

Probing Further: Household electrical power

In the U.S., household current is delivered on three wires, a **neutral wire** and two **hot wires**. The voltage between either hot wire and the neutral wire is somewhere around 110 to 120 volts, RMS (root mean square). The voltage between the two hot wires is around 220 to 240 volts, RMS. The higher voltage is used to drive appliances that need more power, such as washing machines and air conditioners. Here, we examine exactly how this works.

The voltage between the hot wires and the neutral wire is sinusoidal with a frequency of 60 Hz. Thus, for one of the hot wires, it is a function $x: \mathbf{Reals} \rightarrow \mathbf{Reals}$ where the domain represents time and the range represents voltage, and

$$\forall t \in \mathbf{Reals}, \quad x(t) = 170 \cos(60 \times 2\pi t).$$

This 60 Hertz sinusoidal waveform completes one cycle in a period of $P = 1/60$ seconds. Why is the amplitude 170, rather than 120? Because the 120 voltage is **RMS (root mean square)**. That is,

$$\text{voltage}_{RMS} = \sqrt{\frac{\int_0^P x^2(t) dt}{P}},$$

the square root of the average of the square of the voltage. We leave it as an exercise to verify that this evaluates to 120 volts.

The voltage between the second hot wire and the neutral wire is a function $y: \mathbf{Reals} \rightarrow \mathbf{Reals}$ where

$$\forall t \in \mathbf{Reals}, \quad y(t) = -170 \cos(60 \times 2\pi t) = -x(t).$$

It is the negative of the other voltage at any time t . This sinusoidal signal is said to have a **phase shift** of 180 degrees, or π radians, compared to the first sinusoid. Equivalently, it is said to be 180 degrees **out of phase**.

We can now see how to get the higher voltage for driving power-hungry appliances. We simply use the two hot wires rather than one hot wire and the neutral wire. The voltage between the two hot wires is the difference, a function $z: \mathbf{Reals} \rightarrow \mathbf{Reals}$ where

$$\forall t \in \mathbf{Reals}, \quad z(t) = x(t) - y(t) = 340 \cos(60 \times 2\pi t).$$

This corresponds to 240 volts RMS. A plot of the three voltage functions is shown in figure 1.5.

Note that the neutral wire should not be confused with the ground wire in a three-prong plug. The ground wire is not connected to electrical company facilities. The neutral wire is. The ground wire is typically connected to the plumbing or some metallic conductor that goes underground. It is a safety feature to allow current to flow into the earth rather than, say, through a person.

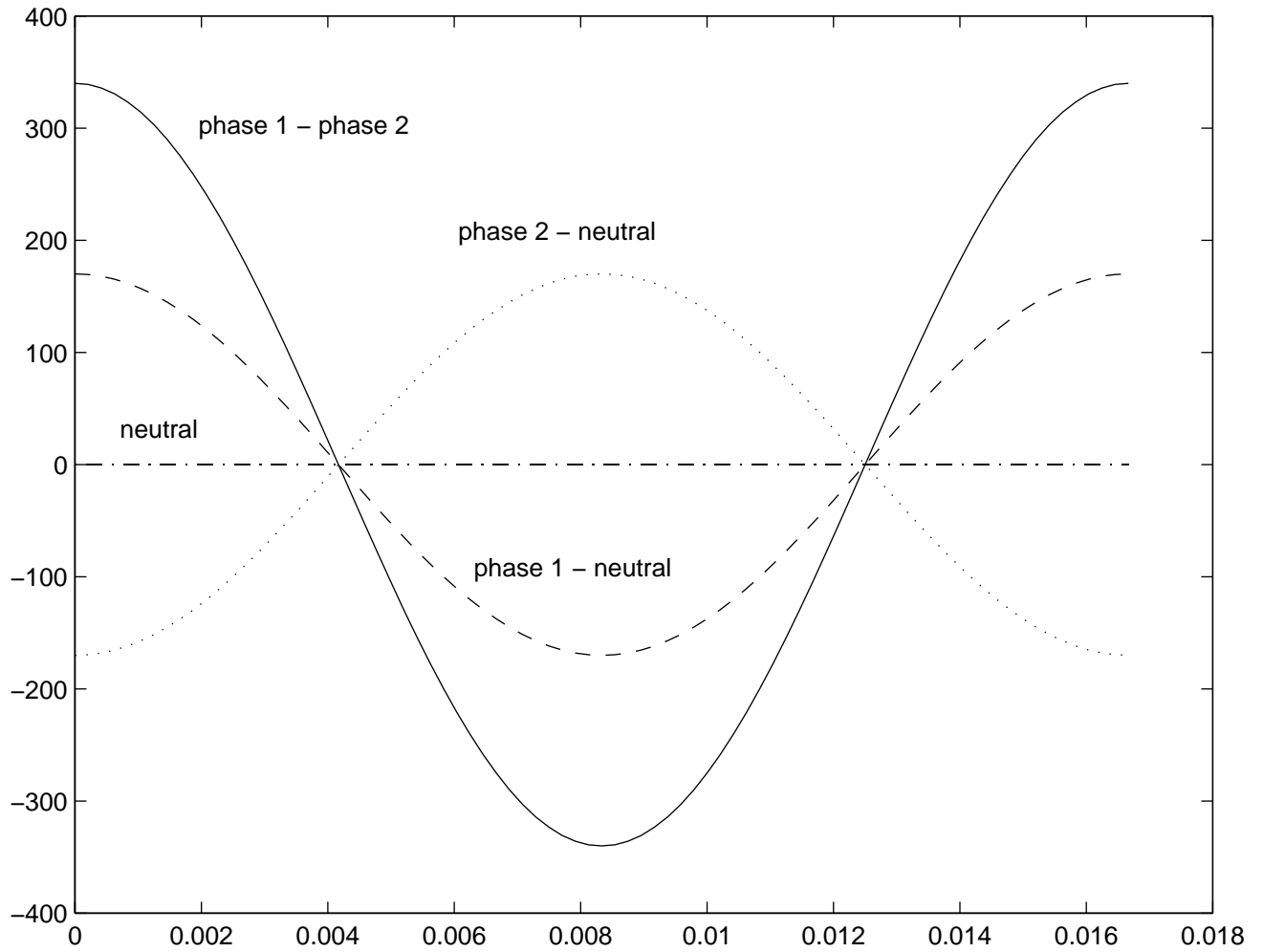


Figure 1.5: The voltages between the two hot wires and the neutral wire and between the two hot wires in household electrical power in the U.S.