) \quad \forall n.$

The fundamental period of y cannot exceed that of x.

Nonlinearity of F is not relevant here, so choice III is not valid.

Q3.2 (15 Points) Consider the discrete-time system $\mathsf{F} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$ shown below:



The output signal y is related to the input signal x as follows:

$$\forall n \in \mathbb{Z}, \quad y(n) = \text{sgn}(x(n)) \triangleq \begin{cases} +1 & \text{if } x(n) > 0 \\ 0 & \text{if } x(n) = 0 \\ -1 & \text{if } x(n) < 0, \end{cases}$$

where sgn($\cdot$) is, according to one popular definition, the *signum function*.

Note that the system F is *memoryless*. That is, a function $f : \mathbb{R} \to \mathbb{R}$ exists such that $y(n) = f(x(n)), \forall n \in \mathbb{Z}$. The function f is the signum function in this case.

Suppose the input signal x is periodic with fundamental period p_x. For some appropriate choice of fundamental frequency ω_x, the discrete Fourier series (DFS) expansion of x is

$$\forall n \in \mathbb{Z}, \quad x(n) = \sum_{k=\langle p_x \rangle} X_k \, e^{ik\omega_x n}.$$

Select the strongest true assertion from the list below. To receive credit, you must explain your selection succinctly, but clearly and convincingly.

(I) The output signal y must be periodic with *fundamental* period $p_y = p_x$.

(II) The output signal y must be periodic with *fundamental* period $p_y \leq p_x$. If this is your selection, provide a periodic input signal x in response to which the system produces an output y with a strictly shorter period (i.e., $p_y < p_x$).

(III) The output signal y must be periodic with *fundamental* period $p_y > p_x$.

(IV) The output signal y cannot be periodic, because the system F is nonlinear.

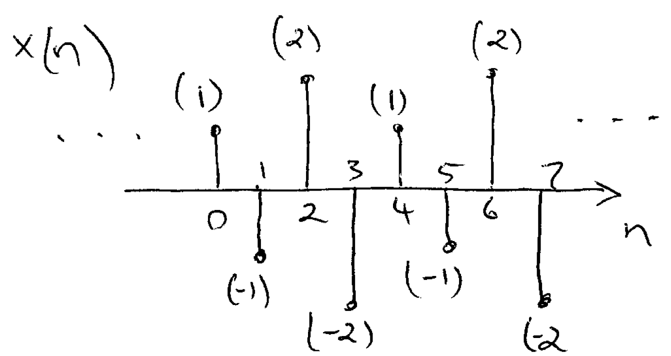
Use the next page to show your work for this problem.

Show your work for Problem Q3.2 on this blank page:

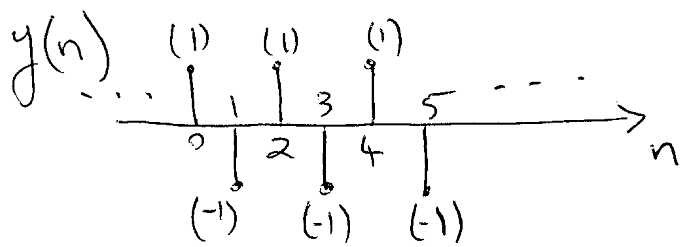
The system is memoryless $\Rightarrow \exists f: \mathbb{R} \to \mathbb{R}$ such that $y(n) = f(x(n))$. In this case, $f(\cdot) = \text{sgn}(\cdot)$.

$$y(n+P_x) = f(x(n+P_x)) = f(x(n)) = y(n).$$

The output's fundamental period cannot exceed that of the input. BUT, it can be shorter. Here's an example:



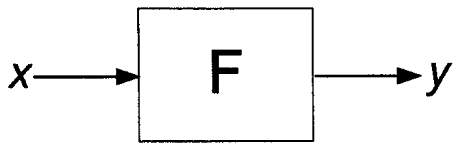
$P_x = 4$



$P_y = 2$

Note that P_y, if it is shorter than P_x, divides P_x, because y must be periodic with period P_x (not necessarily fundamental) due to the memorylessness of F.

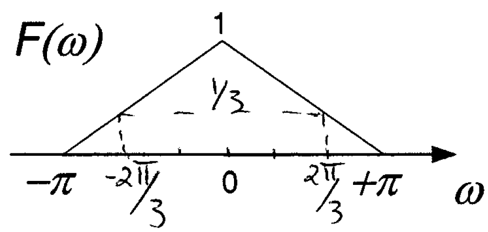
Q3.3 (10 Points) Consider the discrete-time LTI filter $\mathsf{F} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$ shown below:



The input signal x is characterized by

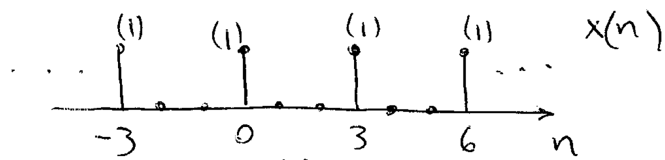
$$\forall n \in \mathbb{Z}, \quad x(n) = \sum_{l=-\infty}^{+\infty} \delta(n - 3l).$$

The filter's frequency response, over the interval $-\pi \le \omega \le +\pi$, is shown below:



Determine a reasonably-simple expression for $y(n), \forall n \in \mathbb{Z}$, that completely specifies the output y corresponding to the input x described above.

To receive credit, you must explain your work succinctly, but clearly and convincingly.



x is an infinite-duration impulse train with fundamental period $p_x = 3$. Therefore, its fundamental frequency is $\omega_x = \frac{2\pi}{3}$

Can write $x(n) = X_{-1} e^{-i\frac{2\pi}{3}n} + X_0 + X_1 e^{i 2\pi/3\, n}$

$X_k = \frac{1}{3}\sum_{n=-1}^{1} x(n) e^{-ik\frac{2\pi}{3}n} = \frac{1}{3}$ (only one term in the sum is nonzero) $\forall k$

$x(n) = \frac{1}{3} e^{-i\frac{2\pi}{3}n} + \frac{1}{3} + \frac{1}{3} e^{i\frac{2\pi}{3}n}$ $\Longrightarrow$

$y(n) = \frac{1}{3} F\left(-\frac{2\pi}{3}\right) e^{-i\frac{2\pi}{3}n} + \frac{1}{3}F(0)^{7} + \frac{1}{3} F\left(\frac{2\pi}{3}\right) e^{i\frac{2\pi}{3}n}$

$F\left(\frac{2\pi}{3}\right) = F\left(\frac{-2\pi}{3}\right) = \frac{1}{3} \Longrightarrow y(n) = \frac{1}{3} + \frac{2}{9}\cos\left(\frac{2\pi}{3}n\right)$

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