Optimal Control of Stochastic Hybrid Systems

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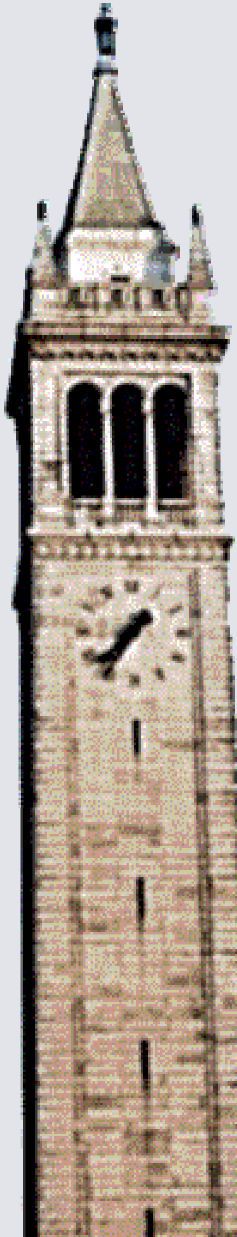

Chess Review
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Introduction - Motivations:

- Present a new method for optimal control of Stochastic Hybrid Systems.
- More flexible than Hamilton-Jacobi, because handles more problem formulations.
- In implementation, up to dimension 4-5 in the continuous state.

Problem Formulation:

Minimize $E[f(X)]$

Subject to $dX_t = u(X_t, m_t)dt + \sigma(X_t, m_t)dB_t$

$u \in \mathcal{U}$

- $\{B_t \in \mathbb{R}^d : t \geq 0\}$ standard Brownian motion.
- $\{X_t \in \mathbb{R}^n : t \geq 0\}$ continuous state. Solves an SDE whose jumps are governed by the discrete state.
- $\{m_t \in \{1, \ldots, M\} : t \geq 0\}$ discrete state: continuous time Markov chain.
- $u : \mathbb{R}^n \times \{1, \ldots, M\} \to \mathbb{R}^n$ control.

Applications:

- Engineering: Maintain dynamical system in safe domain for maximum time.

  Maximize $E[f(X)] = E[\inf_{t \geq 0}\{t : X(t) \notin U\}]$

  Subject to $\frac{dX(t)}{dt} = f(X(t), u(t)) + \sigma(m_t)w(t)$

- Systems biology: Parameter identification.

  Minimize $E[f(X)] = ||E[CX_T] - E_{\text{observed}}||$

  Subject to $\frac{dX(t)}{dt} = f(X(t), \theta) + \sigma(\theta)w(t)$

- Finance: Optimal portfolio selection

  Maximize $E[f(X)] = E[\int_0^{+\infty} e^{-\alpha t} r(X_t)\, dt]$

  Subject to $dX_t = \mu(X_t, t)dt + \sigma(X_t, t)dB_t + dJ_t$

Method: 1st step

1. Derive a PDE satisfied by the objective function in terms of the generator:

$$L_u V(x,m) = \sum_{i=1}^{n} u_i(x,m)\frac{\partial V(x,m)}{\partial x_i} + \frac{1}{2}\sum_{i,j=1}^{n}(\sigma(x,m)\sigma(x,m)^T)_{ij}\frac{\partial^2 V(x,m)}{\partial x_i \partial x_j} + \sum_{k=1}^{M}\lambda_{mk}(x)V(x,k), \quad \forall x \in \mathbb{R}^n,\ \forall m = 1,\ldots,M.$$

- Example 1:

If $V(x) = E^x[\int_0^\infty e^{-\alpha s} r(X_s)\, ds]$

then $L_u V(x) - \alpha V(x) = -r(x)$

- Example 2:

If $V(x) = E^x[\inf_{t \geq 0}\{t : X(t) \notin U\}]$

then $L_u V(x) = -1,\ V(x) = 0,\ \forall x \in \partial U$

Method:

2. Rewrite original problem as deterministic PDE optimization program

$$\begin{array}{ll} \text{Minimize} & E^x[f(X)] \\ \text{subject to} & \text{SHS}(u_t, X_t) = 0 \end{array}$$

$$\Longleftrightarrow \quad \begin{array}{ll} \text{Minimize} & V(x,t) \\ \text{subject to} & \text{PDE}(u(x), V(x)) = 0 \end{array}$$

3. Solve PDE optimization program using adjoint method

Simple and robust…