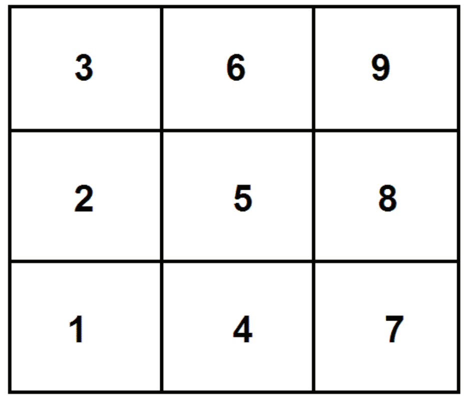

Figure 4.1: Playing Grid for a Cat and Mouse Game.

4.6 Exercises

1. Consider a game between a robotic cat C and a robotic mouse M on a 3×3 grid. Assume that the squares of the grid are numbered 1 to 9, as shown in Figure 4.1.

   The state of the combined cat-and-mouse system, at any time step, is a pair (p_C, p_M) where p_C and p_M are respectively the numbers of the square in which C and M are positioned.

   At any step, C and M can either stay in their current square or go to an adjacent square. Note that C and M make their move simultaneously.

   C wins if a state in which $p_C = p_M$ is reached. In temporal logic, we state C's objective as $\mathbf{F}(p_C = p_M)$. Obviously, M's objective is $\neg\mathbf{F}(p_C = p_M)$, which is $\mathbf{G}(p_C \neq p_M)$.

   Recall that a *controllable state for C* is one from which C is guaranteed to win, no matter what sequence of moves M performs. We similarly define a controllable state for M.

   Suppose that initially $p_C = 1$ (bottom left square) and $p_M = 9$ (top right square). I.e., the initial state is $(1, 9)$.

   Does C have a winning strategy? If so, formally describe this strategy. If not, show that C does not have a winning strategy by proving formally that the initial state is not controllable for C.

   [Hint: begin computing the set of controllable states for C and reason whether this set can contain the initial state.]