Outline

- Part 3: Models of Computation
  - FSMs
  - Discrete Event Systems
  - CFSMs
  - Data Flow Models
  - Petri Nets
  - The Tagged Signal Model

Data-flow networks

- A bit of history
- Syntax and semantics
  - actors, tokens and firings
- Scheduling of Static Data-flow
  - static scheduling
  - code generation
  - buffer sizing
- Other Data-flow models
  - Boolean Data-flow
  - Dynamic Data-flow

Data-flow networks

- Powerful formalism for data-dominated system specification
- Partially-ordered model (no over-specification)
- Deterministic execution independent of scheduling
- Used for
  - simulation
  - scheduling
  - memory allocation
  - code generation

  for Digital Signal Processors (HW and SW)

A bit of history

- Karp computation graphs ('66): seminal work
- Kahn process networks ('58): formal model
- Dennis Data-flow networks ('75): programming language for MIT DF machine
- Several recent implementations
  - graphical:
    - Ptolemy (UCB), Khoros (U. New Mexico), Grape (U. Leuven)
    - SPW (Cadence), COSSAP (Synopsys)
  - textual:
    - Silage (UCB, Mentor)
    - Lucid, Haskell

Data-flow network

- A Data-flow network is a collection of functional nodes which are connected and communicate over unbounded FIFO queues
- Nodes are commonly called actors
- The bits of information that are communicated over the queues are commonly called tokens

Intuitive semantics

- (Often stateless) actors perform computation
- Unbounded FIFOs perform communication via sequences of tokens carrying values
  - integer, float, fixed point
  - matrix of integer, float, fixed point
  - image of pixels
- State implemented as self-loop
- Determinacy:
  - unique output sequences given unique input sequences
  - Sufficient condition: blocking read

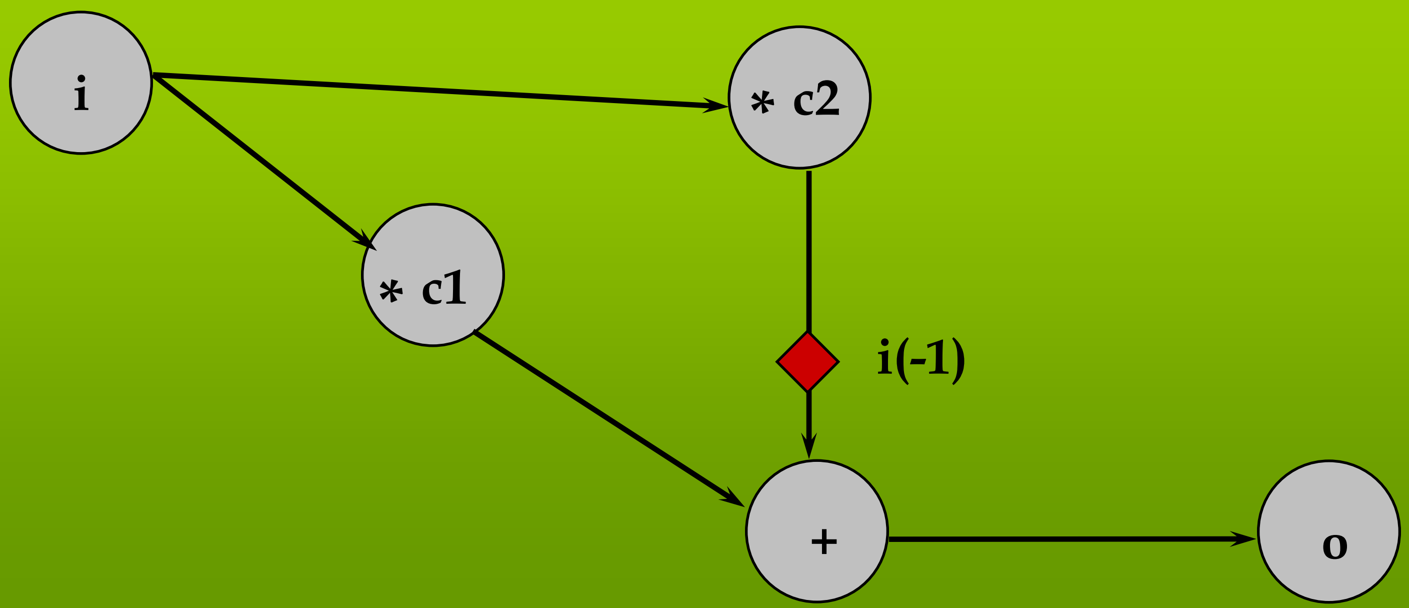
    (process cannot test input queues for emptiness)

Intuitive semantics

- At each time, one actor is fired
- When firing, actors consume input tokens and produce output tokens
- Actors can be fired only if there are enough tokens in the input queues

Intuitive semantics

- Example: FIR filter
  - single input sequence i(n)
  - single output sequence o(n)
  - o(n) = c1 i(n) + c2 i(n-1)



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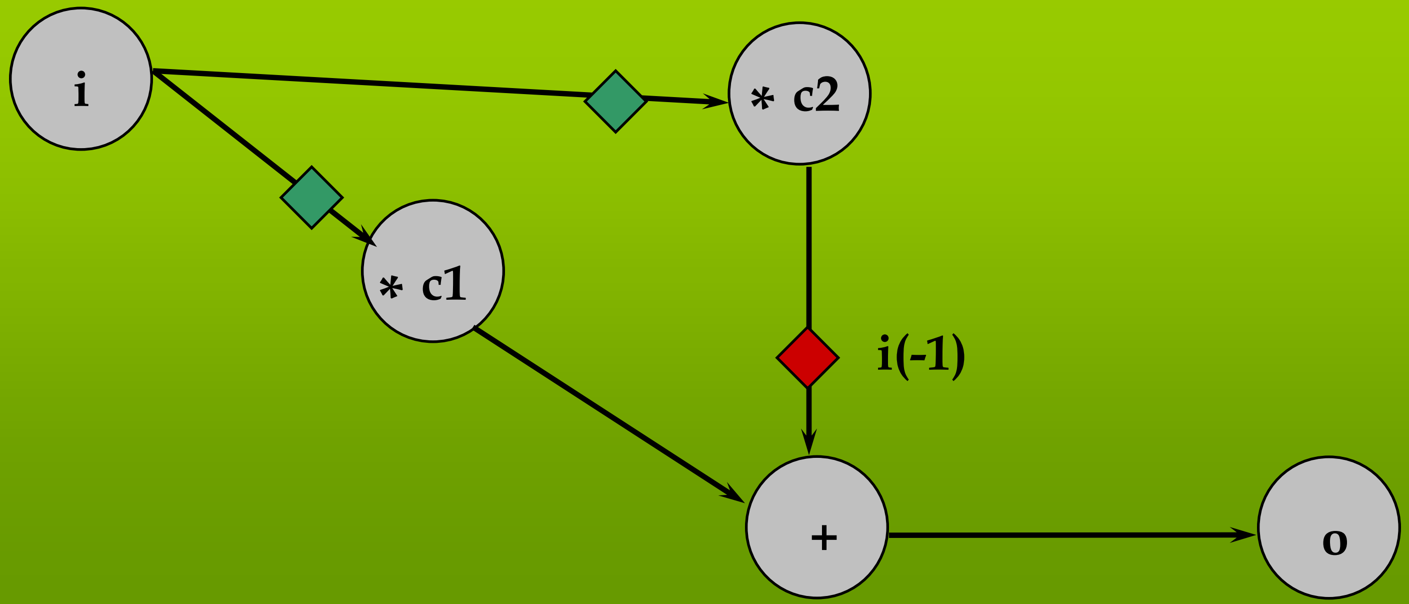

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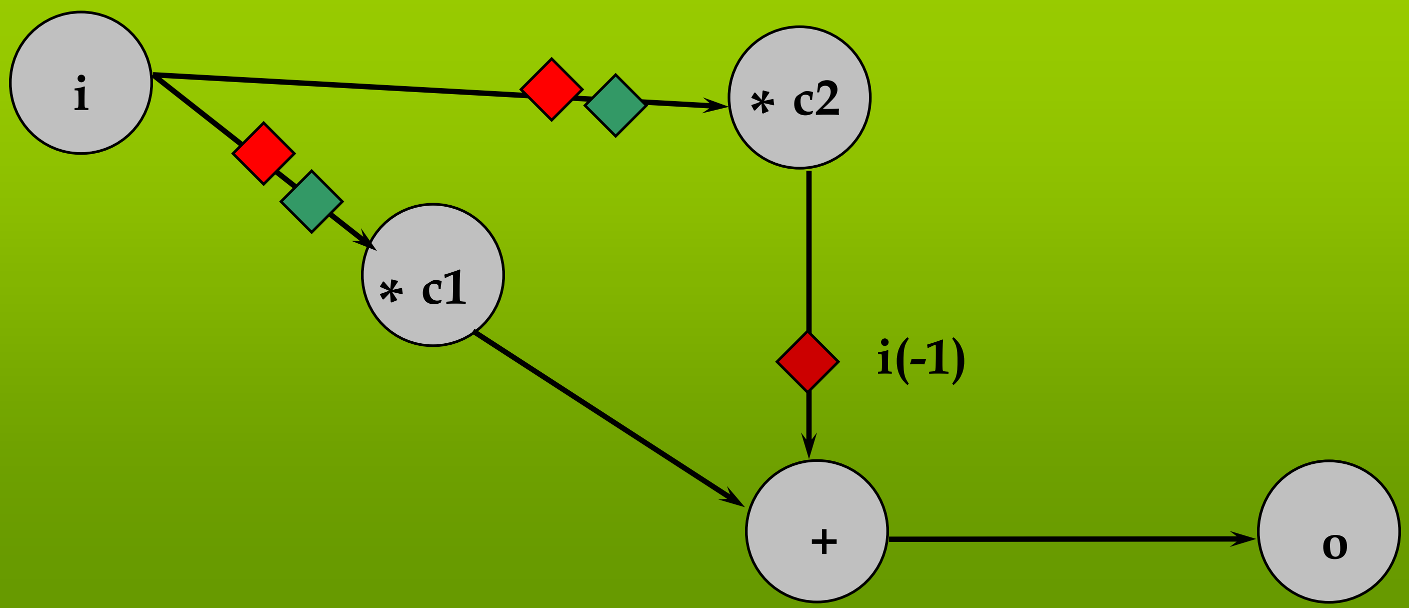

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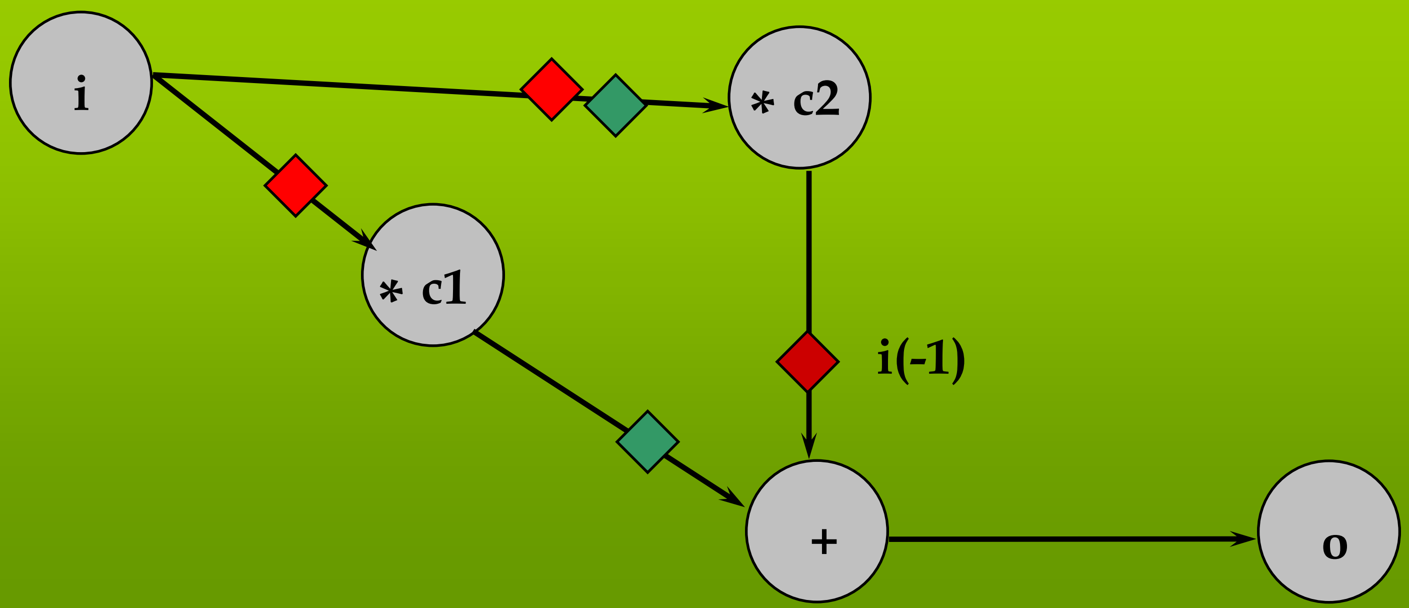

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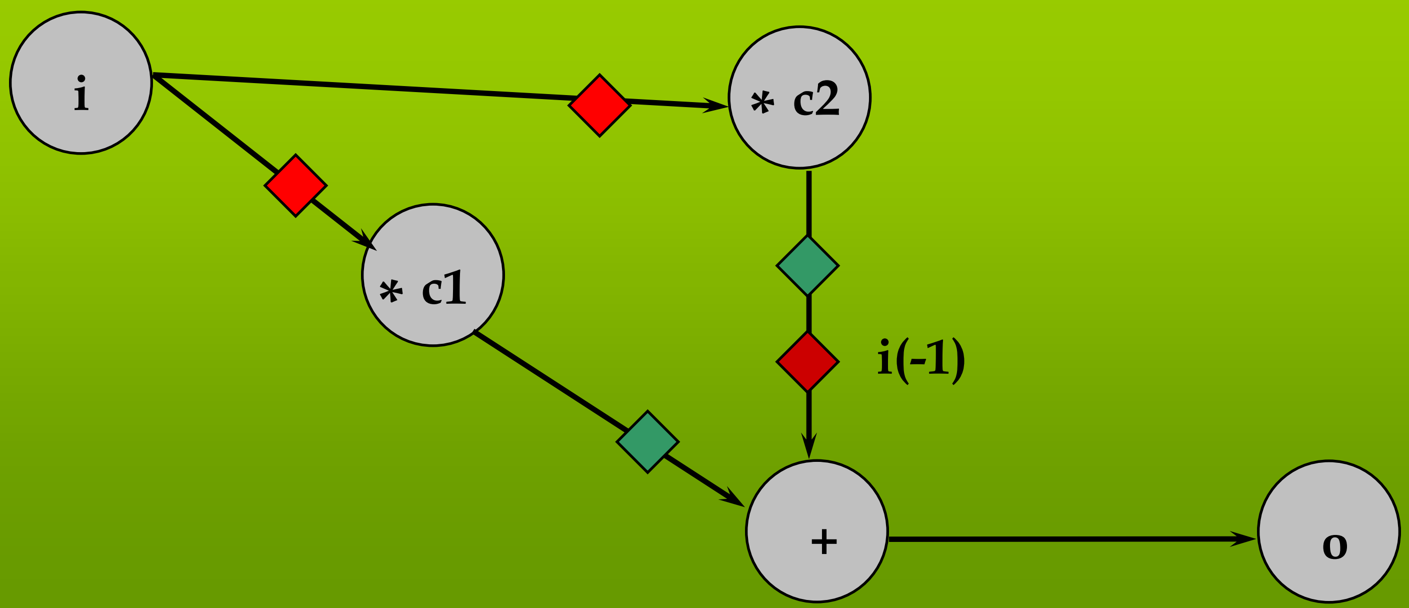

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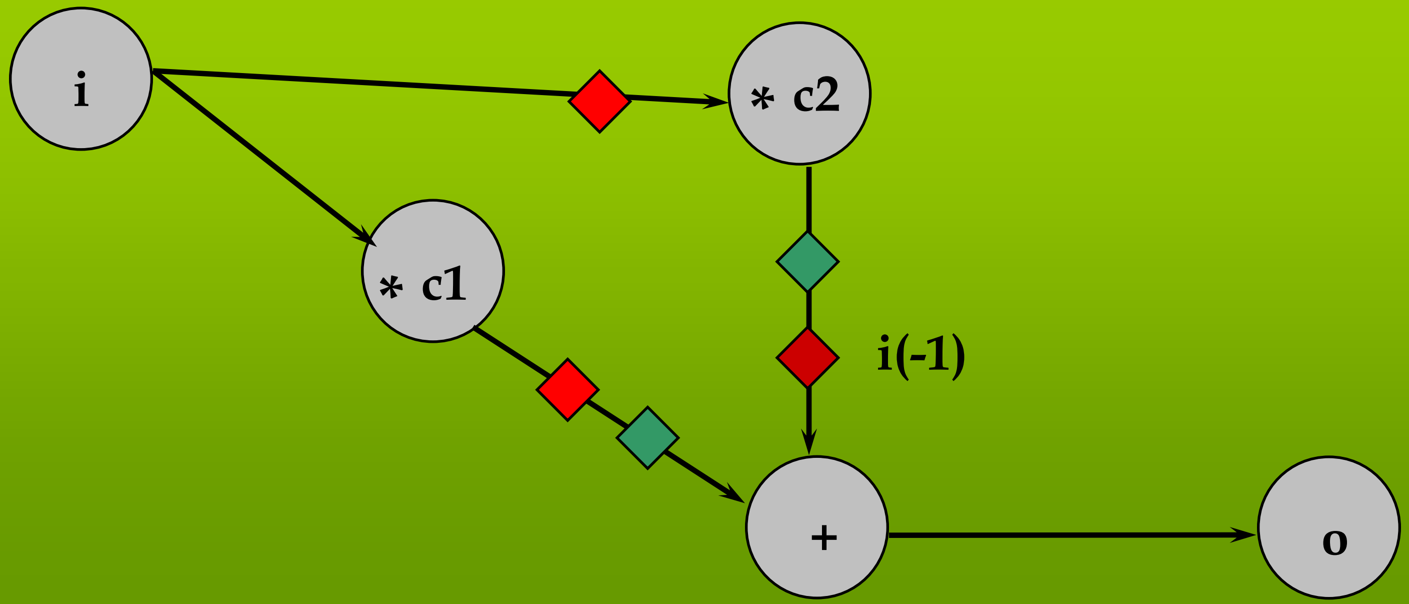

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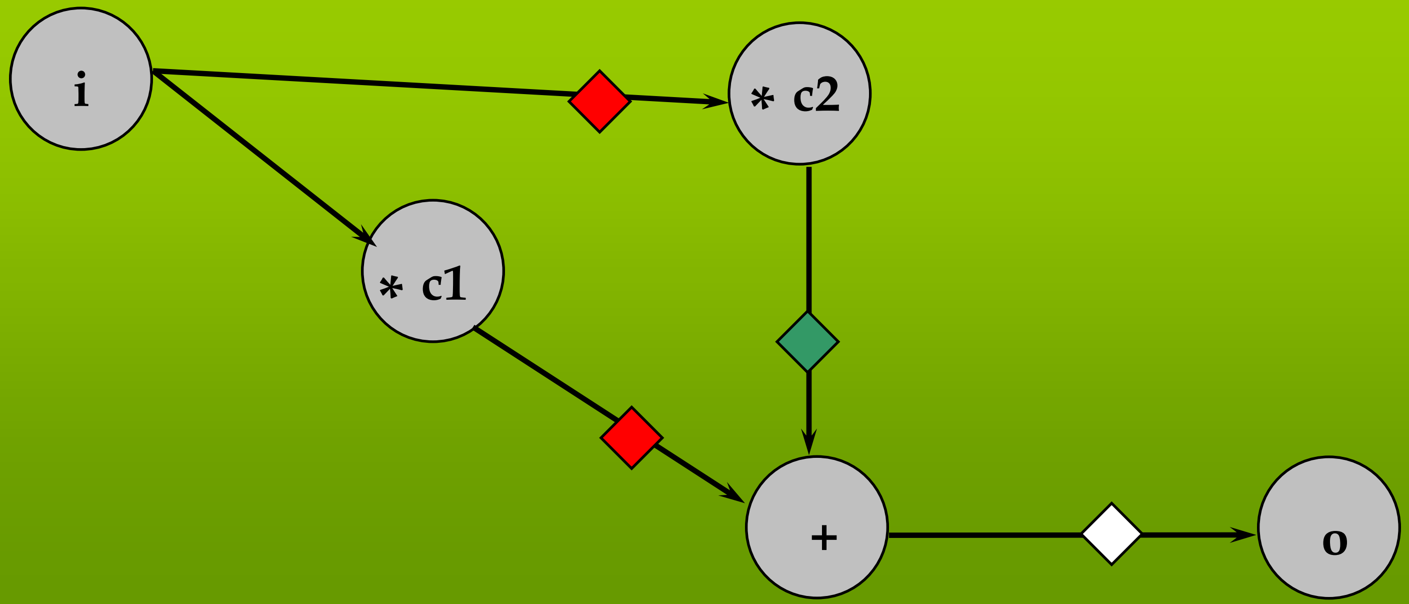

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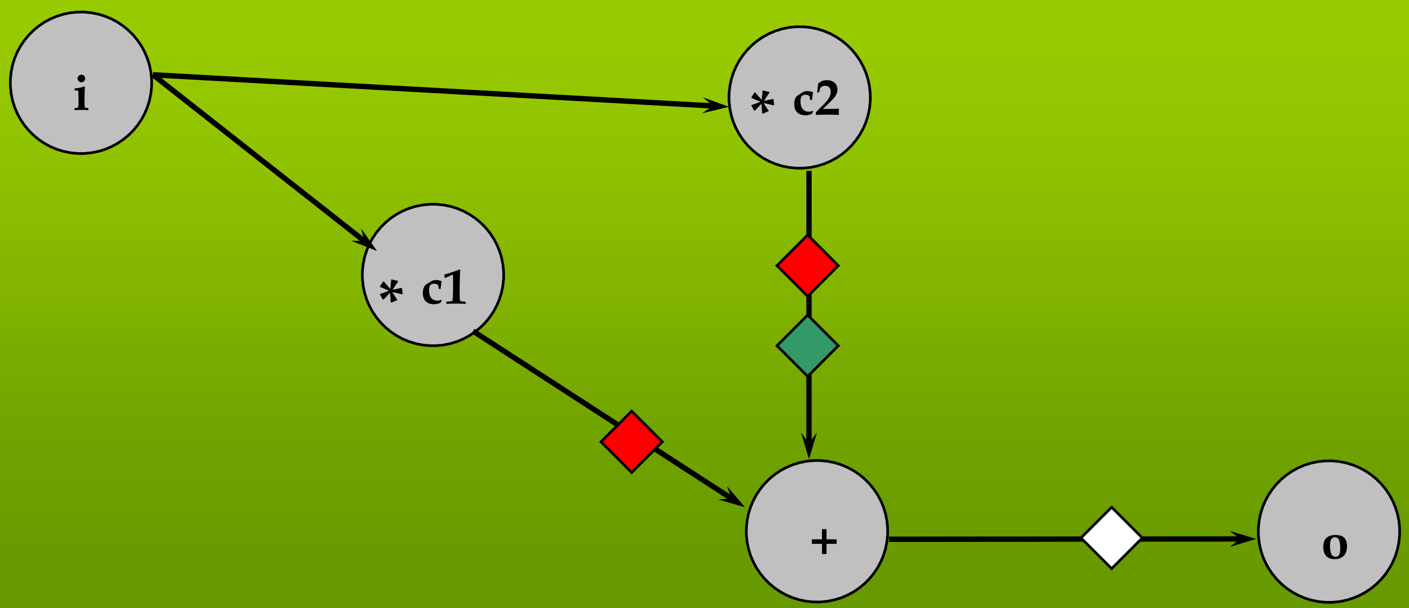

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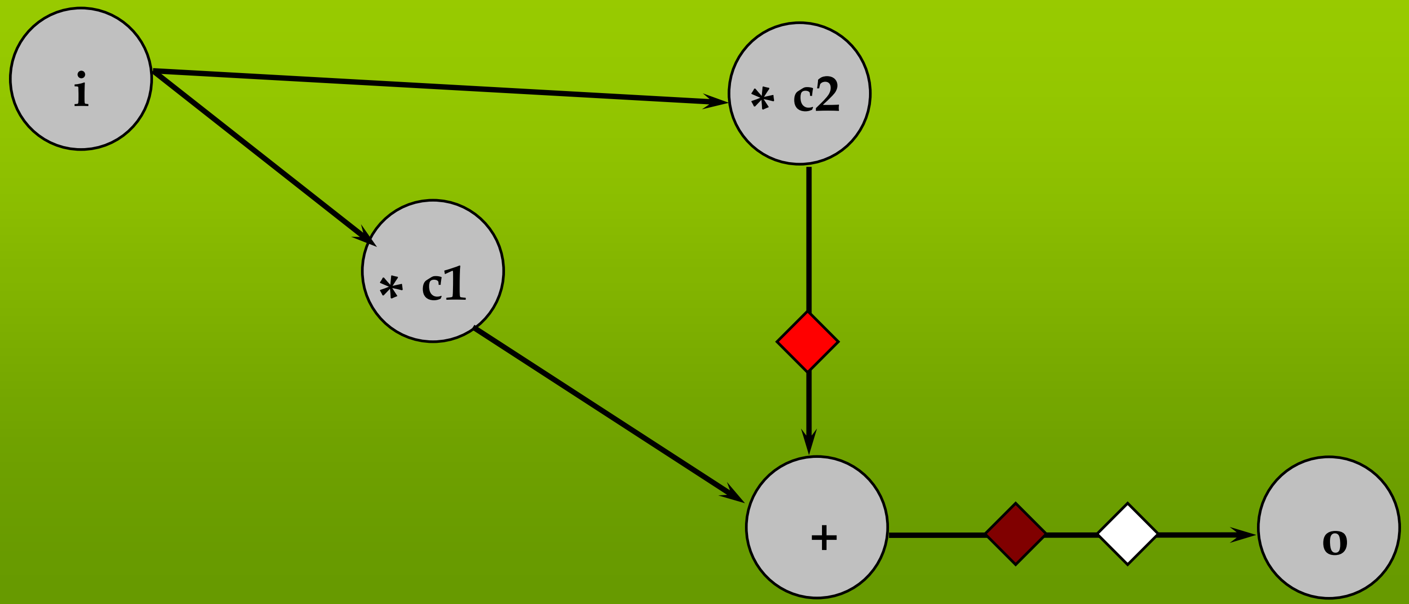

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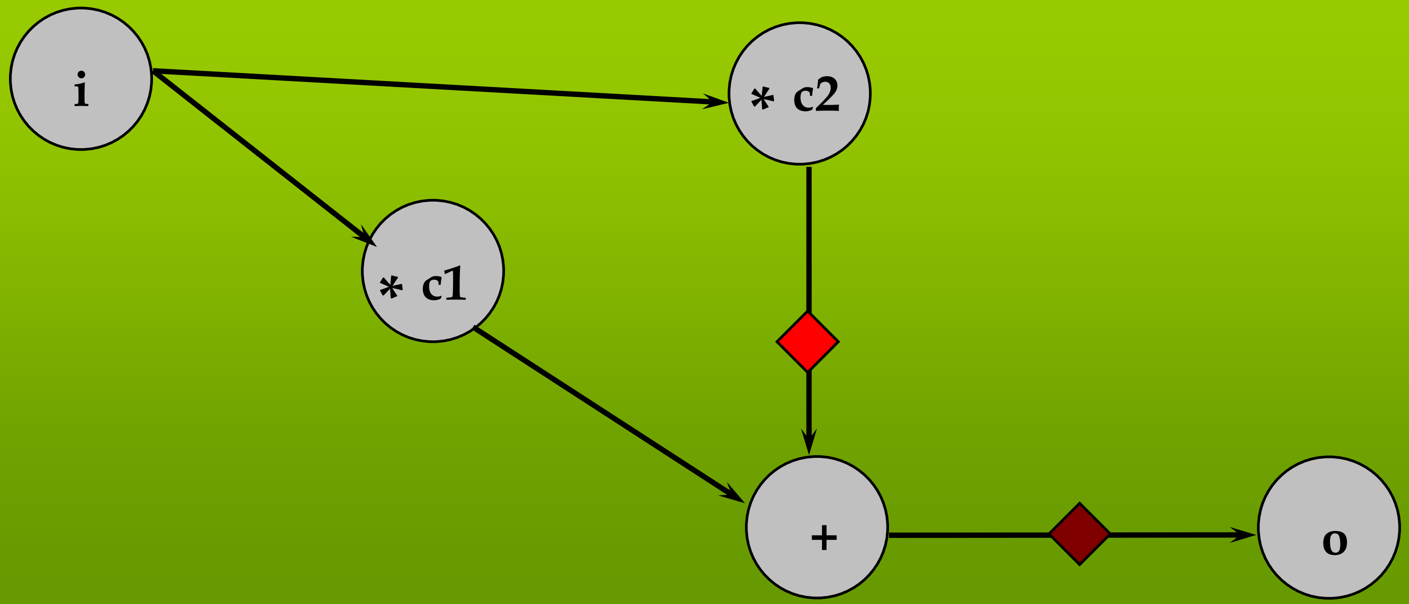


Questions

- Does the order in which actors are fired affect the final result?
- Does it affect the "operation" of the network in any way?
- Go to Radio Shack and ask for an unbounded queue!!

Formal semantics: sequences

- Actors operate from a sequence of input tokens to a sequence of output tokens
- Let tokens be noted by x_1, x_2, x_3, etc…
- A sequence of tokens is defined as

$$X = [\, x_1, x_2, x_3, \ldots]$$

- Over the execution of the network, each queue will grow a particular sequence of tokens
- In general, we consider the actors mathematically as functions from sequences to sequences (not from tokens to tokens)

Ordering of sequences

- Let X_1 and X_2 be two sequences of tokens.
- We say that X_1 is less than X_2 if and only if (by definition) X_1 is an initial segment of X_2
- Homework: prove that the relation so defined is a partial order (reflexive, antisymmetric and transitive)
- This is also called the prefix order
- Example: $[x_1, x_2] <= [x_1, x_2, x_3]$
- Example: $[x_1, x_2]$ and $[x_1, x_3, x_4]$ are incomparable

Chains of sequences

- Consider the set S of all finite and infinite sequences of tokens
- This set is partially ordered by the prefix order
- A subset C of S is called a chain iff all pairs of elements of C are comparable
- If C is a chain, then it must be a linear order inside S (otherwise, why call it chain?)
- Example: { [x_1], [x_1, x_2], [x_1, x_2, x_3], … } is a chain
- Example: { [x_1], [x_1, x_2], [x_1, x_3], … } is not a chain

(Least) Upper Bound

- Given a subset Y of S, an upper bound of Y is an element z of S such that z is larger than all elements of Y
- Consider now the set Z (subset of S) of all the upper bounds of Y
- If Z has a least element u, then u is called the least upper bound (lub) of Y
- The least upper bound, if it exists, is unique
- Note: u might not be in Y (if it is, then it is the largest value of Y)

Complete Partial Order

- Every chain in S has a least upper bound
- Because of this property, S is called a Complete Partial Order
- Notation: if C is a chain, we indicate the least upper bound of C by lub(C)
- Note: the least upper bound may be thought of as the limit of the chain

Processes

- Process: function from a p-tuple of sequences to a q-tuple of sequences

$$F : S^p \to S^q$$

- Tuples have the induced point-wise order:

$Y = (y_1, \ldots , y_p)$, $Y' = (y'_1, \ldots , y'_p)$ in S^p : $Y \le Y'$ iff $y_i \le y'_i$ for all $1 \le i \le p$

- Given a chain C in S^p, F(C) may or may not be a chain in S^q
- We are interested in conditions that make that true

Continuity and Monotonicity

- Continuity: F is continuous iff (by definition) for all chains C, lub(F(C)) exists and

  F(lub(C)) = lub(F(C))

- Similar to continuity in analysis using limits
- Monotonicity: F is monotonic iff (by definition) for all pairs X, X'

  X <= X' => F(X) <= F(X')

- Continuity implies monotonicity
  - intuitively, outputs cannot be "withdrawn" once they have been produced
  - timeless causality. F transforms chains into chains

Least Fixed Point semantics

- Let X be the set of all sequences
- A network is a mapping F from the sequences to the sequences

$$X = F(X, I)$$

- The behavior of the network is defined as the unique least fixed point of the equation
- If F is continuous then the least fixed point exists LFP = LUB($\{ F^n(\perp, I) : n >= 0 \}$)

From Kahn networks to Data Flow networks

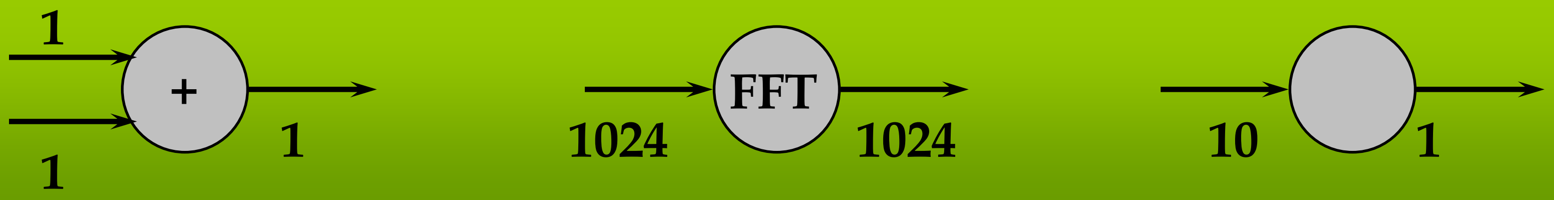
- Each process becomes an *actor*: set of pairs of
  - firing rule

    (number of required tokens on inputs)
  - function

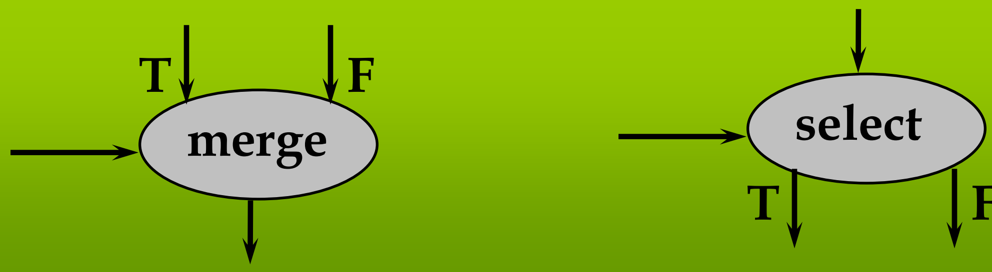
    (including number of consumed and produced tokens)
- Formally shown to be equivalent, but actors with firing are more intuitive
- *Mutually exclusive* firing rules imply monotonicity
- Generally simplified to *blocking read*

Examples of Data Flow actors

- SDF: Synchronous (or, better, Static) Data Flow
  - fixed input and output tokens



- BDF: Boolean Data Flow
  - control token determines consumed and produced tokens



Static scheduling of DF

- Key property of DF networks: output sequences do not depend on *time of firing* of actors
- SDF networks can be *statically scheduled* at compile-time
  - execute an actor when it is *known* to be fireable
  - no overhead due to sequencing of concurrency
  - static buffer sizing
- Different schedules yield different
  - code size
  - buffer size
  - pipeline utilization

Static scheduling of SDF

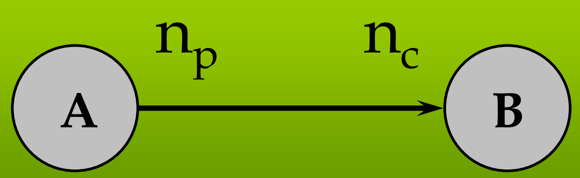
- Based only on *process graph* (ignores functionality)
- Network state: number of tokens in FIFOs
- Objective: find schedule that is *valid*, i.e.:
  - *admissible*

    (only fires actors when fireable)
  - *periodic*

    (brings network back to initial state firing each actor at least once)
- Optimize cost function over admissible schedules

Balance equations

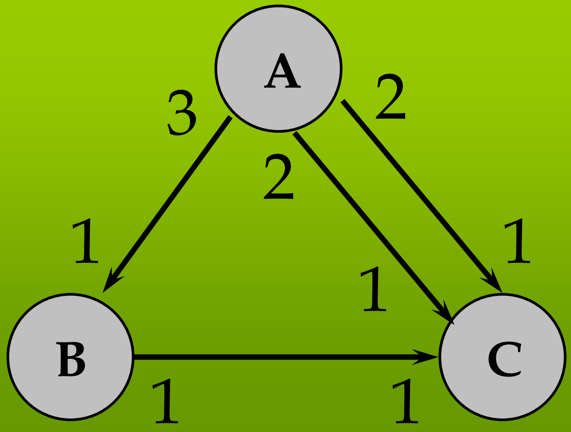
- Number of produced tokens must equal number of consumed tokens on every edge



- Repetitions (or firing) vector v_S of schedule S: number of firings of each actor in S
- $v_S(A)\, n_p = v_S(B)\, n_c$

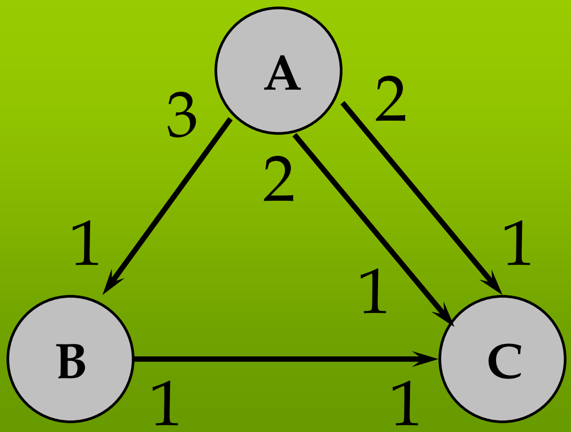
  must be satisfied for each edge

Balance equations



- Balance for each edge:
  - $3\ v_S(A) - v_S(B) = 0$
  - $v_S(B) - v_S(C) = 0$
  - $2\ v_S(A) - v_S(C) = 0$
  - $2\ v_S(A) - v_S(C) = 0$

Balance equations



$$M = \begin{vmatrix} 3 & -1 & 0 \\ 0 & 1 & -1 \\ 2 & 0 & -1 \\ 2 & 0 & -1 \end{vmatrix}$$

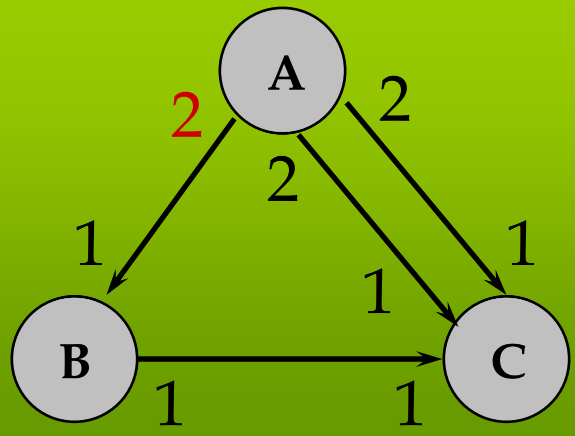
- $M\ v_S = 0$

  iff S is periodic

- Full rank (as in this case)
  - no non-zero solution
  - no periodic schedule

  (too many tokens accumulate on A->B or B->C)

Balance equations



$$M = \begin{vmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ 2 & 0 & -1 \\ 2 & 0 & -1 \end{vmatrix}$$

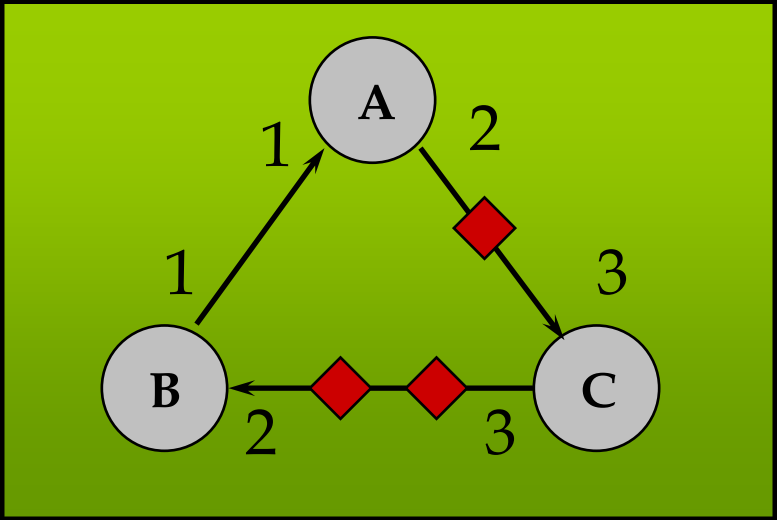
- Non-full rank
  - infinite solutions exist (linear space of dimension 1)
- Any multiple of $q = |1 \quad 2 \quad 2|^T$ satisfies the balance equations
- ABCBC and ABBCC are minimal valid schedules
- ABABBCBCCC is non-minimal valid schedule

Static SDF scheduling

- Main SDF scheduling theorem (Lee '86):
  - A connected SDF graph with *n* actors has a periodic schedule iff its topology matrix M has rank *n-1*
  - If M has rank *n-1* then there exists a unique smallest integer solution q to

    $M\ q = 0$
- Rank must be at least *n-1* because we need at least *n-1* edges (connected-ness), providing each a linearly independent row
- Admissibility is not guaranteed, and depends on initial tokens on *cycles*

Admissibility of schedules



- No admissible schedule:

  BACBA, then deadlock…

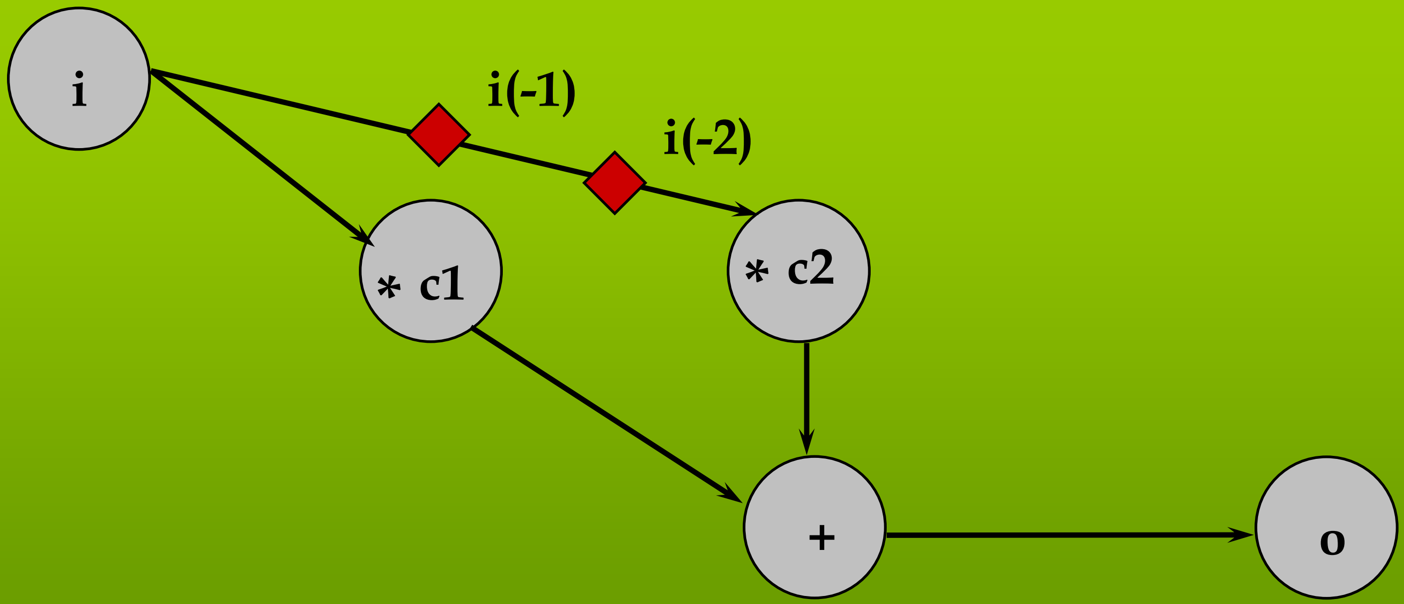
- Adding one token (delay) on A->C makes

  BACBACBA valid

- Making a periodic schedule admissible is always possible, but changes specification...

Admissibility of schedules

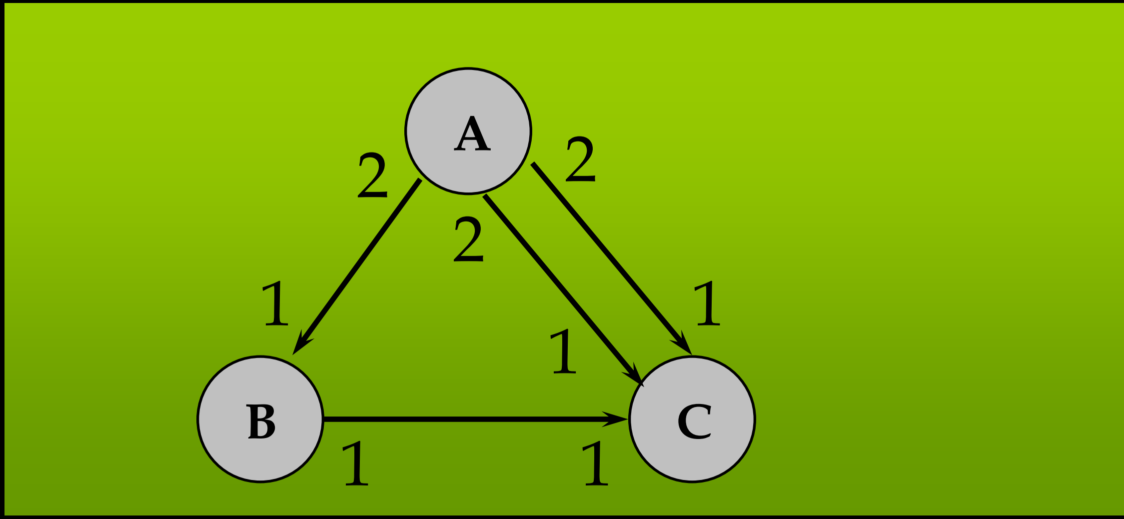
- Adding initial token changes FIR order



From repetition vector to schedule

- Repeatedly schedule fireable actors up to number of times in repetition vector

$q = |1 \quad 2 \quad 2|^T$



- Can find either ABCBC or ABBCC
- If deadlock before original state, no valid schedule exists (Lee '86)

From schedule to implementation

- Static scheduling used for:
  - behavioral simulation of DF (extremely efficient)
  - code generation for DSP
  - HW synthesis (Cathedral by IMEC, Lager by UCB, …)
- Issues in code generation
  - execution speed (pipelining, vectorization)
  - code size minimization
  - data memory size minimization (allocation to FIFOs)
  - processor or functional unit allocation

Compilation optimization

- Assumption: *code stitching*

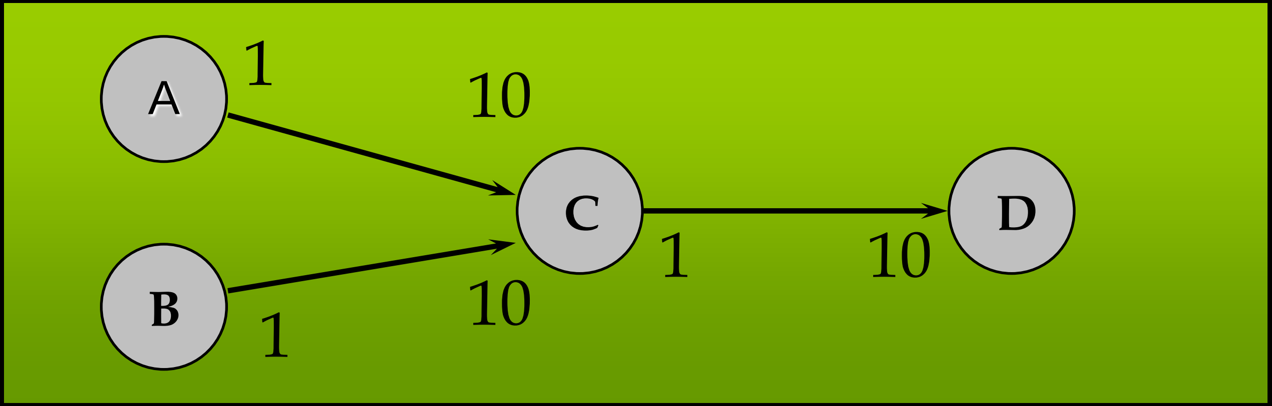
  (chaining custom code for each actor)
- More efficient than C compiler for DSP
- Comparable to hand-coding in some cases
- Explicit parallelism, no artificial control dependencies
- Main problem: memory and processor/FU allocation depends on scheduling, and vice-versa

Code size minimization

- Assumptions (based on DSP architecture):
  – subroutine calls expensive
  – fixed iteration loops are cheap
  ("zero-overhead loops")
- Absolute optimum: *single appearance schedule*
  e.g. ABCBC -> A (2BC), ABBCC -> A (2B) (2C)
  – may or may not exist for an SDF graph…
  – buffer minimization relative to single appearance schedules
  (Bhattacharyya '94, Lauwereins '96, Murthy '97)

Buffer size minimization

- Assumption: no buffer sharing
- Example:



$q = | 100 \;\; 100 \;\; 10 \;\; 1|^T$

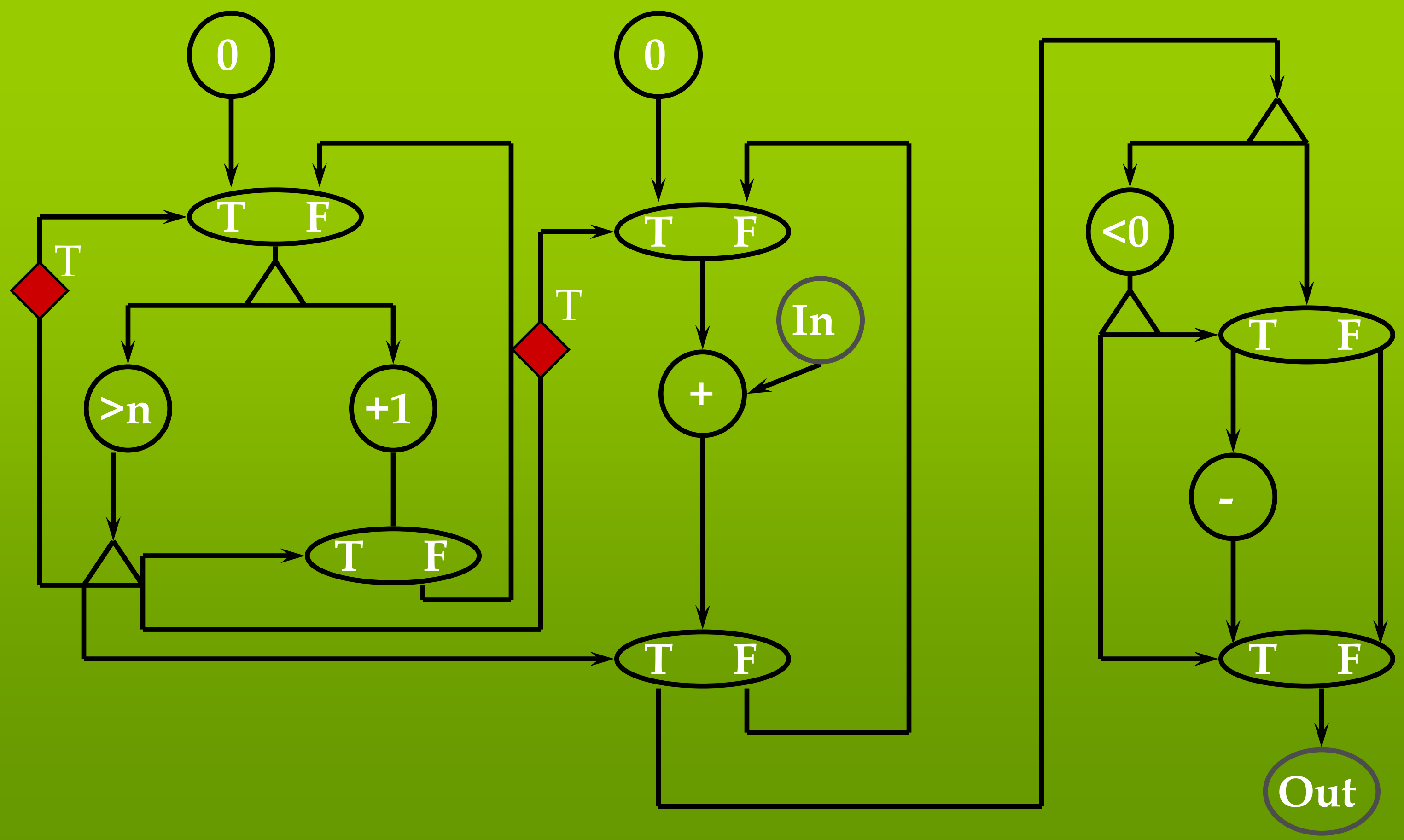
- Valid SAS: (100 A) (100 B) (10 C) D
  - requires 210 units of buffer area
- Better (factored) SAS: (10 (10 A) (10 B) C) D
  - requires 30 units of buffer areas, but…
  - requires 21 loop initiations per period (instead of 3)

Dynamic scheduling of DF

- SDF is limited in modeling power
  - no run-time choice
    - cannot implement Gaussian elimination with pivoting
- More general DF is too powerful
  - non-Static DF is Turing-complete (Buck '93)
    - bounded-memory scheduling is not always possible
- BDF: semi-static scheduling of special "patterns"
  - if-then-else
  - repeat-until, do-while
- General case: thread-based dynamic scheduling
  - (Parks '96: may not terminate, but never fails if feasible)

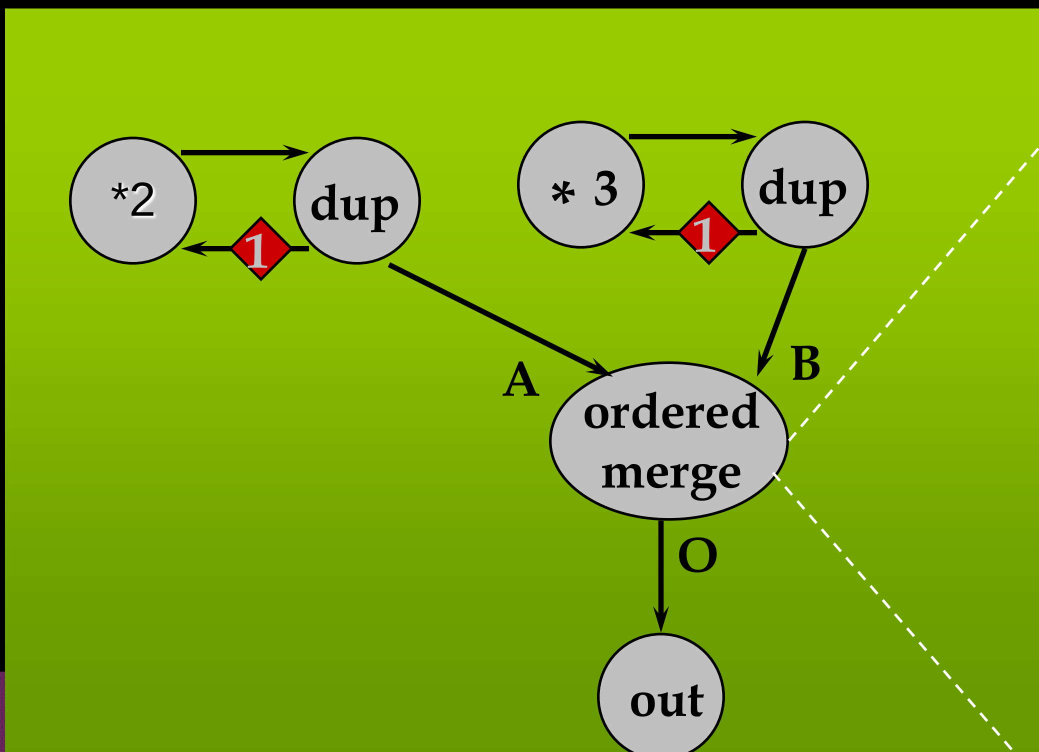
Example of Boolean DF

- Compute absolute value of average of *n* samples



Example of general DF

- Merge streams of multiples of 2 and 3 in order (removing duplicates)




```
a = get (A)
b = get (B)
forever {
    if (a > b) {
        put (O,  a)
        a = get (A)
    } else if (a < b) {
        put (O,  b)
        b = get (B)
    } else {
        put (O, a)
        a = get (A)
        b = get (B)
    }
}
```

- Deterministic merge

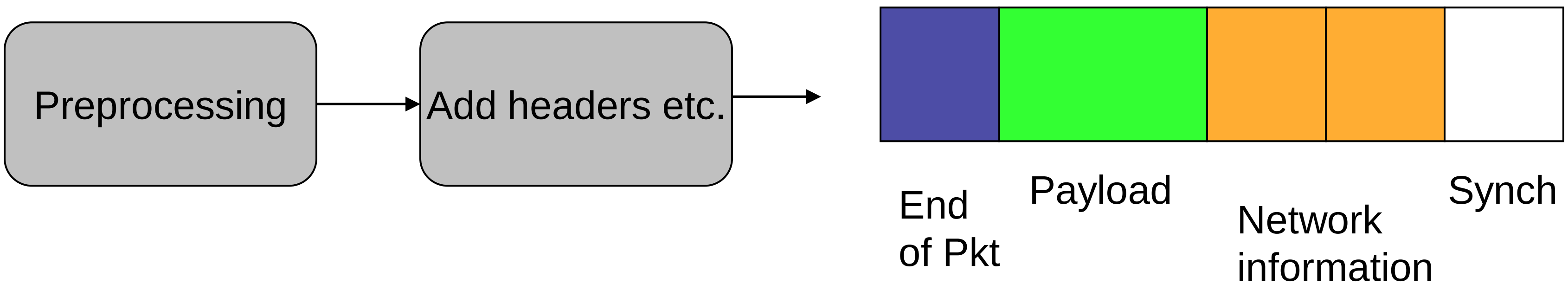
  (no “peeking”)

Summary of DF networks

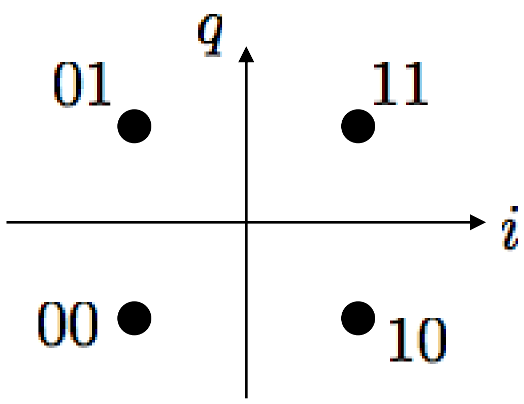
- Advantages:
  - Easy to use (graphical languages)
  - Powerful algorithms for
    - verification (fast behavioral simulation)
    - synthesis (scheduling and allocation)
  - Explicit concurrency
- Disadvantages:
  - Efficient synthesis only for restricted models
    - (no input or output choice)
  - Cannot describe reactive control (blocking read)

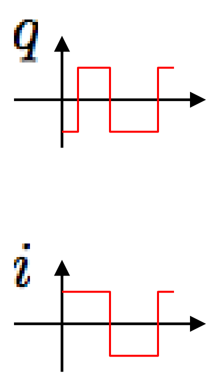
Base-band Processing in Cell Phones

Frame to transmit
(stream of bits)

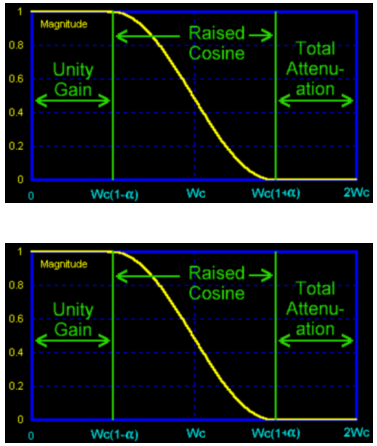


Mapping on a Constellation (QPSK)

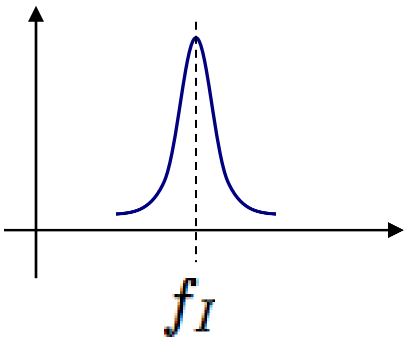




Filtering



Modulation



Base-band Processing: Denotation

Composition of functions = overall base-band specification

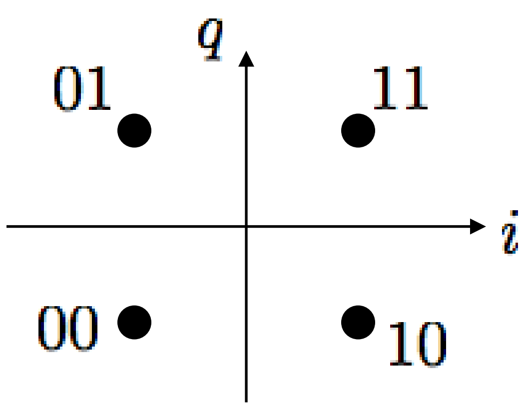
$$x[n] \quad = \quad (Map_i(s) * h)[n]\sin(2\pi f_I nT) + (Map_q(s) * h)[n]\cos(2\pi f_I nT)$$

$$\begin{aligned} i[n] &= Map_i(s[n]) \\ q[n] &= Map_q(s[n]) \end{aligned}$$

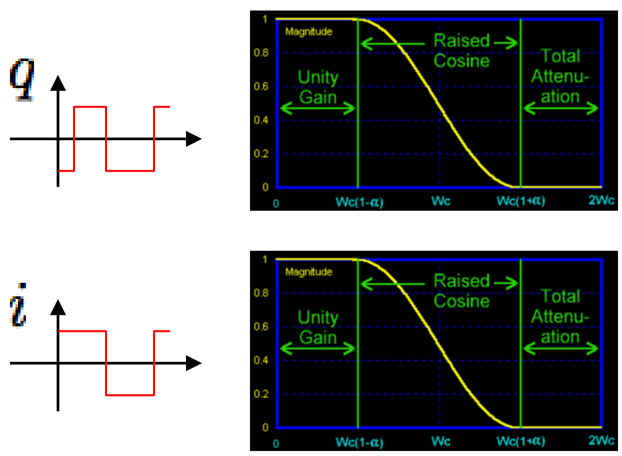
$$\begin{aligned} i_f[n] &= \sum_{k=1}^{N} h[k-1] i_f[n-k] \\ q_f[n] &= \sum_{k=1}^{N} h[k-1] q_f[n-k] \end{aligned}$$

$$\begin{aligned} x[n] &= i_f[n]\sin(2\pi f_I nT) + \\ &+ q_f[n]\cos(2\pi f_I nT) \end{aligned}$$

Mapping on a Constellation (QPSK)



Filtering



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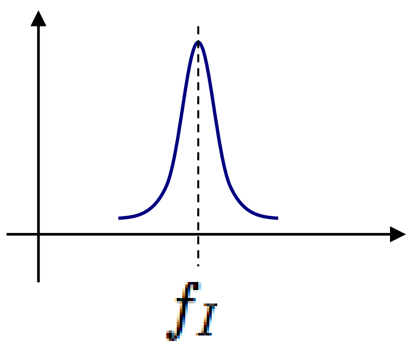
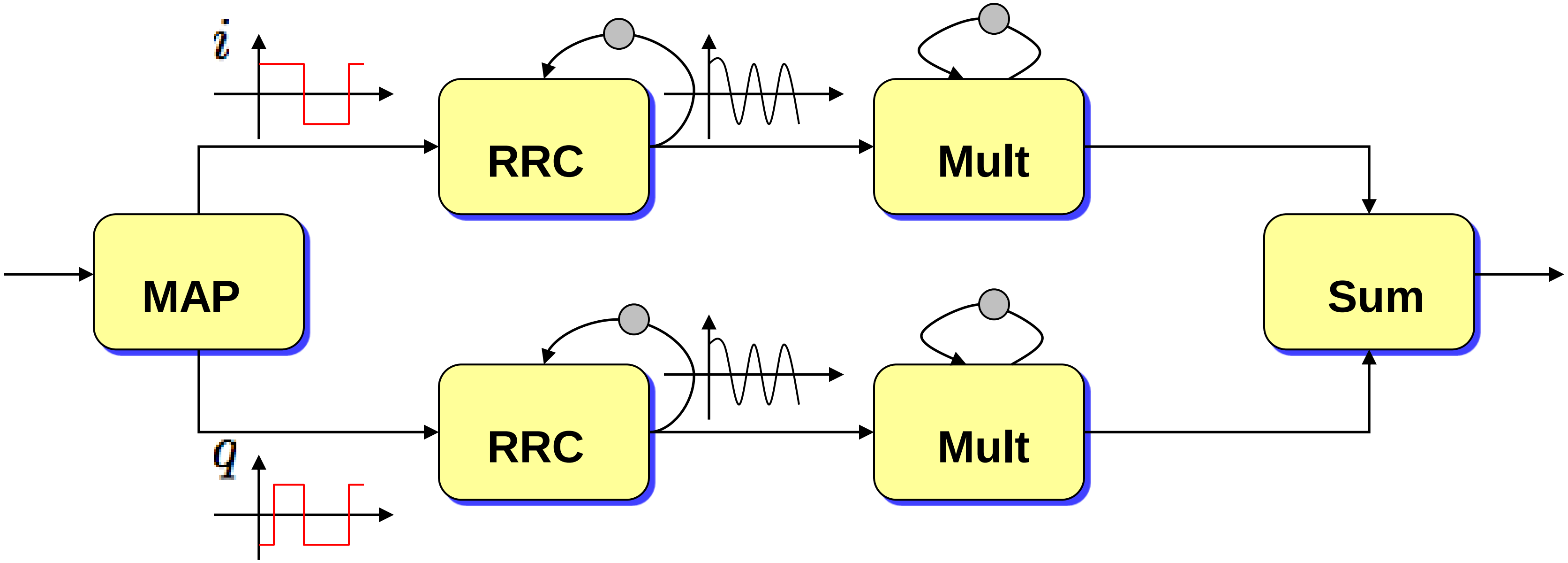
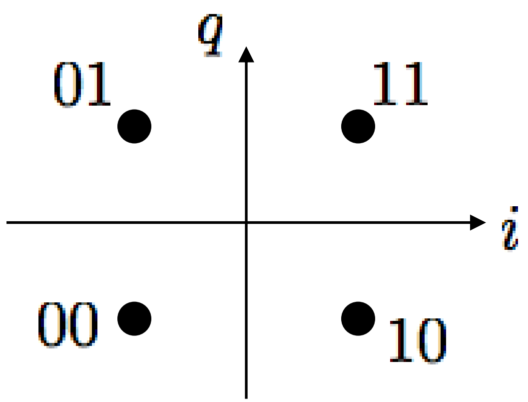


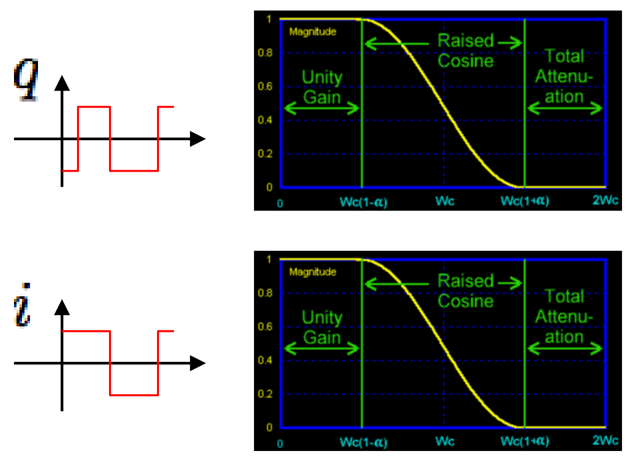
Base-band Processing: Data Flow Model



Mapping on a Constellation (QPSK)



Filtering



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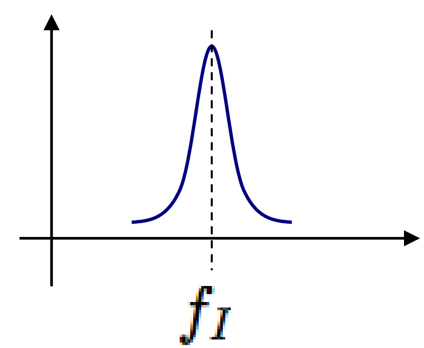


Remarks

- Composition is achieved by input-output connection through communication channels (FIFOs)
- The operational semantics dictates the conditions that must be satisfied to execute a function (actor)
- Functions operating on streams of data rather than states evolving in response to traces of events (data vs. control)
- Convenient to mix denotational and operational specifications

Telecom/MM applications

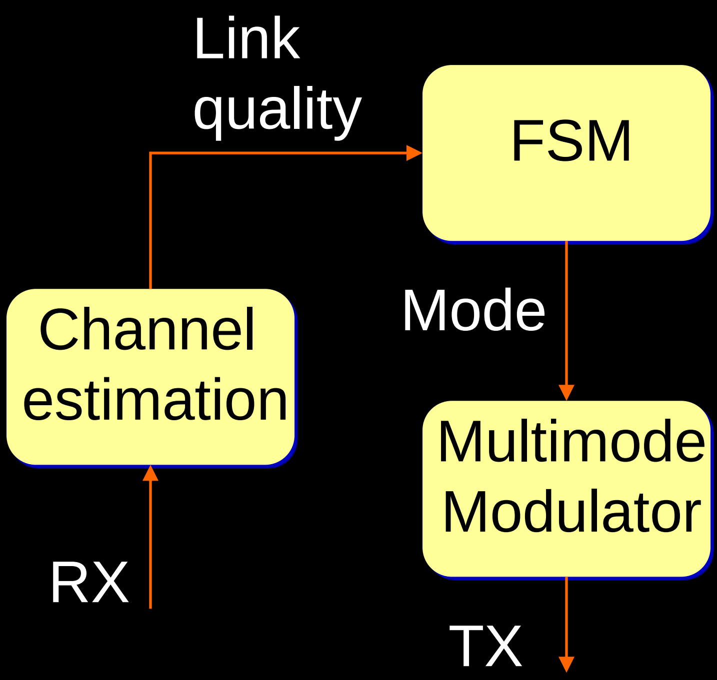
- Heterogeneous specifications including
  - data processing
  - control functions
- Data processing, e.g. encryption, error correction…
  - computations done at regular (often short) intervals
  - efficiently specified and synthesized using DataFlow models
- Control functions (data-dependent and real-time)
  - say when and how data computation is done
  - efficiently specified and synthesized using FSM models
- Need a common model to perform global system analysis and optimization

Mixing the two models: 802.11b

- State machine for control
  - Denotational: processes as sequence of events, sequential composition, choice etc.
  - Operational: state transition graphs
- Data Flow for signal processing
  - Functions
  - Data flow graphs
- And what happens when we put them together?

802.11b: Modes of operation

Data rate *(Mbit/s)*	Modulation	Coding rate	*Ndbps*	1472 byte transfer duration*(µs)*
6	BPSK	1/2	24	2012
9	BPSK	3/4	36	1344
12	QPSK	1/2	48	1008
18	QPSK	3/4	72	672
24	16-QAM	1/2	96	504
36	16-QAM	3/4	144	336
48	64-QAM	2/3	192	252
54	64-QAM	3/4	216	224



- Depending on the channel conditions, the modulation scheme changes
- It is natural to mix FSM and DF (like in figure)
- Note that now we have real-time constraints on this system (i.e. time to send 1472 bytes)

Outline

- Part 3: Models of Computation
  - FSMs
  - Discrete Event Systems
  - CFSMs
  - Data Flow Models
  - Petri Nets
  - The Tagged Signal Model