Discrete-Event Systems:
Generalizing Metric Spaces and Fixed-Point Semantics

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Chess Review
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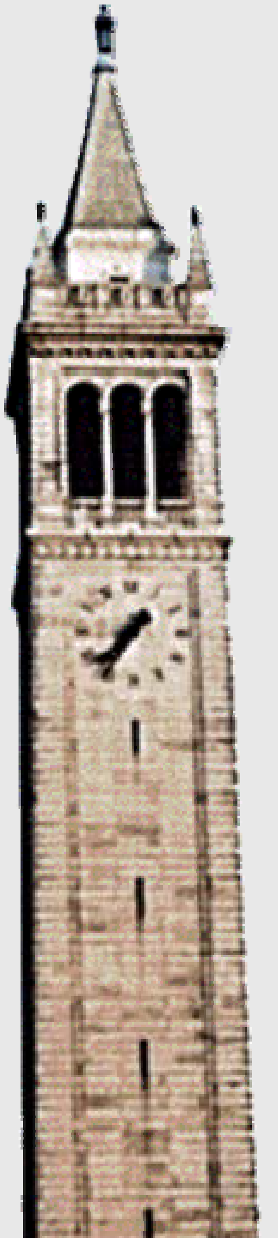
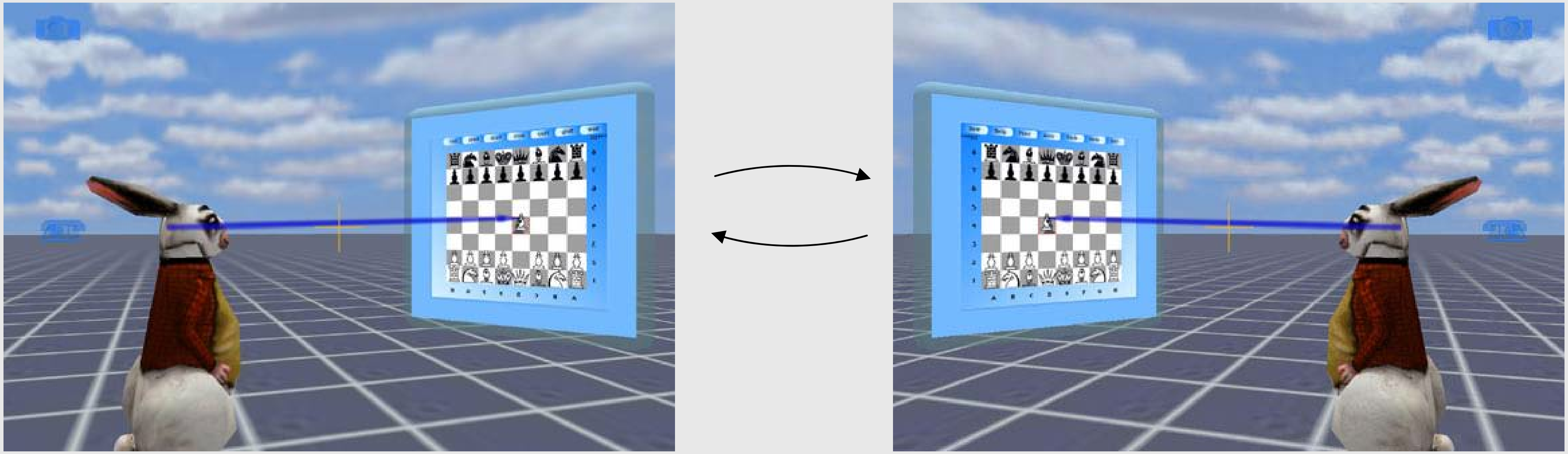


Discrete-Event (DE) Systems

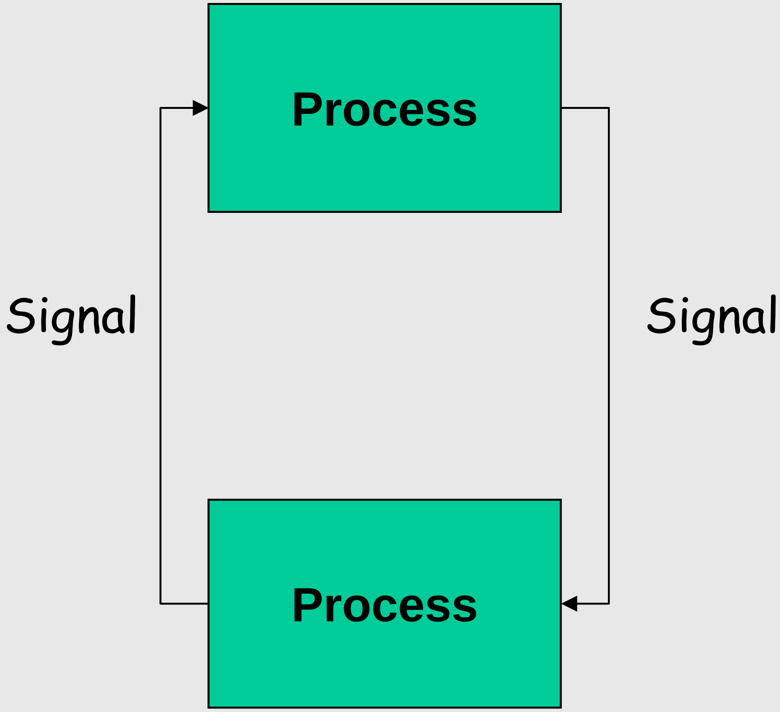
- Traditional Examples
  - VHDL
  - OPNET Modeler
  - NS-2
- Distributed systems
  - TeaTime protocol in Croquet

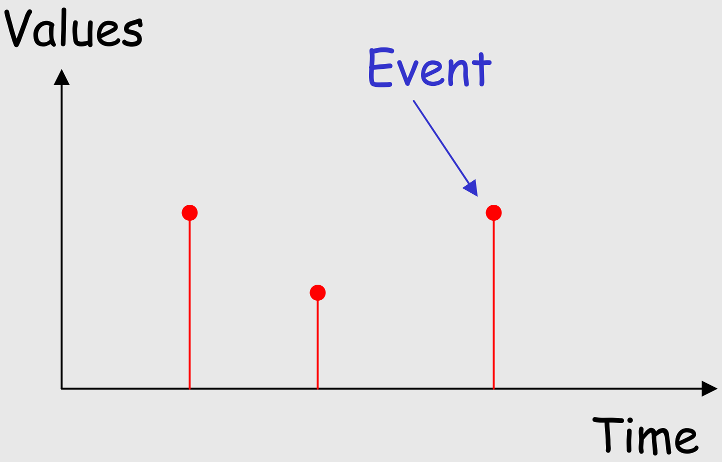


(two players vs. the computer)

Introduction to DE Systems

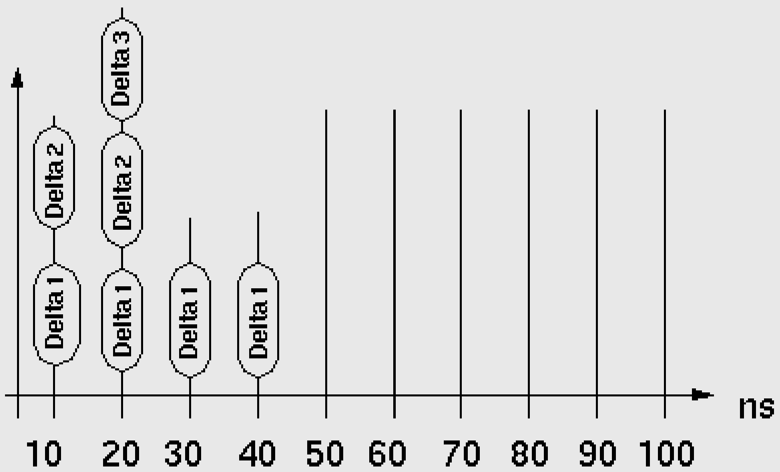
- In DE systems, concurrent objects (processes) interact via signals





What is the semantics of DE?

- Simultaneous events may occur in a model
  - VHDL Delta Time

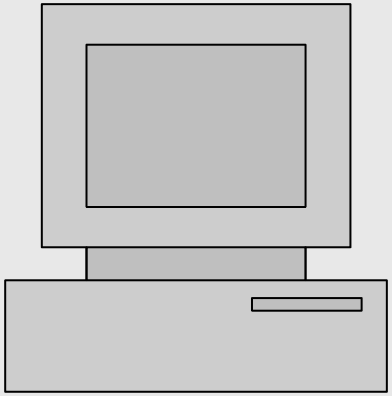


- Simultaneity absent in traditional formalisms
  - Yates
  - Chandy/Misra
  - Zeigler

Time in Software

- Traditional programming language semantics lack time

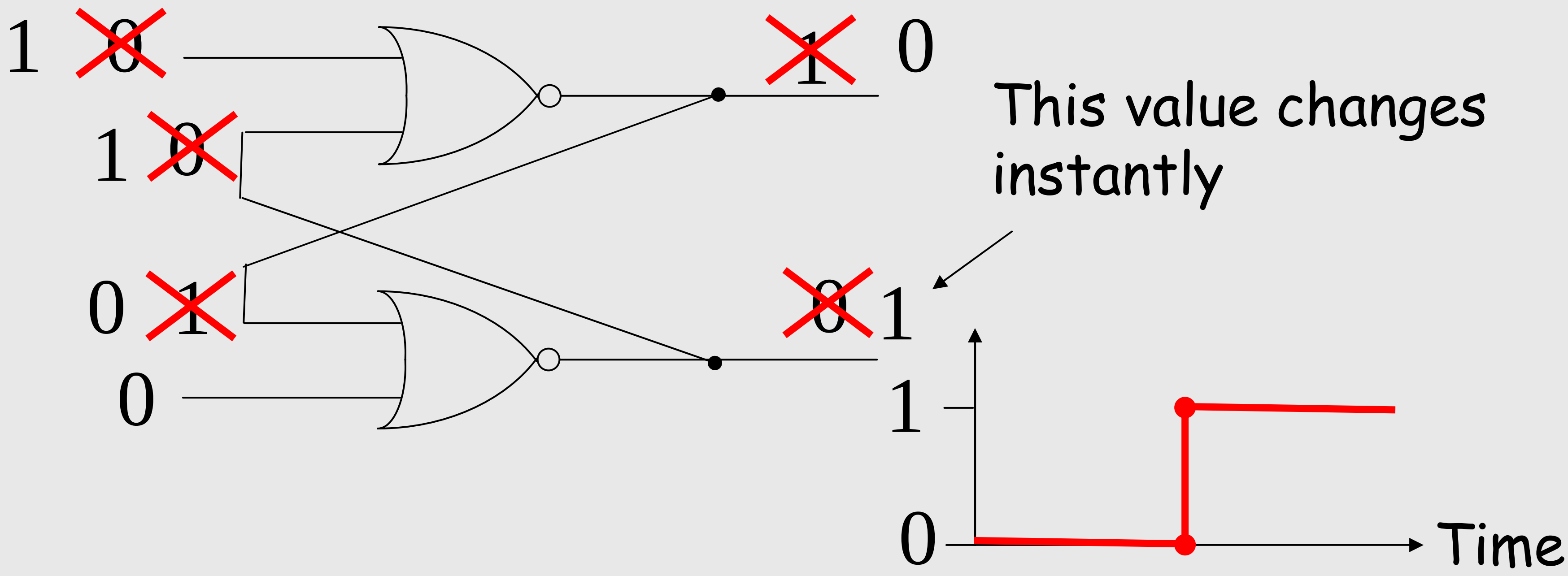
- When a physical system interacts with software, how should we model time?
- One possiblity is to assume some computations take zero time, e.g.
  - Synchronous language semantics
  - GIOTTO logical execution time



Simultaneity in Hardware

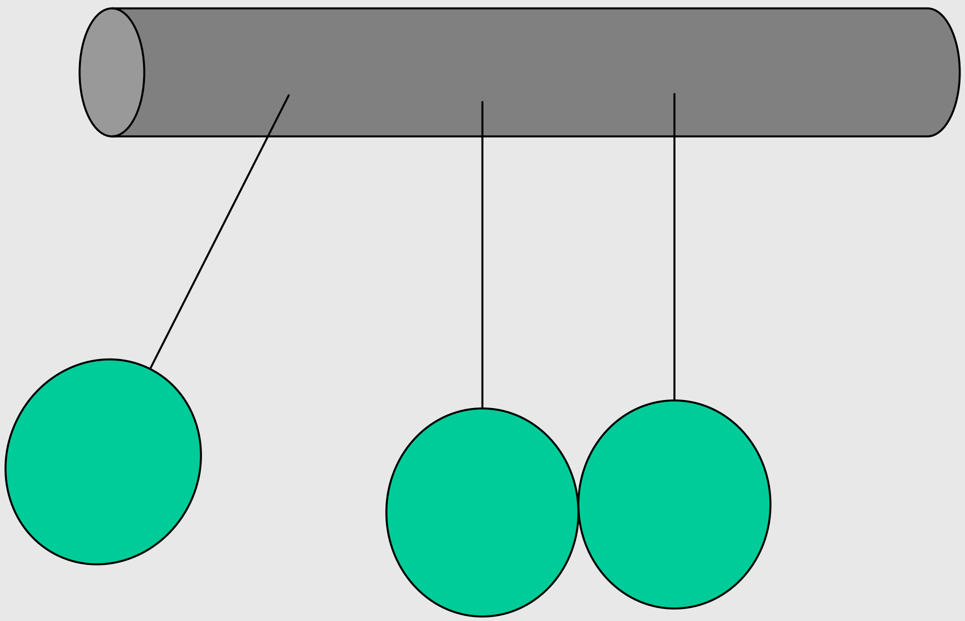
- Simultaneity is common in synchronous circuits

- Example:



Simultaneity in Physical Systems

[Biswas]

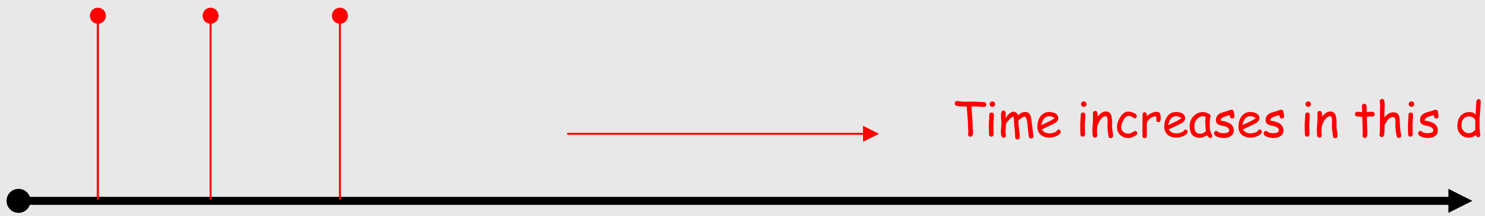


Our Contributions

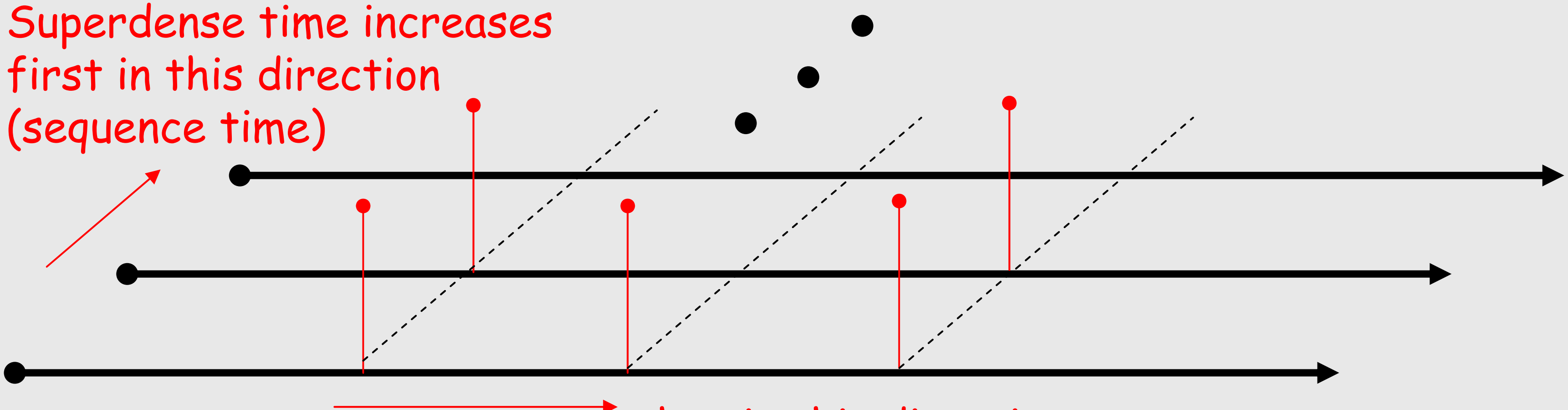
- We generalize DE semantics to handle simultaneous events
- We generalize metric space concepts to handle our model of time
- We give uniqueness conditions and conditions for avoidance of Zeno behavior

Models of Time

- Time (real time)

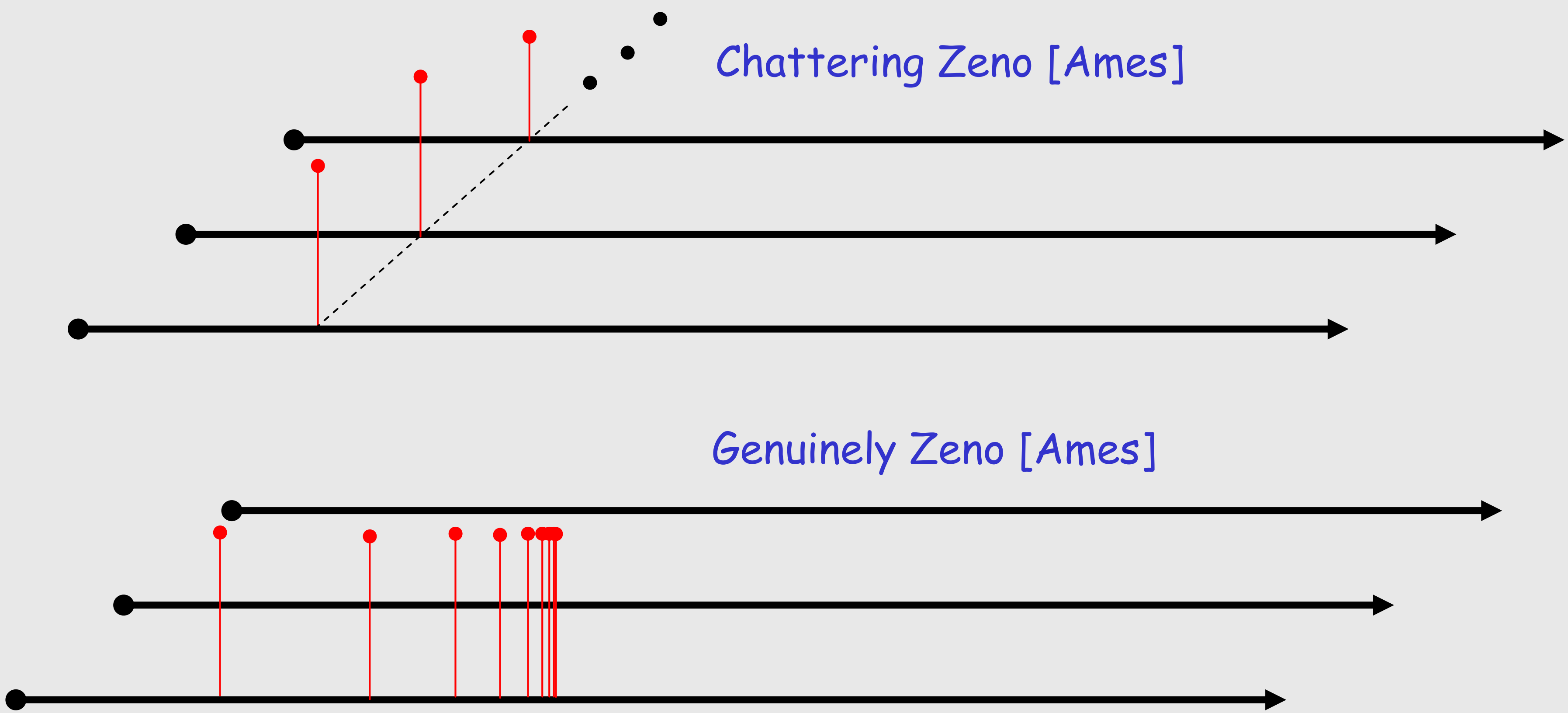


- Superdense time [Maler, Manna, Pnueli]



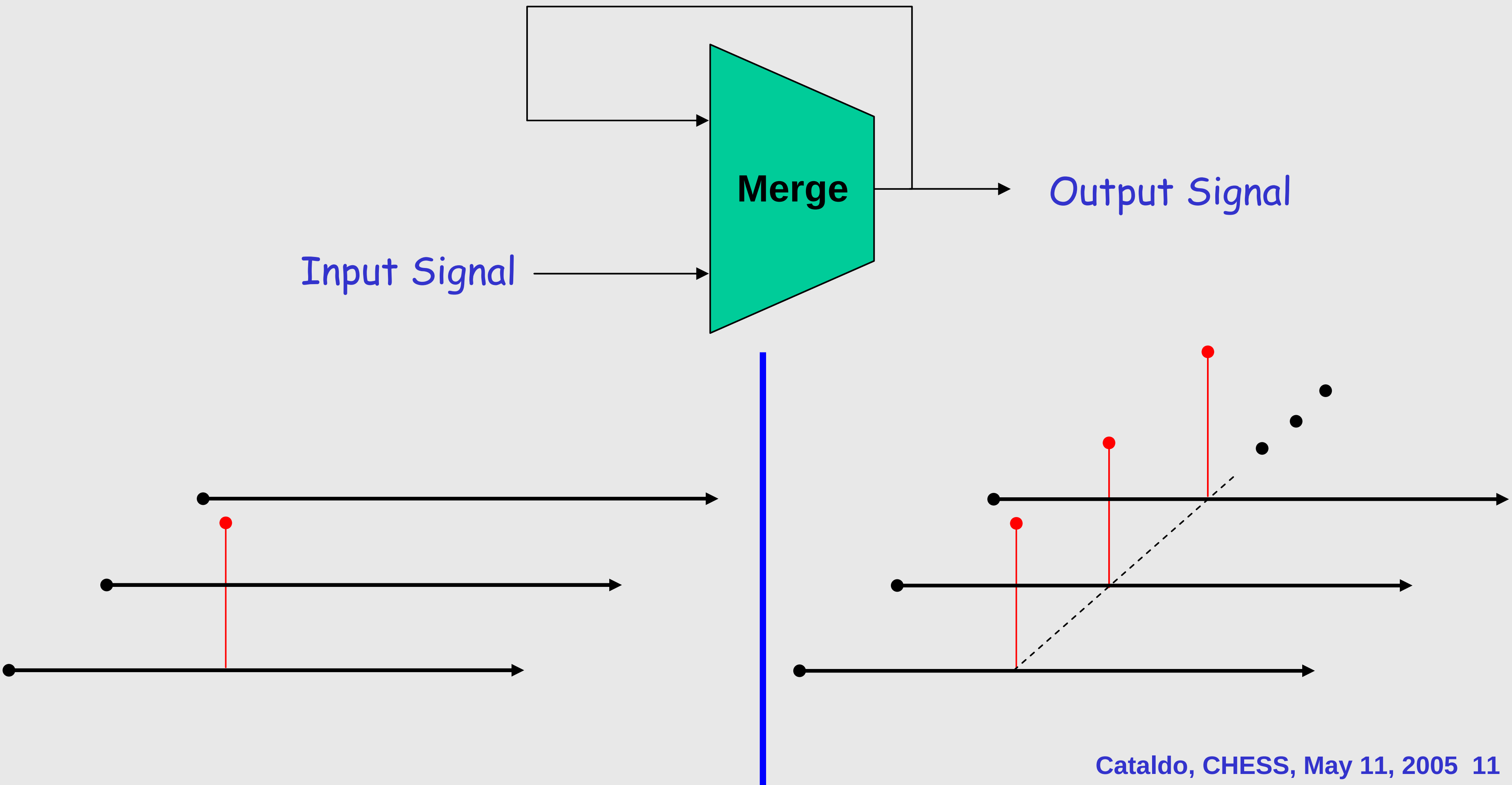
Zeno Signals

- Definition: *Zeno Signal*
  infinite events in finite real time



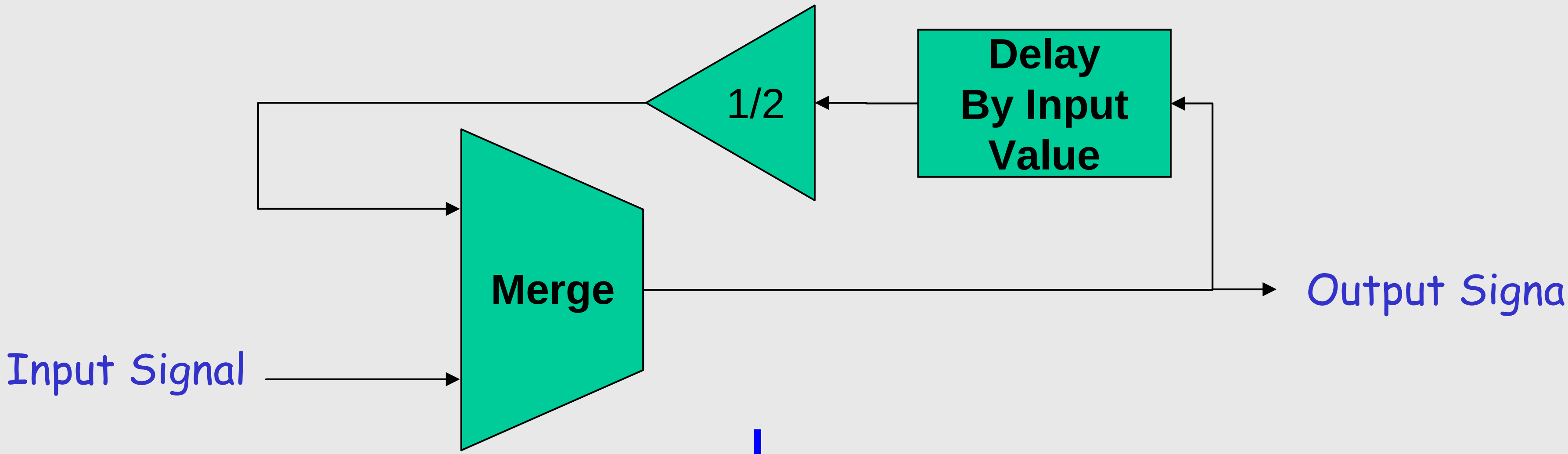
Source of Zeno Signals

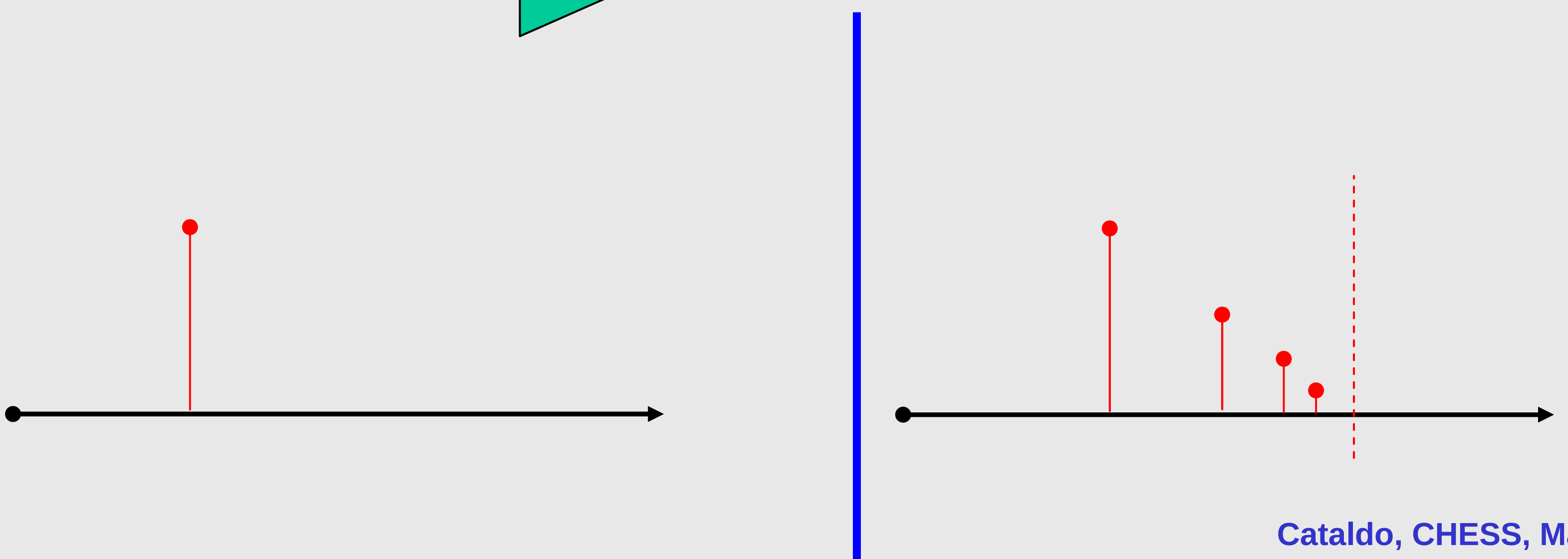
- Feedback can cause Zeno



Genuinely Zeno

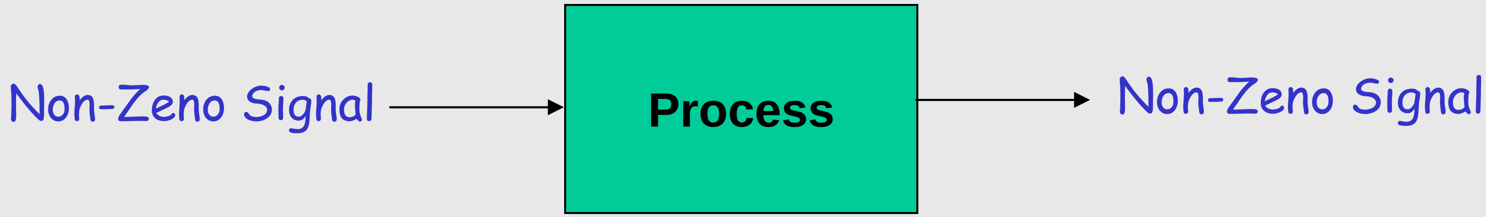
- A source of genuinely Zeno signals



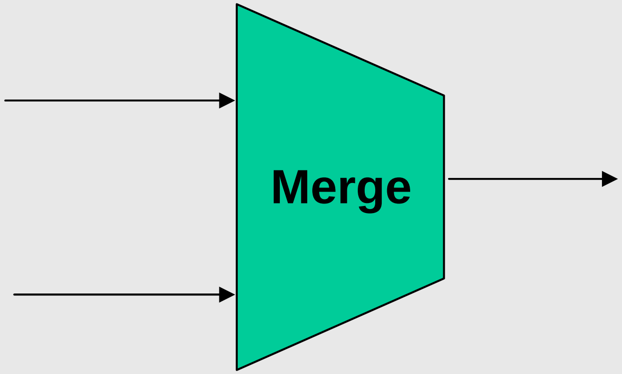


Simple Processes

- Definition: *Simple Process*

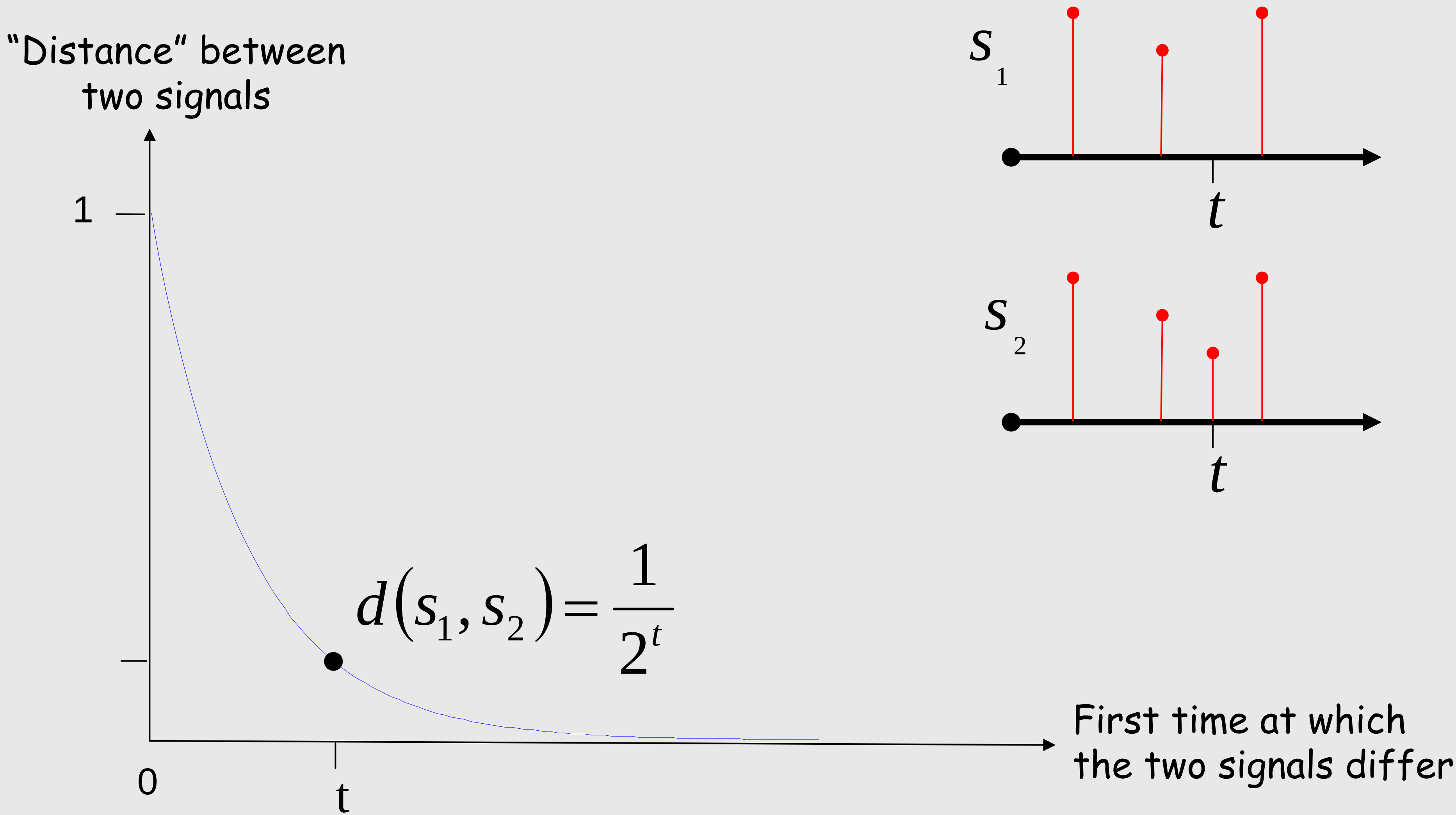


- Merge is simple, but it has Zeno feedback solutions



- When are compositions of simple processes simple?

Cantor Metric for Signals



$$d(s_1, s_2) = \frac{1}{2^t}$$

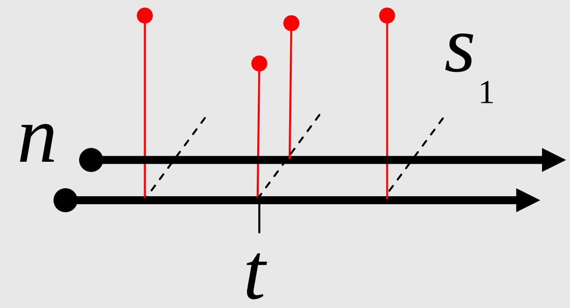
Tetrics: Extending Metric Spaces

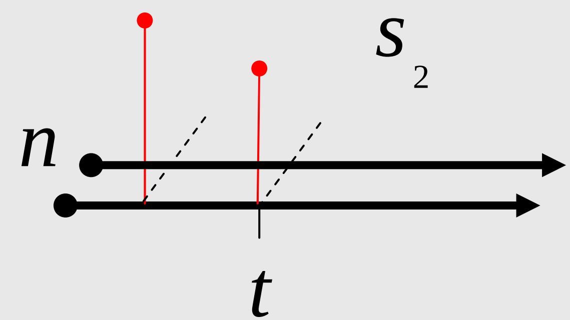
- Cantor metric doesn't capture simultaneity
- We can capture simultaneity with a tetric
- Tetrics are generalized metrics
- We generalized metric spaces with "tetric spaces"
- Our tetric allows us to deal with simultaneity

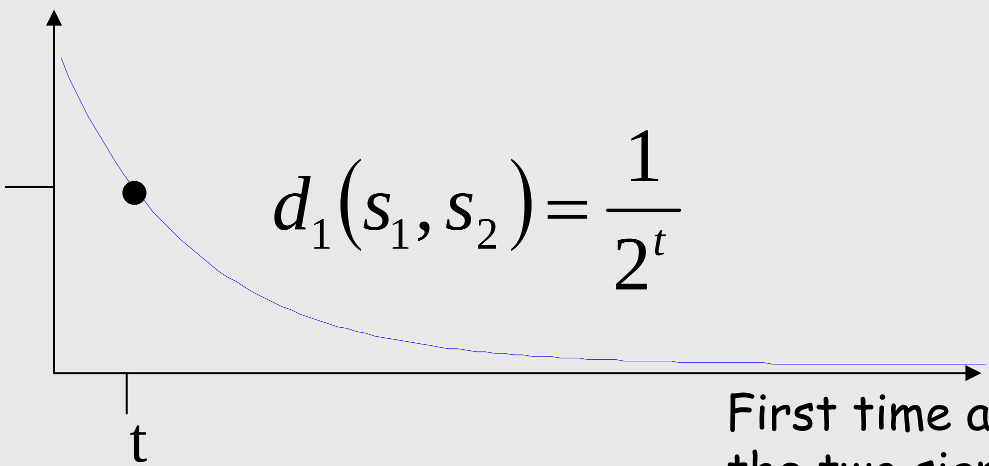
Our Tetric for signals

"Distance" between two signals:

$$d(s_1, s_2) = \left(\frac{1}{2^t}, \frac{1}{2^n} \right)$$

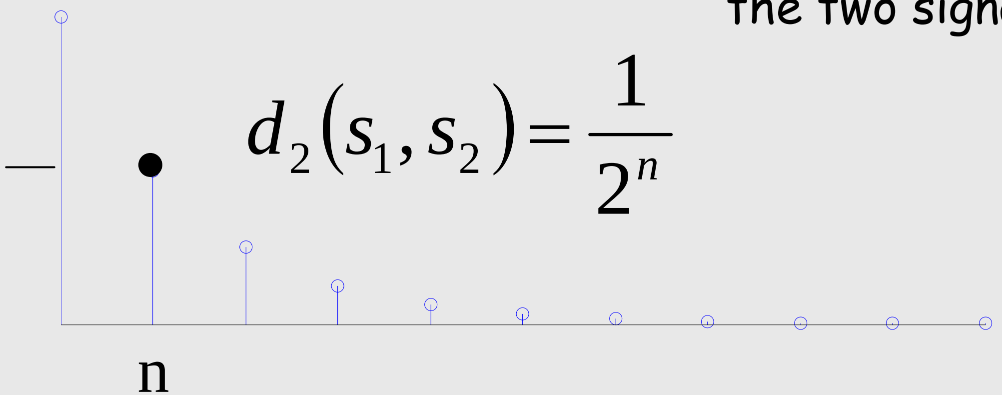






$$d_1(s_1, s_2) = \frac{1}{2^t}$$

First time at which
the two signals differ

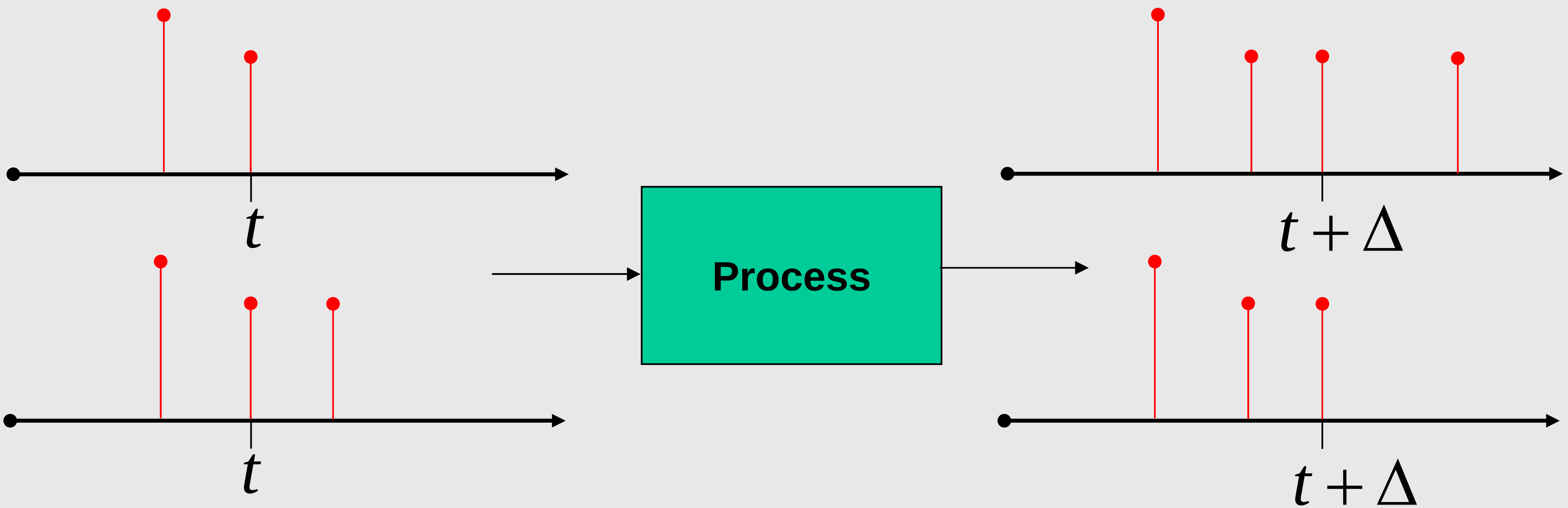


$$d_2(s_1, s_2) = \frac{1}{2^n}$$

First sequence number at
which the two signals differ

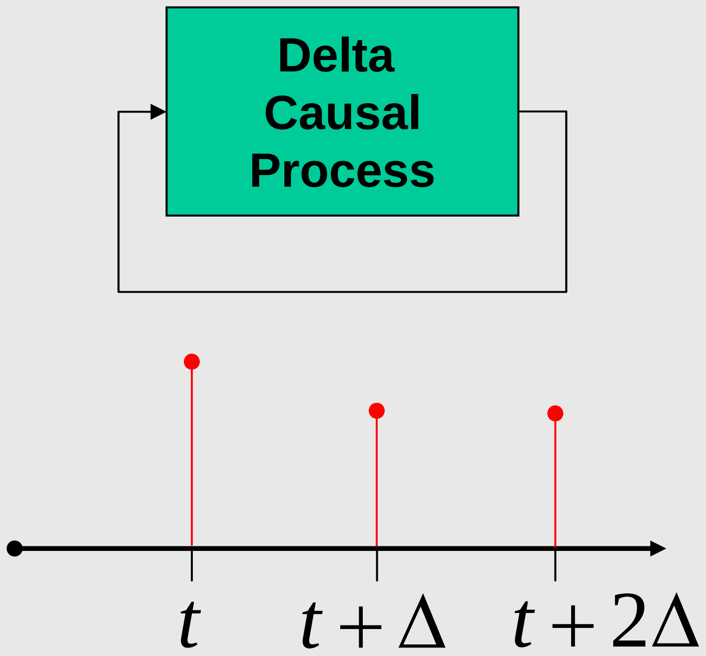
Delta Causal

Definition: *Delta Causal*
Input signals agree up to time t implies
output signals agree up to time $t + \Delta$



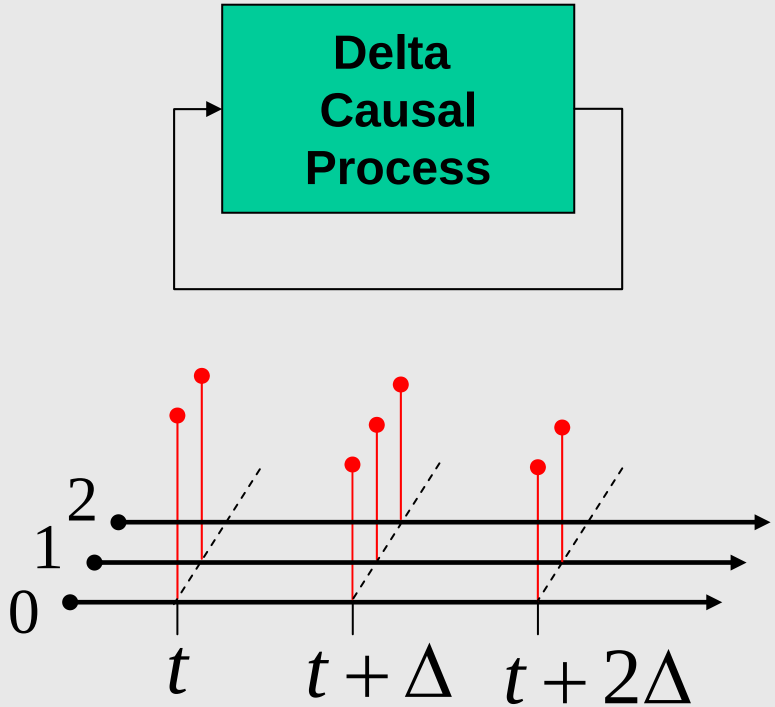
What Delta Causal Means

- Signals which delay their response to input events by delta will have non-Zeno fixed points



Extending Delta Causal

- The system should be allowed to chatter

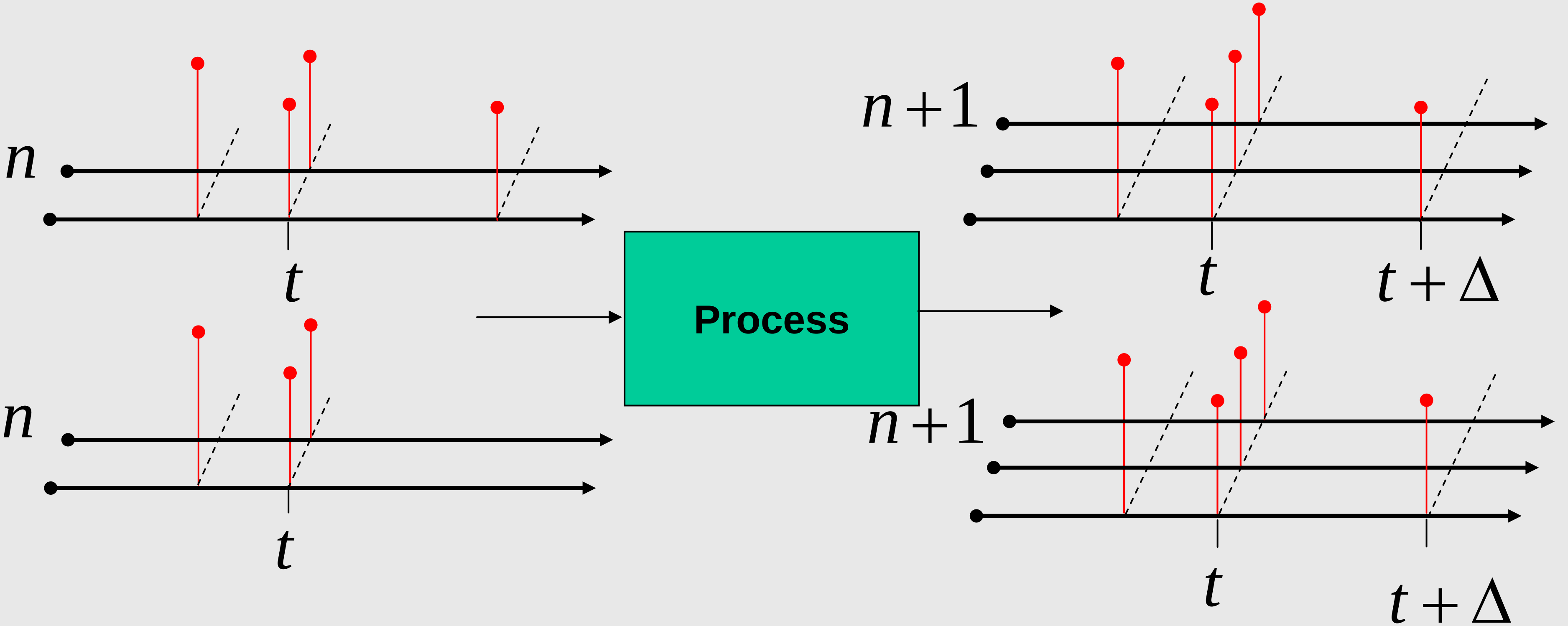


- As long as time eventually advances by delta

Tetric Delta Causal

Definition: *Tetric Delta Causal*

1) Input signals agree up to time (t,n)
implies output signals agree up to time $(t, n+1)$

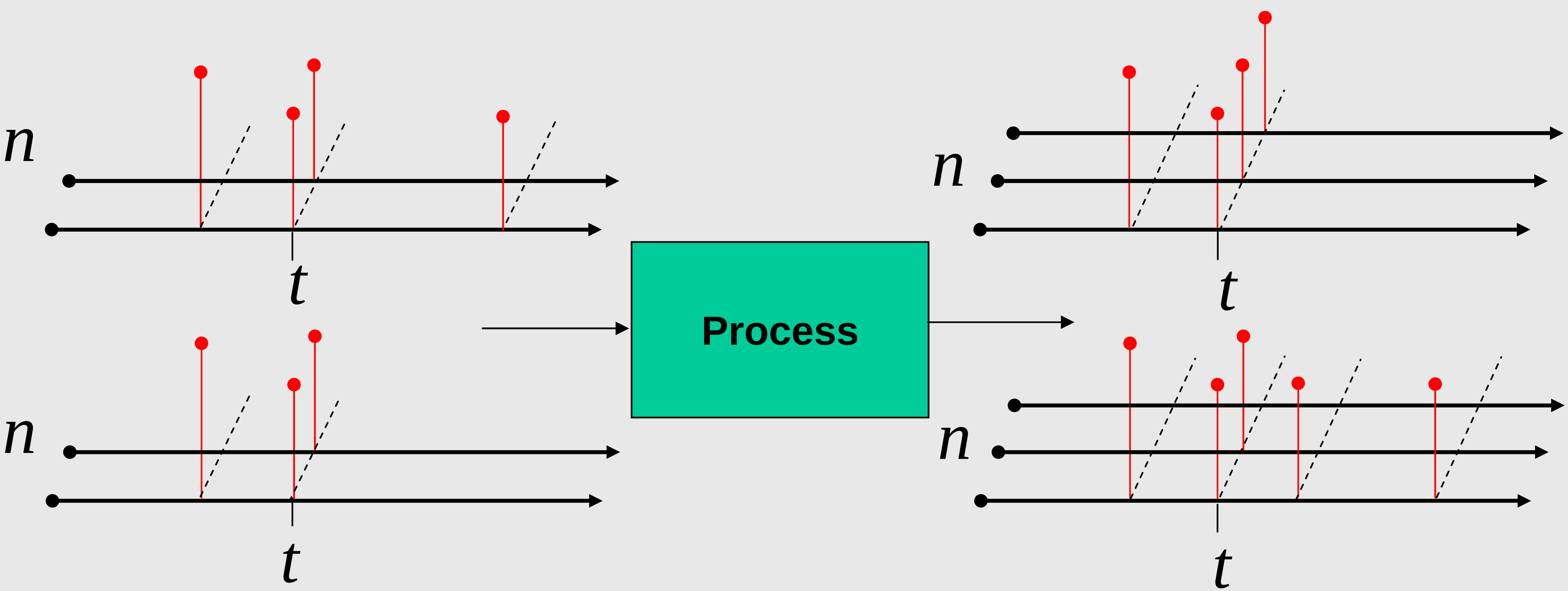


2) If n is large enough, this also
implies output signals agree up to time $(t+\Delta,0)$

Causal

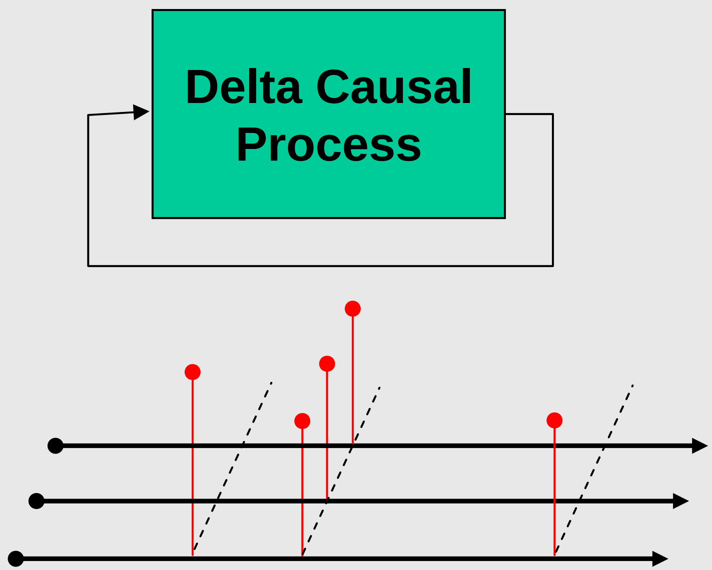
Definition: *Causal*

If input signals agree up to supertime (t, n) then the output signals agree up to supertime (t, n)



Result 1

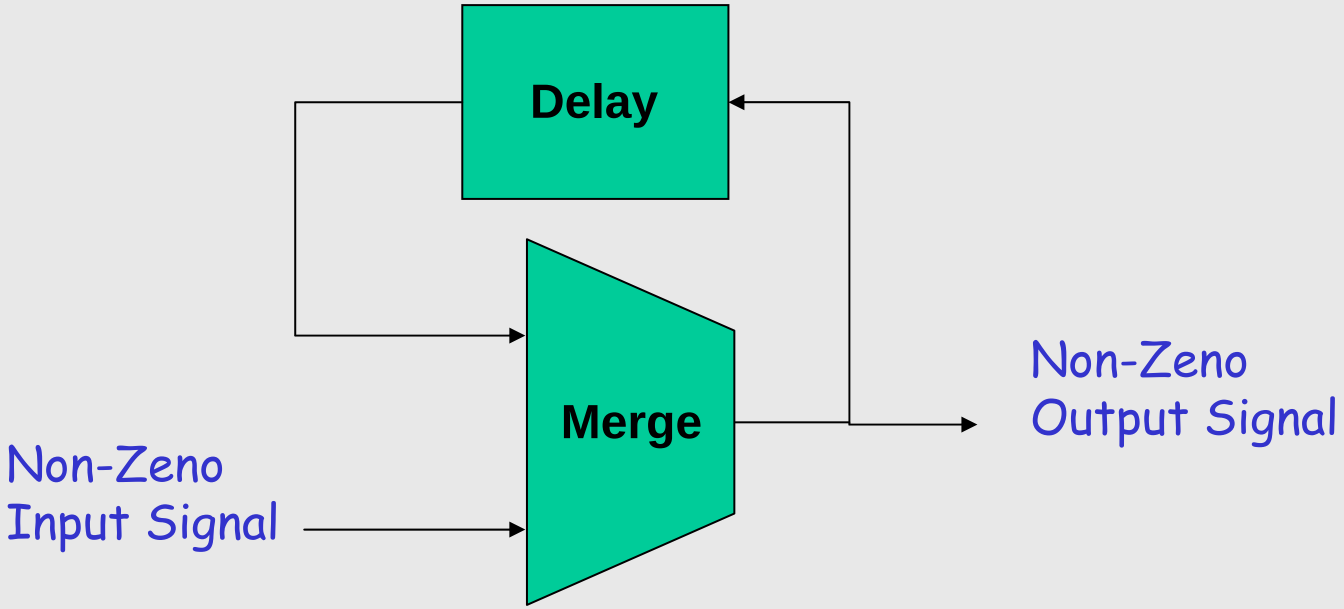
- Every tetric delta causal process has a unique feedback solution



Unique feedback solution

Result 2

- Every network of simple, causal processes is a simple causal process, provided in each cycle there is a delta causal process
- Example



Conclusions

- We broadened DE semantics to handle superdense time
- We invented tetric spaces to measure the distance between DE signals
- We gave conditions under which systems will have unique fixed-point solutions
- We provided sufficient conditions under which this solution is non-Zeno
- http://ptolemy.eecs.berkeley.edu/papers/05/DE_Systems

Acknowledgements

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