

Introduction to Embedded Systems

Edward A. Lee & Sanjit A. Seshia

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EECS 149
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Lecture 11: Simulation of Discrete-Event Systems

Material drawn from book by Banks et al., notes by M. Harchol-Balter

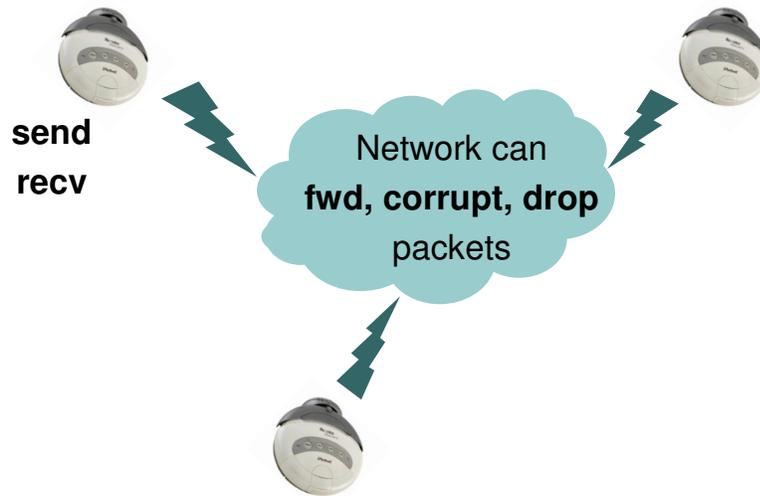
Discrete-Event System

A dynamical system whose evolution is governed by the *occurrence of events at discrete time points*, at possibly irregularly-spaced intervals (Informal defn)

Many cyber-physical systems are modeled as discrete-event systems:

- Communication networks
- Microprocessors
- Manufacturing facilities
- Communicating robots

Example: Communicating Robots/Sensor Nodes



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This Lecture

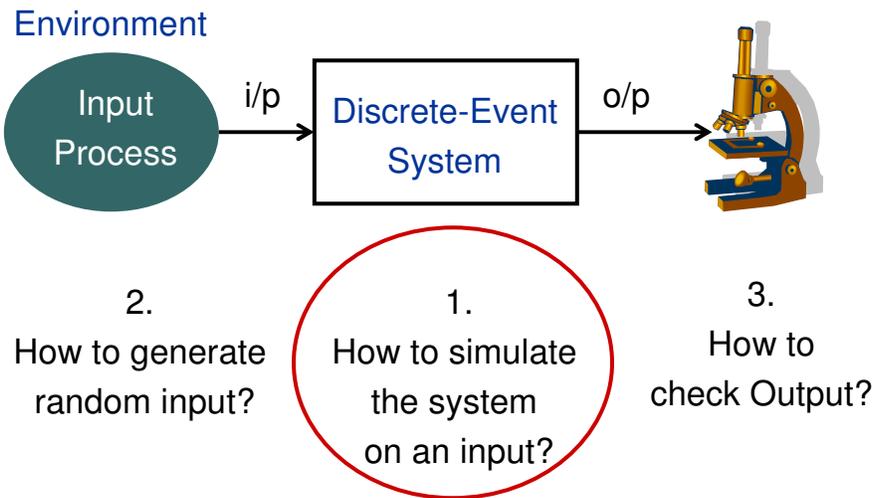
How to build a simulator for a discrete-event system
– the basics

Examples of such simulators:

- ns-2 (for simulating computer networks)
- ModelSim (for simulating digital circuit designs)

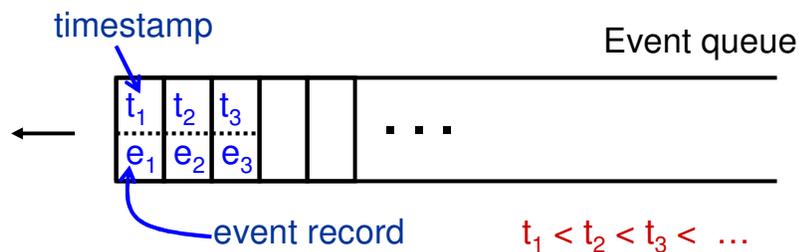
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Simulating a Discrete-Event System (DES)



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Simulating the System with an Event Queue



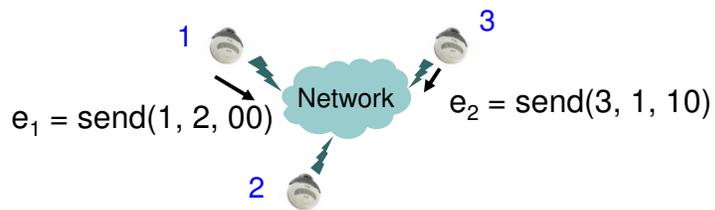
Simulation Timer, $T = 0$

Repeat while there are events in the event queue:

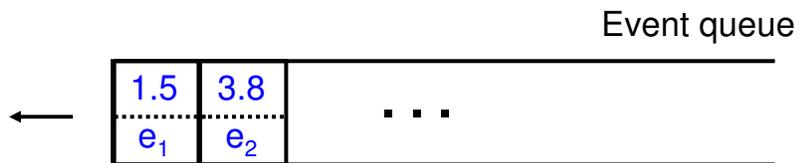
1. Dequeue event at head of queue ("imminent event")
2. Advance simulation timer to time of imminent event
3. Execute imminent event: update system state
4. Generate future events and enqueue them

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Example of Simulation with Event Queue

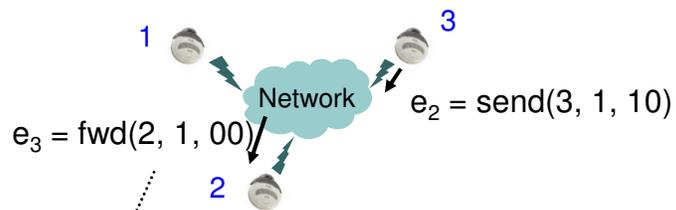


$T = 0$

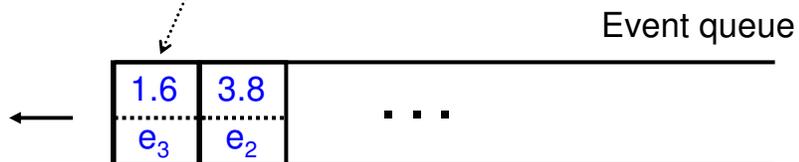


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Example of Simulation with Event Queue

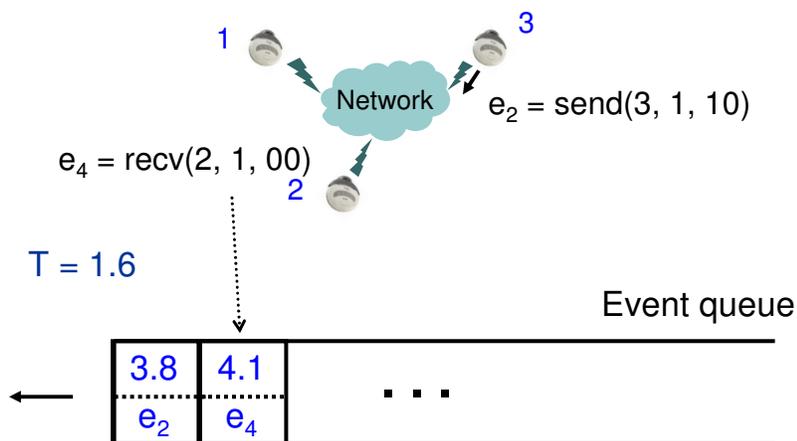


$T = 1.5$



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Example of Simulation with Event Queue



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Implementing the Event Queue

- Event with smallest time-stamp must be dequeued
- New events must be inserted into sorted order according to their timestamps
- Efficient Data Structure: Priority Queue
- Particular version: Calendar Queue
[R. Brown, Comm. of the ACM, 1988, vol. 31(10)]

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Simulating a Discrete-Event System (DES)

Environment



2.
How to generate
random input?

1.
How to simulate
the system?

3.
How to
check Output?

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Generating Random Inputs

Suppose we have an input signal taking values in a set of "events": $\{e_1, e_2, e_3, \dots, e_n\}$

Suppose event e_i occurs with probability p_i

$$\sum_i p_i = 1$$

How do we generate events randomly?

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Inverse Transform Method

- Generate u in $[0,1)$ – uniformly at random
 - using your programming environment's built-in pseudo-random number generator
 - e.g. `drand48()` in C
- Set $p_0 = 0$. Add up p_i 's until we get to
$$\sum_{i=0}^j p_i \leq u < \sum_{i=0}^{j+1} p_i$$

Is this an efficient procedure?
- Generate input event e_{j+1}

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Analysis of Inverse Transform Method

Inefficient if n is large

Why?

Because we need to compute many partial sums $\sum_i p_i$ in order to figure out where u lies

- worst case: $n-1$ such sums

There's one special case where sampling from the p_i 's is easy: what is it?

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Solution: The Accept/Reject Method

Easy to generate events uniformly at random efficiently

But in general, we have an arbitrary probability mass function p_1, p_2, \dots, p_n .

How can we leverage the ease of generating uniformly at random to generate according to the p_i 's?

→ The accept/reject method gives us a way to do this

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General Setup

Given:

Efficient method for sampling from n events according to p.m.f. $\{q_1, q_2, \dots, q_n\}$

- e.g. the pmf is uniform random, $q_i = 1/n$ for all i

Need:

Efficient method for sampling from same n events but according to non-uniform p.m.f. $\{p_1, p_2, \dots, p_n\}$

Constraint: $\forall i, q_i > 0$ iff $p_i > 0$

Any ideas on how to do this?

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Idea #1 for Accept/Reject Method

Do two steps:

1. Select index i randomly according to easy distribution $\{q_1, q_2, \dots, q_n\}$
2. Then *accept* the index i with probability p_i
 - generate event e_i
 - otherwise go back to step 1

How do we implement this?

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Idea #1 for Accept/Reject Method

Do two steps:

1. Select index i randomly according to easy distribution $\{q_1, q_2, \dots, q_n\}$
2. Then *accept* the index i with probability p_i
 - generate event e_i

Two questions:

1. Is this correct? **YES, if each $q_i = 1/n$**
 - Are we really generating e_i according to p_1, p_2, \dots, p_n ?
2. Is this efficient?
 - What is the expected #trials before we generate e_i ?

On the order of n

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Idea #2 for Accept/Reject Method

Do three steps:

1. Initially: select c such that $p_i/q_i \leq c \forall i$ s.t. $p_i > 0$
2. Select index i randomly according to easy distribution $\{q_1, q_2, \dots, q_n\}$
3. Then *accept* the index i with probability $p_i/(c q_i)$
 - generate event e_i

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Idea #2 for Accept/Reject Method

Do three steps:

1. Initially: select c such that $p_i/q_i \leq c \forall i$ s.t. $p_i > 0$
2. Select index i randomly according to easy distribution $\{q_1, q_2, \dots, q_n\}$
3. Then *accept* the index i with probability $p_i/(c q_i)$
 - generate event e_i

Claim: This method is

- Correct: $\Pr(e_j \text{ is generated}) = p_j$
- Efficient: Expected #trials = c

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Accept/Reject works the same way for Continuous Random Variables, too!

Given: How to generate Y with “easy” probability density function (pdf) $f_Y(t)$

Need: To generate X with pdf $f_X(t)$

Constraint: $\forall t, f_Y(t) > 0$ iff $f_X(t) > 0$

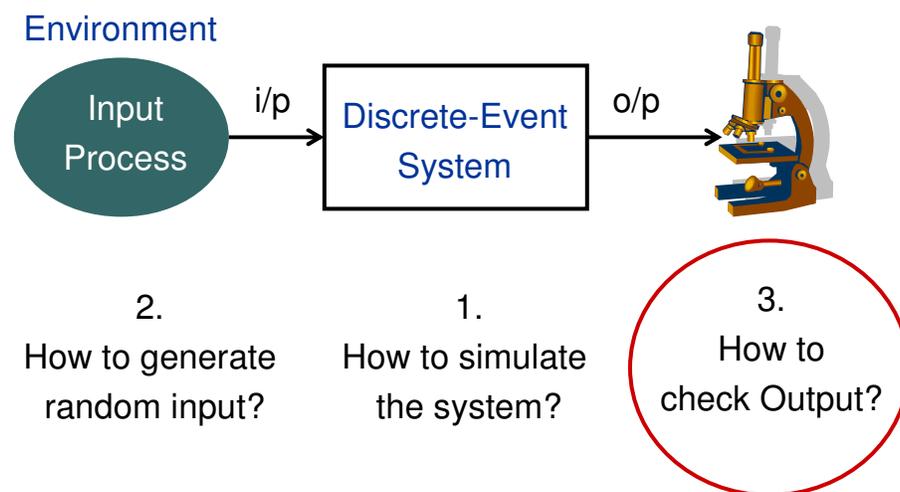
Algorithm:

1. Pick constant c such that $f_X(t) \leq c f_Y(t) \quad \forall t$ s.t. $f_Y(t) > 0$
2. Sample t according to f_Y
3. Accept $X = t$ with probability $f_X(t) / [c f_Y(t)]$

Useful for generating inter-arrival times of events

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Simulating a Discrete-Event System (DES)

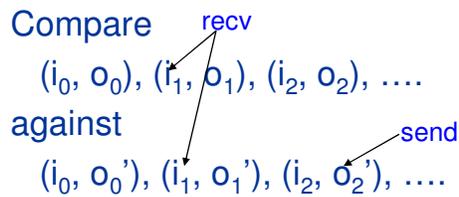


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How to check the output

Need to compare the **observed** input-output sequence with the **expected** input-output sequence

Compare $(i_0, o_0), (i_1, o_1), (i_2, o_2), \dots$
against $(i_0, o_0'), (i_1, o_1'), (i_2, o_2'), \dots$



Note: can only compare finite-length sequences!

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How to check the output

Synthesize an FSM that “observes” the output of simulation and enters an “ERROR” state iff the expected output differs from observed output

Formally, all input-output sequences for which the FSM enters the ERROR state are incorrect behaviors, and all other sequences are considered correct system behaviors.

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