Hybrid Geometric Mechanics: Reduction and Going Beyond Zeno

Edited and presented by
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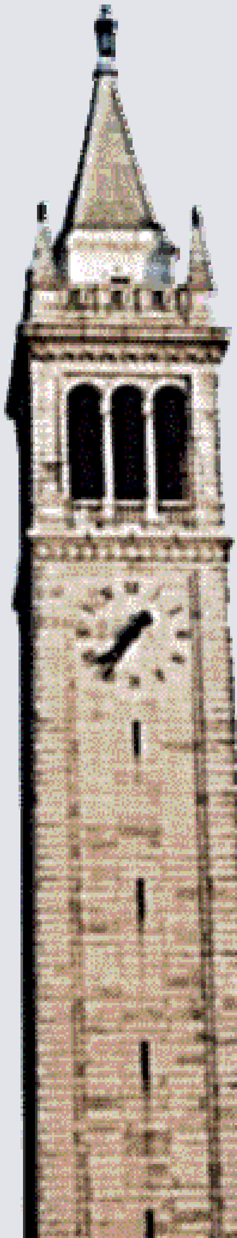


Chess Review
November 21, 2005
Berkeley, CA

Introduction

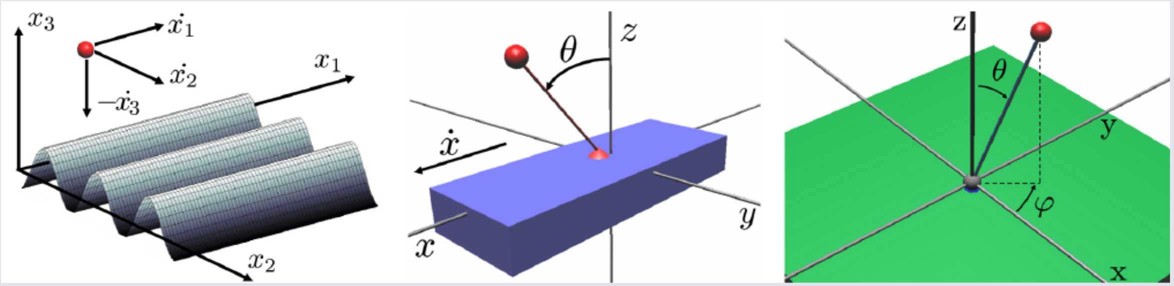
- Mechanical systems provide the quintessential example of dynamical systems
- Hybrid mechanical systems provide the quintessential example of hybrid systems
- By studying these systems, we can better understand general hybrid systems
  - Reduction of hybrid systems (how to reduce the dimensionality of a hybrid system)
  - Going beyond Zeno (how to carry executions of hybrid systems *past* the Zeno point)
  - Applications, e.g., bipedal walking

Educational Outreach

- Hybrid Mechanical Systems Provide a unique opportunity for educational outreach:
  - Teaching (with S. Sastry) special (undergraduate) research course:
    - *Bipedal Robotic Walking: From Theory to Application.*
  - Coauthored paper with Robert Gregg (undergraduate, UC Berkeley).
    - Is there Life after Zeno? Taking Executions past the Breaking (Zeno) Point. Submitted to ACC 2006.
    - Research was the result of the NSF sponsored SUPERB program.

Motivation for Hybrid Reduction

- Hybrid Systems describe a large class of physical systems



- Why should I care about hybrid reduction?
  - Reduction reduces the dimensionality of the systems aiding in analysis and simulation.
  - Analyzing and simulating hybrid systems is harder than for continuous systems
    - reduction is more important for hybrid systems!

Hybrid Reduction

- Reduction can be generalized to a hybrid setting
  - All of the major ingredients necessary for classical reduction can be hybridized via hybrid category theory
  - This theory has applications beyond reduction
    - hybrid stability theory
    - composition
    - Zeno detection
    - etc.
- Applications of hybrid reduction
  - Bipedal robotic Walking

Hybrid Reduction Theorem

Classical Reduction Theorem

- *Given a symplectic manifold M (the phase space), there exists a symplectic manifold M_μ such that M_μ inherits the symplectic structure from that of M.*
- *Dynamical trajectories of the Hamiltonian H on M determine corresponding trajectories on the reduced space.*

Hybrid Reduction Theorem

- *Given a hybrid symplectic manifold $\mathbf{M}$ (the hybrid phase space), there exists a hybrid symplectic manifold $\mathbf{M}_{\boldsymbol{\mu}}$ such that $\mathbf{M}_{\boldsymbol{\mu}}$ inherits the hybrid symplectic structure from that of $\mathbf{M}$.*
- *Dynamical hybrid trajectories of the hybrid Hamiltonian $\mathbf{H}$ on $\mathbf{M}$ determine corresponding hybrid trajectories on the reduced hybrid space.*

Hybrid Lagrangians

Definition

A simple hybrid Lagrangian is defined to be a tuple

$$\mathbf{L} = (Q, L, h)$$

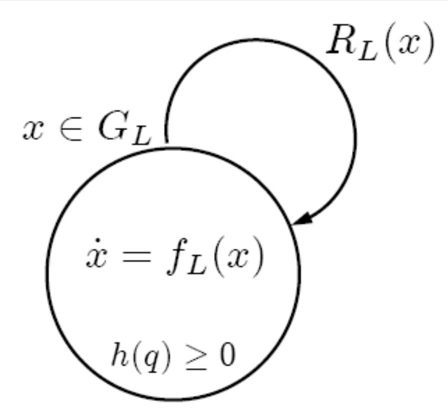
where

Continuous Data
- Q is the configuration space.
- $L : TQ \to \mathbb{R}$ is a (hyperregular) Lagrangian.

Discrete Data
- $h : Q \to \mathbb{R}$ provides unilateral constraints on the configuration space; we assume that $h^{-1}(0)$ is a manifold.

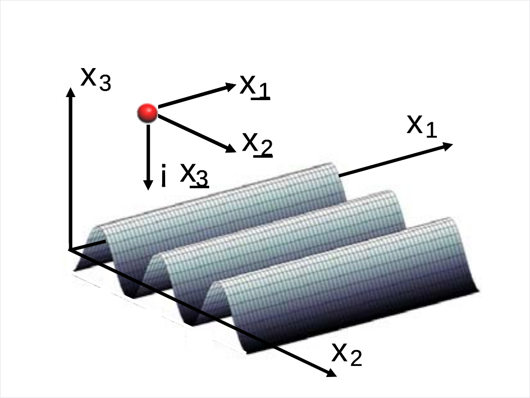
- Associated to a hybrid Lagrangian is a hybrid system

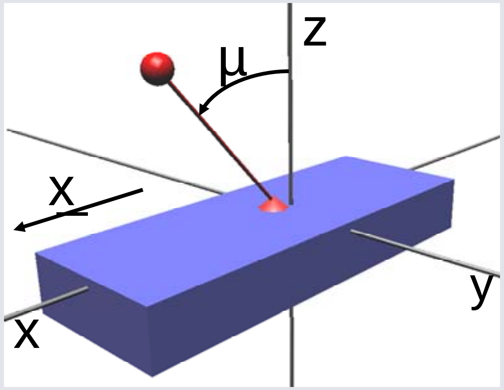
$$H_L = (D_L; G_L; R_L; f_L)$$

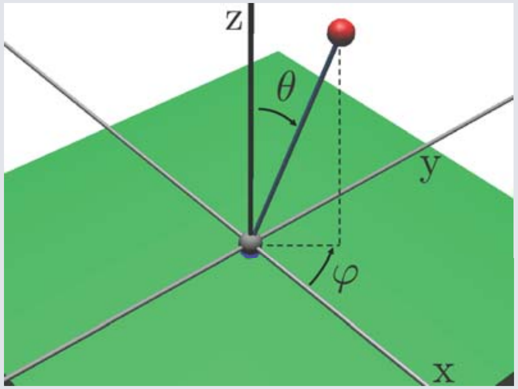


Examples

Ball on Sinusoidal Surface	Pendulum on a Cart	Spherical Pendulum on the Ground







$h_B(x_1; x_2; x_3)$
$= x_3 \; i \; \cos(x_2)$

$h_C(\mu; x) = \cos(\mu)$

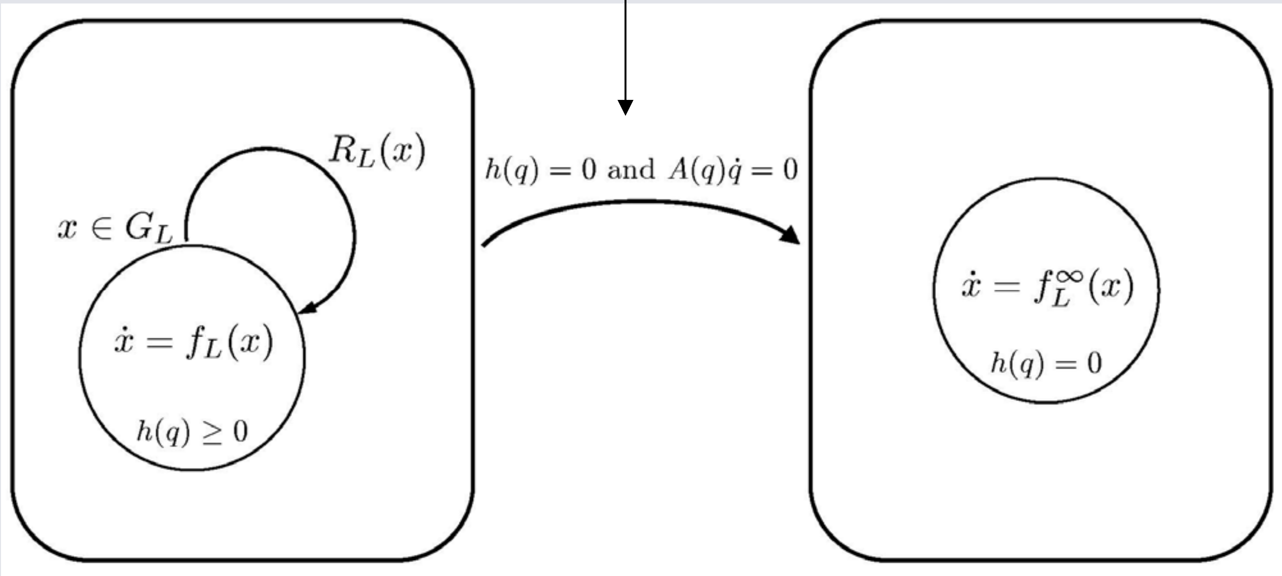
$h_P(\mu; ' \,) = \cos(\mu)$

Going Beyond Zeno

- Zeno behavior occurs when there are an infinite number of discrete transitions in a finite amount of time
- A hybrid system can be *completed* so that it goes beyond a Zeno point
  - A post-Zeno state is added
  - At (or "near") the Zeno point, the system switches to a holonomically constrained dynamical system
  - A hybrid system is completed by composing it with a dynamical system obtainable from the original hybrid system

Completed Hybrid System

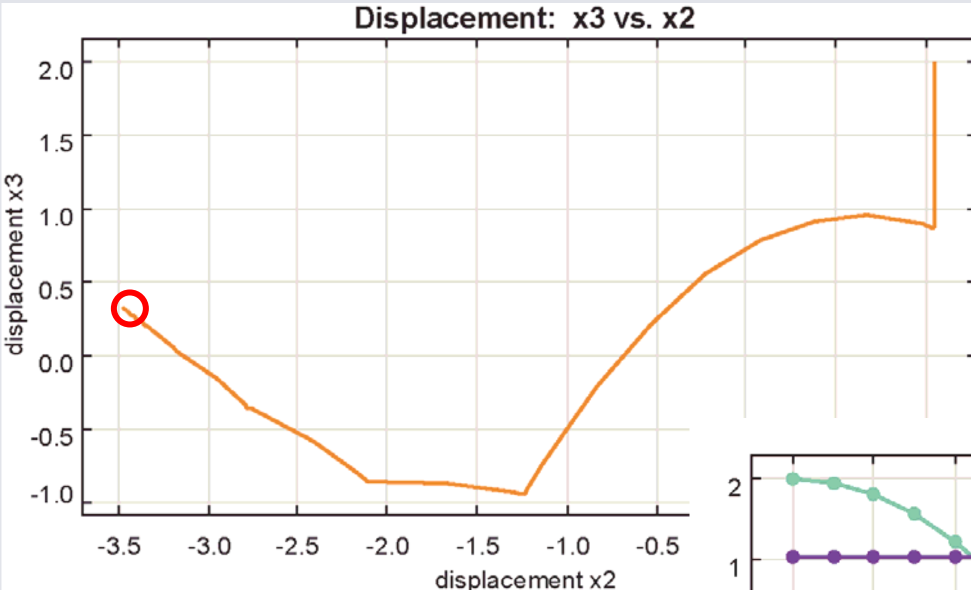
Conditions Satisfied at Zeno Point



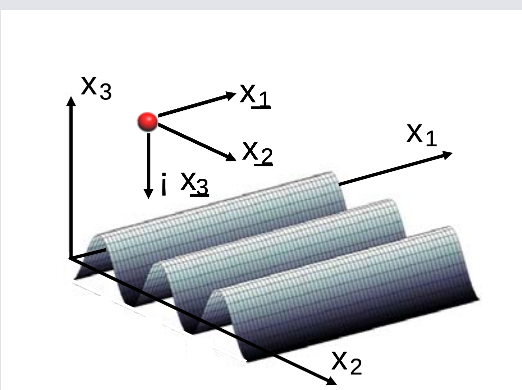
Original Hybrid System

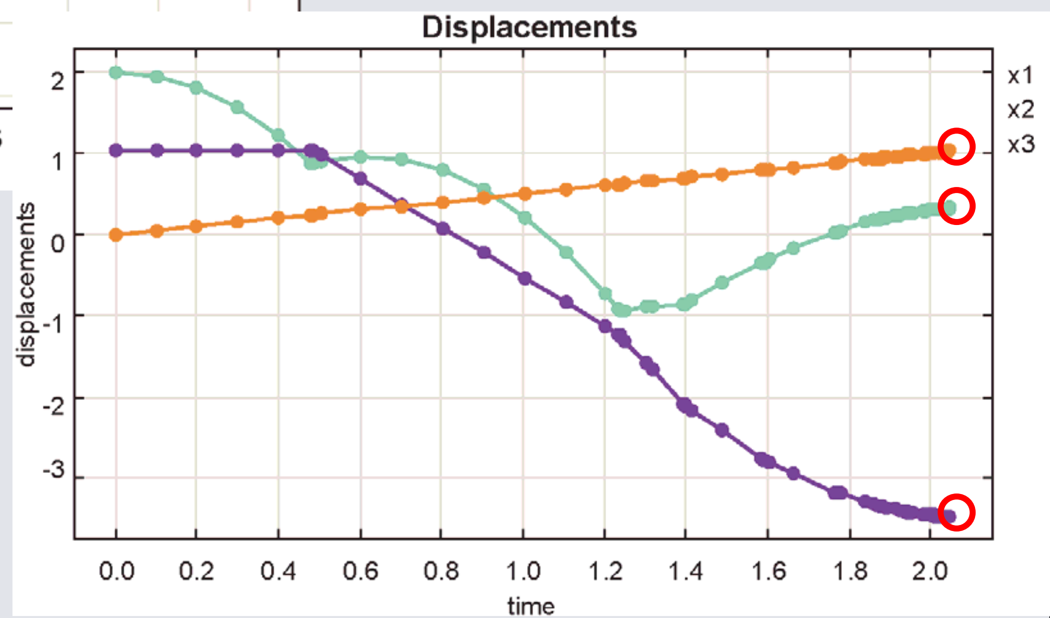
Constrained Dynamical System

Ball on a Sinusoidal Surface

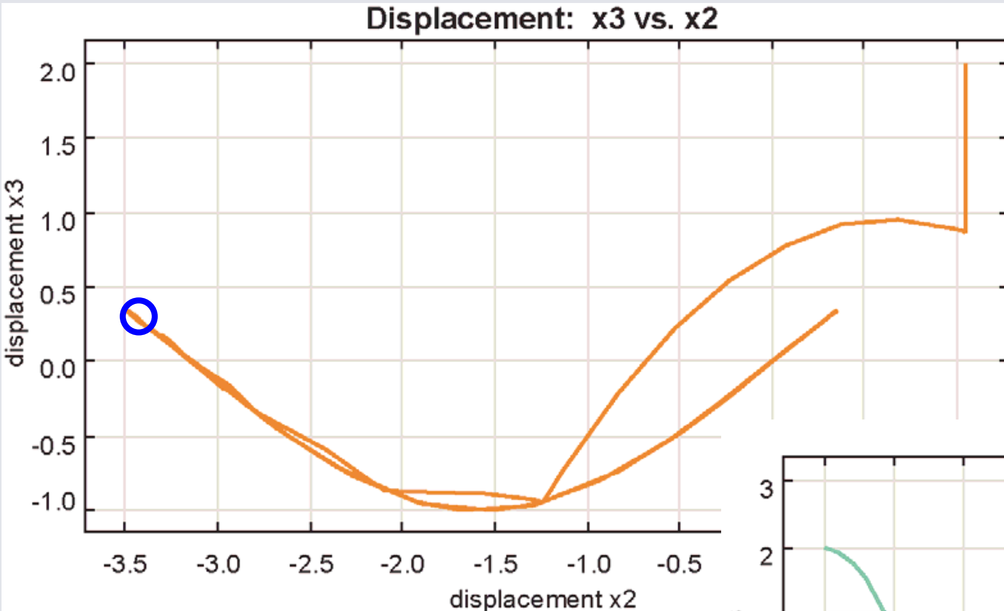


Without Completed Hybrid System, simulation **Stops** before the Zeno point.

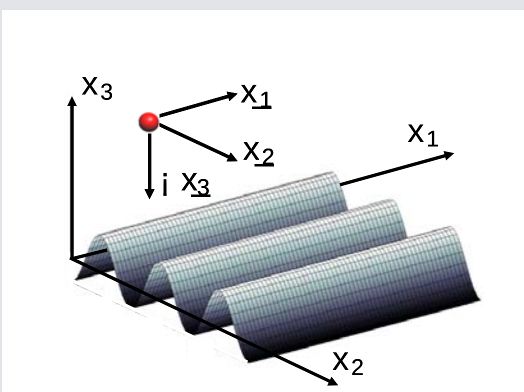


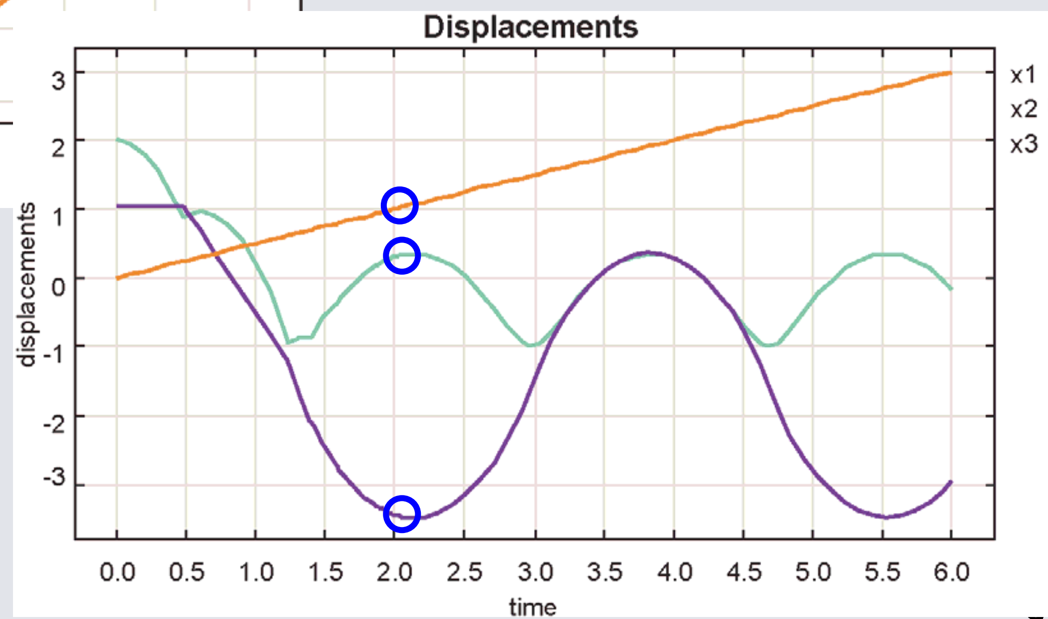


Ball on a Sinusoidal Surface

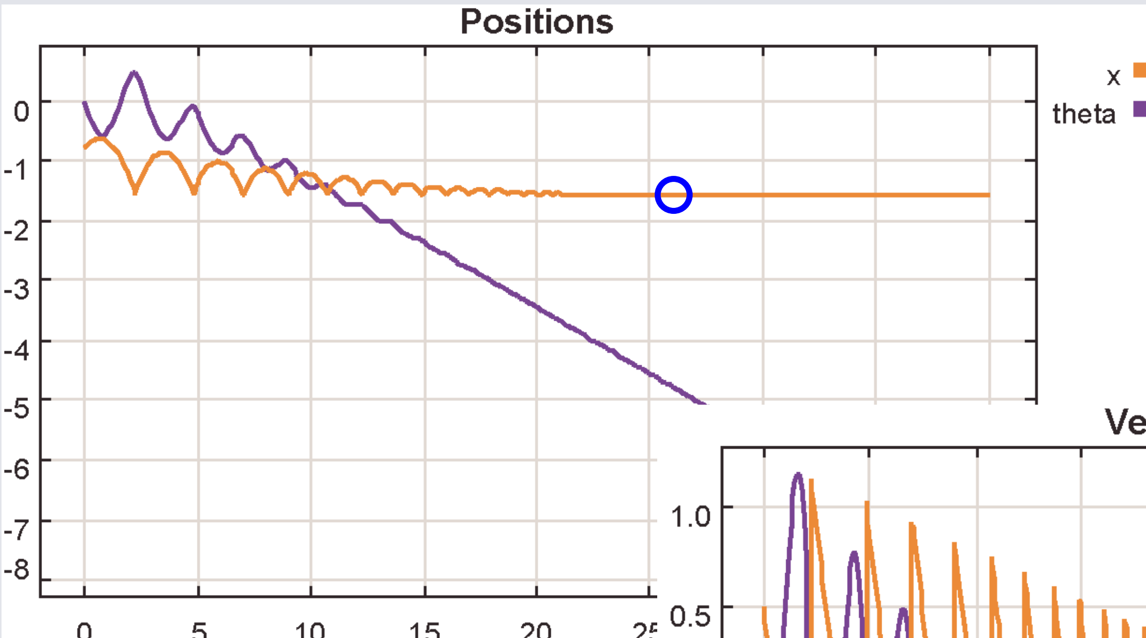


With Completed Hybrid System, simulation goes beyond Zeno point.





Pendulum on a Cart



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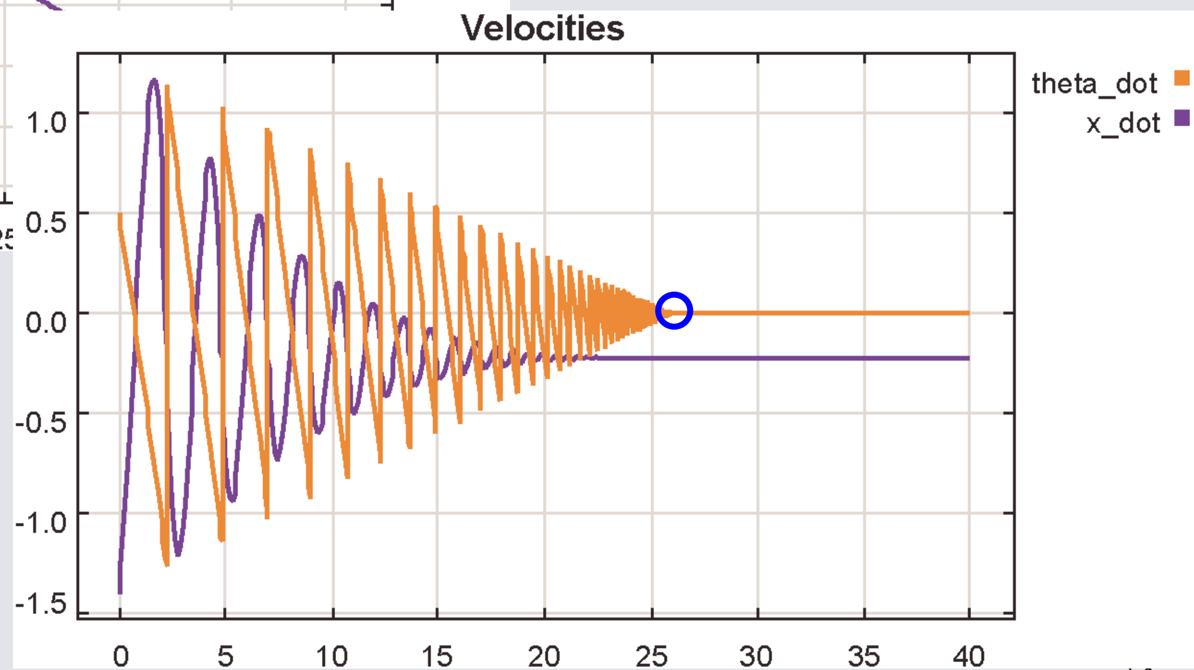
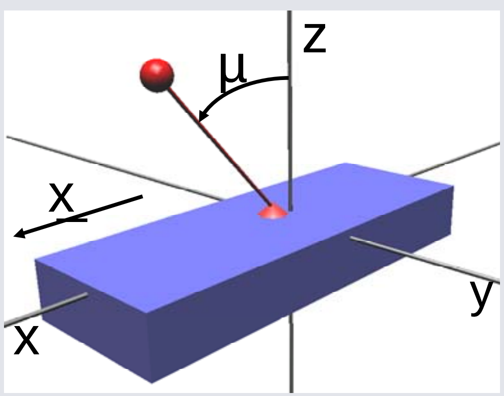
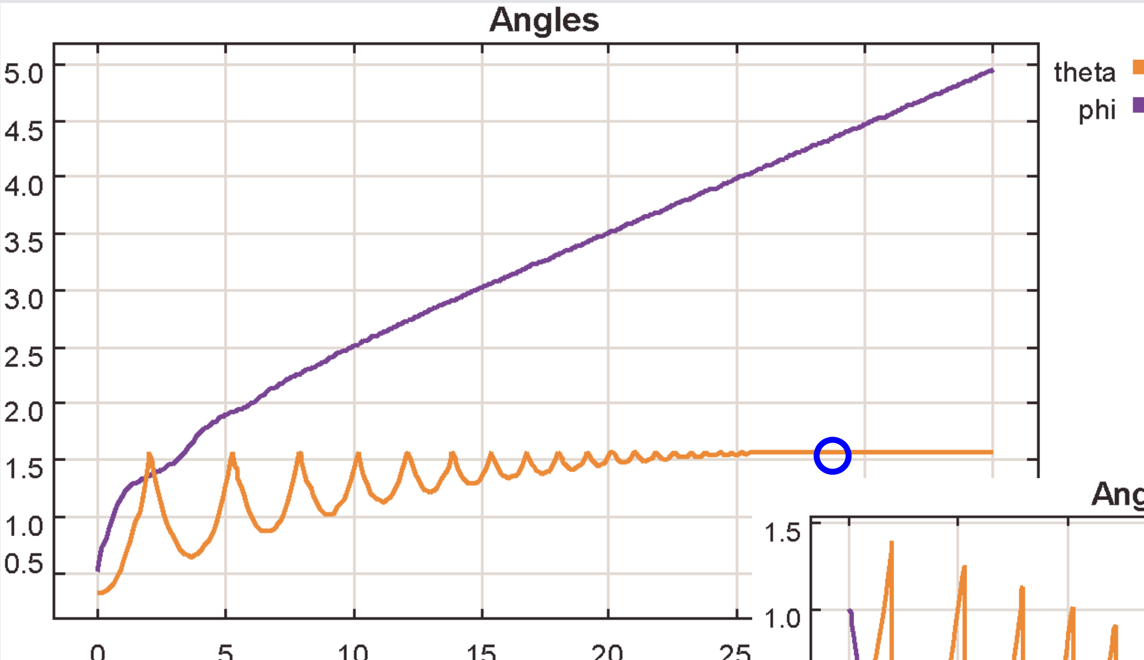




Spherical Pendulum



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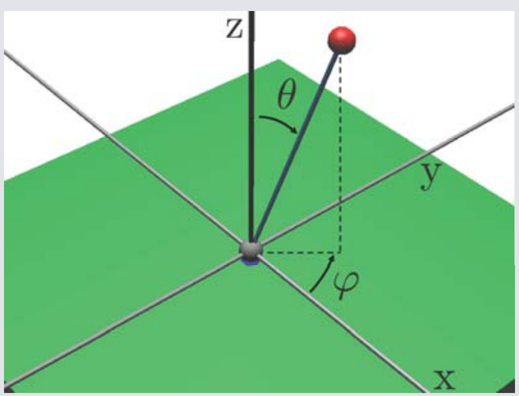

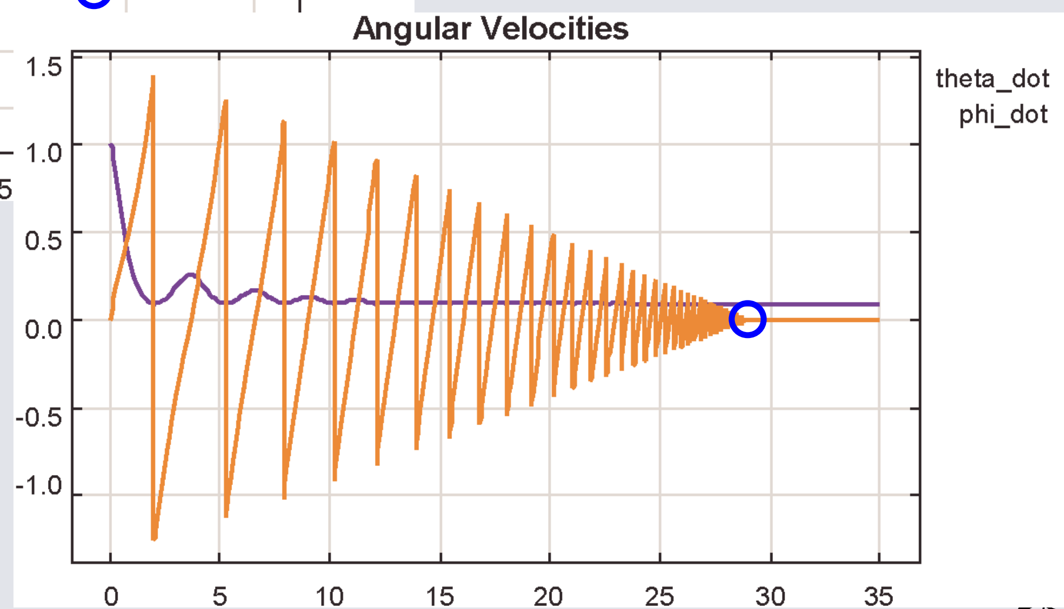

Conclusion

- Introduced hybrid mechanical systems
  - Generalized classical reduction to a hybrid setting
  - Extended executions past the Zeno point
- The special structure of these systems makes them theoretically interesting and practically important.
- Future Research Plans
  - Physical realization of theory
    - build a robotic bipedal walker
  - Continued educational outreach
    - Dissemination of principles on both the undergraduate and graduate level

Collaborators

- Hybrid Reduction
  - Shankar Sastry
- Going Beyond Zeno
  - Edward A. Lee
  - Shankar Sastry
  - Haiyang Zheng
  - Robert Gregg
- Undergraduate Research Course Participants
  - Robert Gregg
  - Jonathan Tesch
  - Eric Wendel