Thermal Modeling for Buildings
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Motivation

- Buildings account for approximately 40% of world energy use -> 21% of greenhouse gas emissions.
- In the U.S., buildings contribute 1 billion metric tons of greenhouse gas emissions.
- On an annual basis, buildings Consume:
  - 39% of total U.S. Energy
  - 68% of U.S. Electricity
  - 38% of U.S. carbon dioxide (primary greenhouse gas associated with climate change)
  - 25% of the nitrogen oxides found in the air
- The only energy end-use sector showing growth in energy intensity.

Source: National Institute of Building Sciences 2009, EE249 Lecture, Fall 2008 Report
Motivation

- According to the National Institute of Building Sciences, an Intelligent Buildings must seek to:
  - Reduce heating, cooling and lighting loads through climate-responsive design and conservation practices
  - Employ renewable energy sources (daylighting, passive solar heating, photovoltaics, geothermal, and groundwater cooling)
  - Specify efficient HVAC and lighting systems that consider part-load conditions and utility interface requirements
  - Optimize building performance by employing energy modeling programs and optimize system control strategies
  - Monitor project performance
- Energy codes provide minimum building requirements that are cost-effective in saving energy
Motivation

- The UC Merced campus: full scale test-bed for advanced energy management in campus buildings and systems
- Test-bed facility involves design, implementation and evaluation of a sensor network prototype
- Sensors include:
  - Light
  - Temperature
  - Humidity
  - Light intensity
  - Mobility patterns of humans inside building
- New measurements can help improve and design control Energy Management and Control Systems (EMCS)

UC Merced Campus

Total Energy Use, per Capita 1960-2001 (Estimated for CA for 2000 and 2001)
National Institute of Building Sciences 2009
Problem Statement

- In order to investigate how new technologies can help improve energy usage at the UC-Merced buildings, an scalable thermal model of the building needs to be created, verified and validated.

- Scalability is important when analyzing control systems for large buildings. Relevant to hybrid control system models

Objectives
- Create a scalable model (in Simulink) of building thermodynamics
- Model UC Merced building
- Validate model using reference data of UC Merced
Maasoumy, Holcomb [Fall 2008]. High Performance Building. Simulink modeling and control. More emphasis on control systems.

Schijndel [2003]. FemLab (Physical system simulator) and Simulink are evaluated as solvers for HVAC problems based on PDEs. Proposes the use in combination of both system, but doesn’t address scalability.

Mendes, et al [2001]. Mathematical model applied to both building thermal analysis and control system design using MatLab and Simulink. Approach incorporates multilayer walls, but does not address scalability.

Felgner, et al [2002]. Modelica is used. Modelica has libraries for building thermal characteristics. Paper makes a good argument about the power of Modelica. However, feedback from Control Engineering at the ME department reveals that it is not an easy tool to use.

Others: Address in detail HVAC systems, but do not address plant modeling or control systems.
Heat Transfer Basics

- **Conduction**
  
  \[ q = kA \frac{t_1 - t_2}{\delta x} \]

- **Convection**
  
  \[ \frac{q}{A} = h(t_s - t_f) \]

- **Radiation**
  
  \[ E = \varepsilon \sigma T^4 \]

**Nomenclature**

- \( C_p \): Specific Heat
- \( \rho \): Density
- \( t \): Temperature, K
- \( \tau \): Time, s
- \( m \): Mass, kg
- \( q \): Heat flow
- \( h \): Heat transfer coefficient of the material
- \( k \): Conduction coefficient
- \( L \): Length of conductor, m
- \( A \): Area, \( m^2 \)

**Heat Storage**

- Heat capacity describes how much energy is required to increase the temperature by a specified amount. Proportional to the mass of the object

\[ mC_p \frac{dt}{d\tau} = q \]
Heat Transfer Basics: Thermal Circuits

- Useful concept in the representation of thermal transfer
- Thermal circuit is a representation of the resistance to heat flow as though it were a resistor
- Heat storage elements can be represented as capacitors
- Temperature can be represented as potential

Network nodal analysis, the following must be satisfied at each node

$$\sum_j \frac{t_j - t_i}{R_{ij}} + q_i = 0$$

Where $q_i$ represents heat added to the node by means other than surface convection. If internal heat is present, the $q_i$'s are known.
Heat Transfer Basics: Thermal Circuits

- For conduction the resistance is:
  \[ R = \frac{\delta x_{ij}}{k A_{k_{ij}}} \]

- For convection it is:
  \[ R = \frac{1}{h_{ij} A_{c_{ij}}} \]
Proposed Approach

- **Simscape from MathWorks**
  - Extends the Simulink with tools for modeling systems spanning mechanical, electrical, hydraulic and other physical domains and physical networks
  - For our approach, use the electrical physical components to simulate a room and investigate its scalability

- **CHALLENGES**
  - No UC Merced model data for validation
  - Mitigation: model using other tool (ComSol, Modelica or PDE Matlab)
  - Other tools have higher learning curve. Verified the Model against Matlab PDEs
Model: Analytical

- Since building is a complex system, a complete theoretical approach is impractical.
- Assumptions:
  - Air is the zone is fully mixed. Temperature distribution is uniform and the dynamics can be expressed in a lump capacity model
  - Effect of each wall is the same
  - Ground and roof have no effect on the zone temperature
  - The density of the air is assumed to be constant and is not influenced by changing the temperature and humidity ratio of the zone
Model: Description

- **State variables:**
  - Zone temperature and wall temperatures

- **People, lights and extreme weather conditions are uncontrolled inputs.**

- **Input variables:**
  - Air flow rate for each zone
Model: Analytical Model

\[
\frac{dt_i}{d\tau} = \frac{1}{C_w} \left[ q_{rad} + \frac{t_o - t_i}{R_i w_i} + \frac{t_a - t_i}{R_o w_i} \right]
\]

(1)

Where

\[
C_w = L A \rho_w C_{pw}
\]

(2)

\[
R_i w_i = \frac{R w_i}{2} + R_{in}
\]

(3)

\[
R w_i = \frac{L}{K A}
\]

(4)

\[
R_{in} = \frac{1}{h_{in} A}
\]

(5)

\[
R_o w_i = \frac{R w_i}{2} + R_{out}
\]

(6)

\[
R_{out} = \frac{1}{h_{out} A}
\]

(7)

\[
\frac{dt_a}{d\tau} = \frac{1}{C_a} \left[ \sum_{i=1}^{n} \frac{t_i - t_a}{R_i w_i} + q_{in} + q_{other} \right]
\]

(8)

\[
q_{in} = \dot{m} C_{pa} (t_{input} - t_a)
\]

(9)

- Other heat gains can be set to zero.

\[
D(\tau) = \sum_{j=1}^{m} \frac{t_{eqj} - t_{Aj}(\tau)}{R_j} + q_p + q_t + q_o
\]

Fall 2008 Room Model
Model: Simulink (1 and 3 Rooms)

Top Level on Simulink
Model: Simulink (1 and 3 Rooms)

Thermal Model Level
Model: Simulink (1 and 3 Rooms)
Model: Simulink (1 and 3 Rooms)

Control Block
Model: Simulink 8 rooms
Model: PDEs MatLab

```matlab
% Global variables
global v_outside
global v_input
global Rw
global R_out
global Ra
global Cw
global Ca
global Cp_a
global input

h_in = 5; % W/(m^2 K)
h_out = 20; % W/(m^2 K)
K = 0.4;
rho = 1500; % density
rho_air = 1;
C_p_W = 1000; % J/(Kg K) specific heat capacitance wall
C_p_a = 1000; % J/(Kg K) specific heat capacitance Air

% Initial voltage for capacitors
v_out = 20; % Volts
v_input = 30; % Volts
v_air = 20; % Volts

% Wall Dimensions
L = 0.2; % Length m (thickness)
W = 4; % Width m
H = 4; % Height m
A = W*H; % Area m^2

% Inputs to Room Subsystem
Ra = 1/(h_in*A); % Resistance to Room Subsystem
R_out = 1/(h_out*A); % Resistance Out
Rw = (L*K)/A; % Resistance of wall
Ca = W^2*H*rho_air*Cp_a;

% State
x = [v_out; v_input; v_air; v_air; v_o; v_o; v_o; v_o; v_o; v_o; v_o; v_o; tspan; options);

function x_dot = deriv2(dt,x)

% Global variables
global v_outside
global v_input
global Rw
global R_out
global Ra
global Cw
global Ca

% Constants
To = v_outside;
Ta_1 = x(1);
Ta_2 = x(2);
Ta_3 = x(3);

% Equations
v_out = (To - Ta_1)/Rw + v_outside;
Rw/2 + R_out;
Rw/2 + R_out;

% Differential equation of the walls
% Room 1

dT(1,1) = (1/CW)*(grad + (To - T_room(1))/RiW1 + (Ta_1 - T_room(1))/Row1);
dT(2,1) = (1/CW)*(grad + (To - T_room(2))/RiW1 + (Ta_1 - T_room(2))/Row1);
dT(3,1) = (1/CW)*(grad + (To - T_room(3))/RiW1 + (Ta_1 - T_room(3))/Row1);
dT(4,1) = (1/CW)*(grad + (To - T_room(4))/RiW1 + (Ta_1 - T_room(4))/Row1);

% Room 2

dT(3,1) = (1/CW)*(grad + (To - T_room2(1))/RiW1 + (Ta_2 - T_room2(1))/Row1);
dT(4,1) = (1/CW)*(grad + (To - T_room2(2))/RiW1 + (Ta_2 - T_room2(2))/Row1);
dT(5,1) = (1/CW)*(grad + (To - T_room2(3))/RiW1 + (Ta_2 - T_room2(3))/Row1);

% Room 3

dT(7,1) = (1/CW)*(grad + (To - T_room3(1))/RiW1 + (Ta_3 - T_room3(1))/Row1);
dT(8,1) = (1/CW)*(grad + (To - T_room3(2))/RiW1 + (Ta_3 - T_room3(2))/Row1);
dT(9,1) = (1/CW)*(grad + (To - T_room3(3))/RiW1 + (Ta_3 - T_room3(3))/Row1);
```

```
Model: Results PDE vs. Simulink (constant input)

- Error is in the order of e-9, probably due to numerical integration errors.
Set point is 25 degrees C. Compute time increases, but model is easily scalable on Simulink. If wall properties are similar, Matlab model is easily scalable as well.
Control Effort

- PID Model for room uses the desired temperature errors as input. Very easy to implement on Simulink.
- Other advance control techniques (LQR, CLQR, MPC) require state space representation. Scalability can be a problem and state-space representation is not easily implementable.
- No direct state-space extension from circuit analysis
Control Effort

- For the 3-room model system we can define the state space as:

  \[ \dot{x} = Ax + \text{diag}(x^T R_1) R_2 u + Bu \]

- Where \( x \) is the state and \( u \) is the input vector. This non-linear system can be linearized about a nominal trajectory \((x_n, u_n)\) for implementation of LQR or CLQR. The constraint is the input since the amount of airflow should be restricted in the circuit analysis.
Control Effort

- The matrices $A$, $R_1$, $R_2$ and $B$ are defined by the dynamics of the system. $R_1$ and $R_2$ are necessary to represent the non-linearity in the model.
- Linearization is straight forward. Selection of nominal trajectory is not.
- Future work should focus on providing the correct system dynamics to the controller.
Summary

- A scalable model of a thermal system was presented. The model was verified using another modeling tool. Behavior of the systems to same inputs produce same outputs. Errors are only due to numerical integration.

- Validation and model tuning can be done once UC Merced data is available.

- Advanced control techniques require a different definition of model (state-space). A simple representation was shown, which can be linearized about a nominal trajectory.

- Future work should include the proper representation of the non-linear model for advanced control techniques.