



# Introduction to Embedded Systems

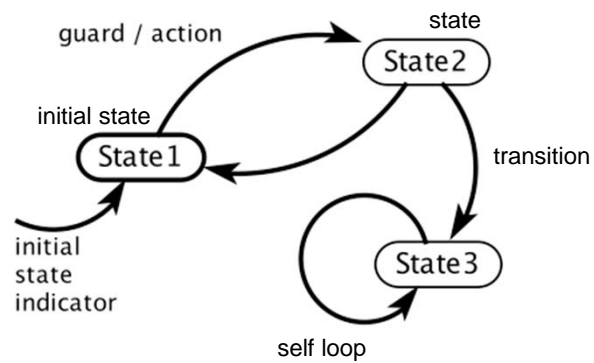
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UC Berkeley  
EECS 149/249A  
Fall 2015

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Chapter 3, 4: Extended Finite State Machines, Timed Automata, Hybrid Automata

## Recall FSM Notation

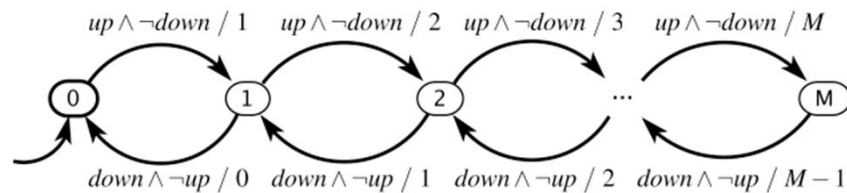


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## Garage Counter Example

Recall this example, which counts cars in a parking garage:

inputs:  $up, down \in \{present, absent\}$   
 output  $\in \{0, \dots, M\}$

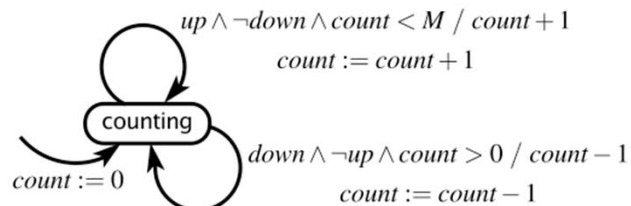


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## Extended State Machines

Extended state machines augment the FSM model with *variables* that may be read or written. E.g.:

variable:  $count \in \{0, \dots, M\}$   
 inputs:  $up, down \in \{present, absent\}$   
 output  $\in \{0, \dots, M\}$

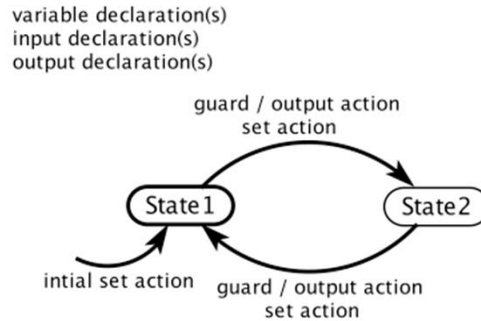


Question: What is the size of the state space?

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## General Notation for Extended State Machines

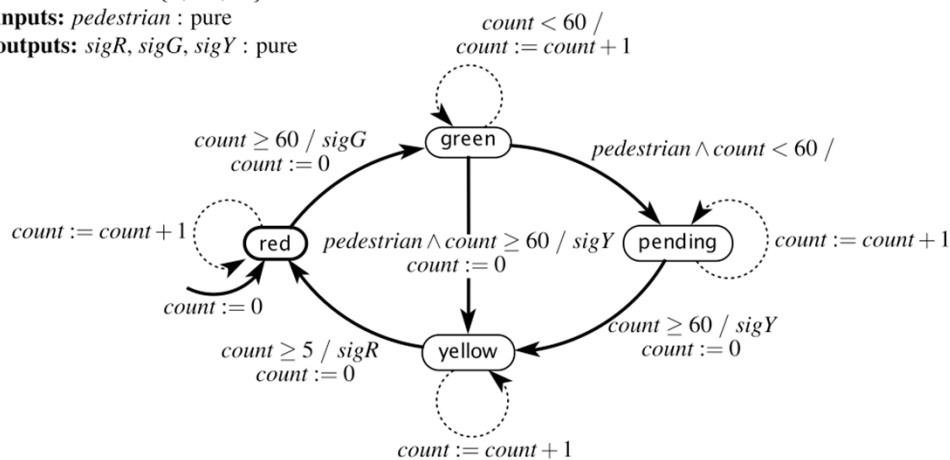
We make explicit declarations of variables, inputs, and outputs to help distinguish the three.



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## Extended state machine model of a traffic light controller at a pedestrian crossing:

**variable:**  $count: \{0, \dots, 60\}$   
**inputs:**  $pedestrian: pure$   
**outputs:**  $sigR, sigG, sigY: pure$



This model assumes one reaction per second  
(a *time-triggered* model)

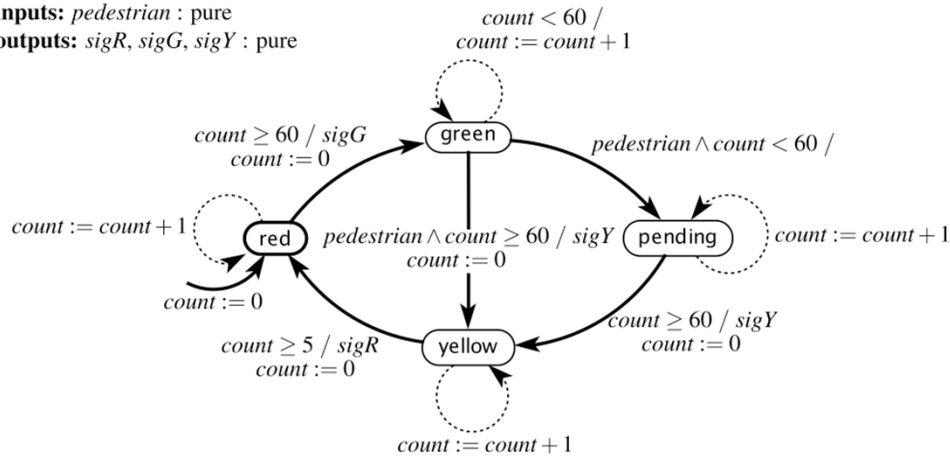
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## Quiz: What is the Size of the State Space for the Traffic Light Controller?

variable:  $count: \{0, \dots, 60\}$

inputs:  $pedestrian$  : pure

outputs:  $sigR, sigG, sigY$  : pure

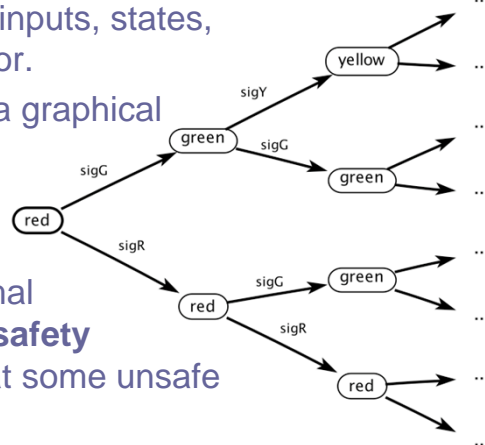


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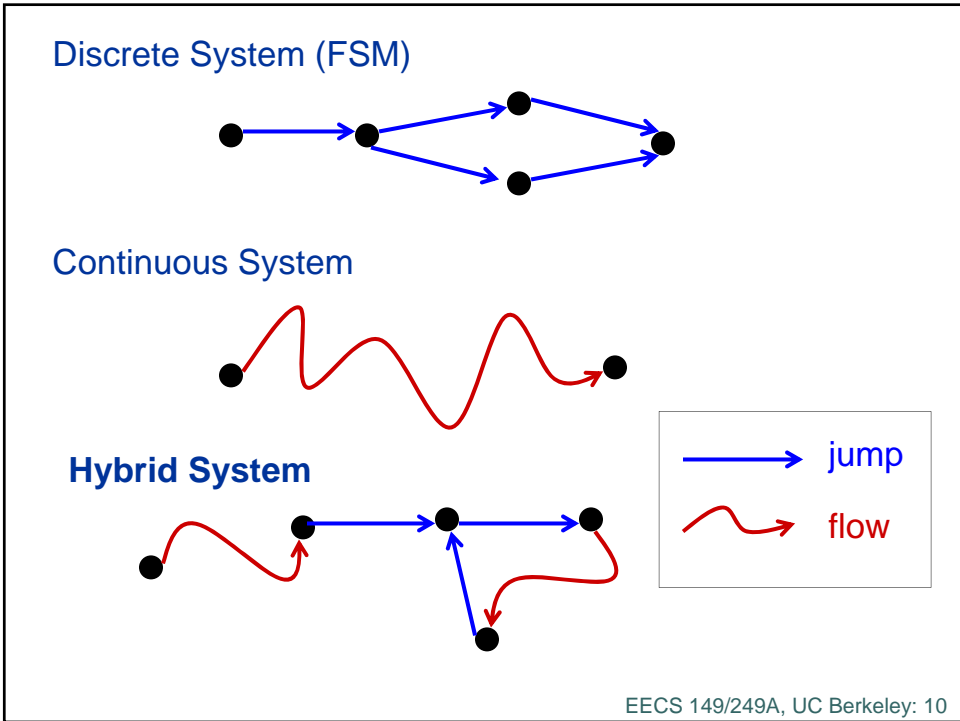
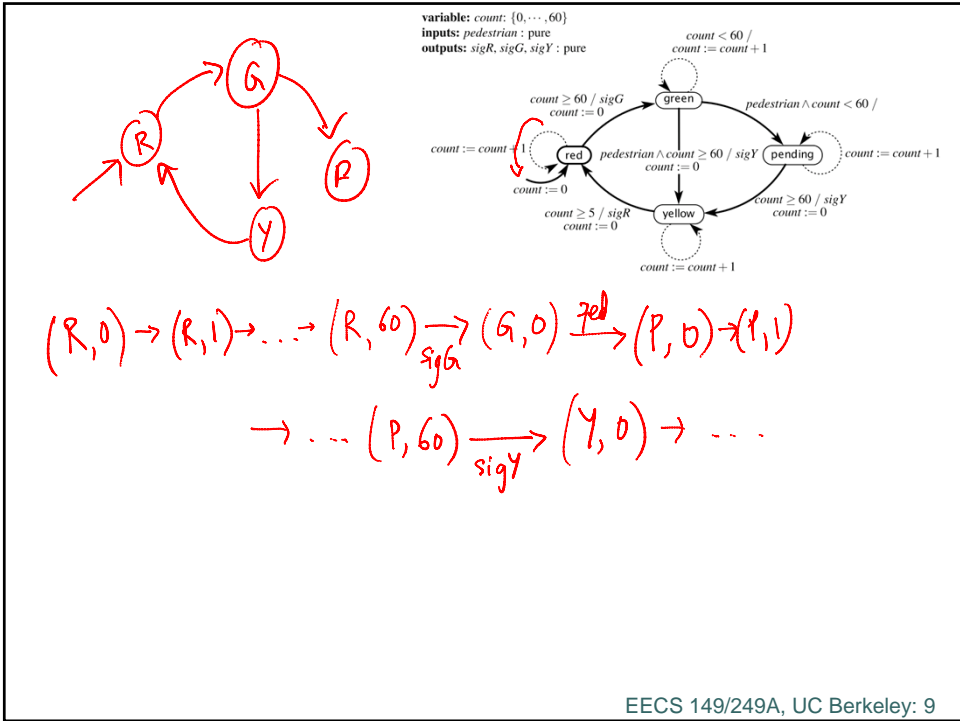
## Behaviors and Traces

- FSM **behavior** is a sequence of (non-stuttering) steps.
- A **trace** is the record of inputs, states, and outputs in a behavior.
- A **computation tree** is a graphical representation of all possible traces.

FSMs are suitable for formal analysis. For example, **safety** analysis might show that some unsafe state is not reachable.



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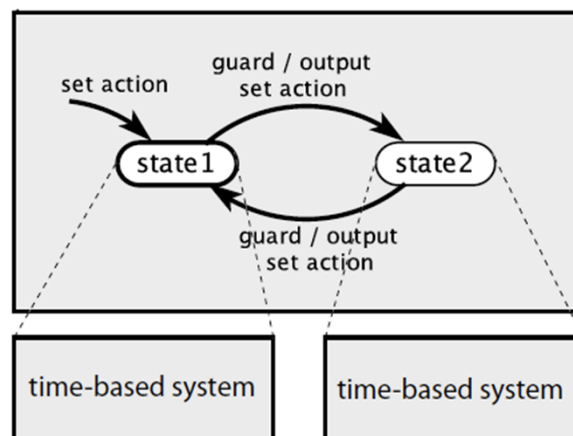


## Where do Hybrid Systems arise?

- ❑ Digital controller of physical “plant”
  - thermostat
  - intelligent cruise/powertrain control in cars
  - aircraft auto pilot
- ❑ Phased operation of natural phenomena
  - bouncing ball
  - biological cell growth
- ❑ Multi-agent systems
  - ground and air transportation systems
  - interacting robots

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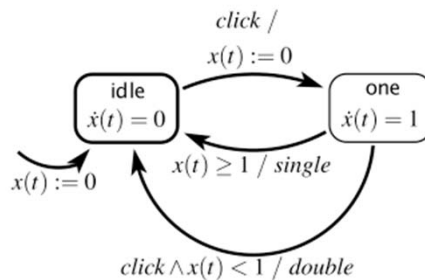
An alternative to FSMs that is explicit about the passage of time: **Timed automata**, a special case of hybrid systems.



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## Example: Mouse Double Click Detector

continuous variable:  $x(t) \in \mathbb{R}$   
 inputs:  $click \in \{present, absent\}$   
 outputs:  $single, double \in \{present, absent\}$

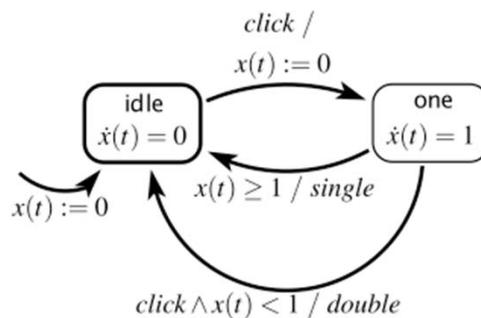


This simple form of hybrid system is called a timed automaton, where the dynamics is just passage of time.

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## Quiz: How many states does this automaton have?

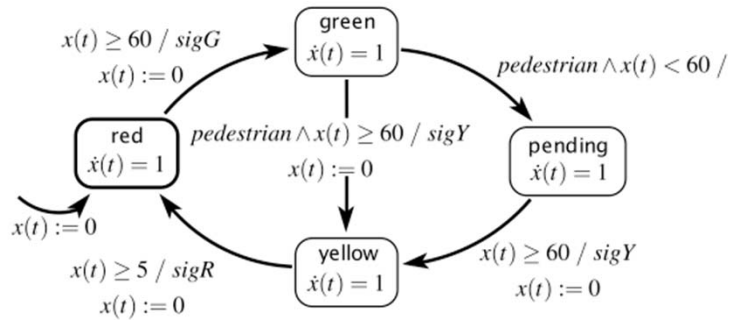
continuous variable:  $x(t) \in \mathbb{R}$   
 inputs:  $click \in \{present, absent\}$   
 outputs:  $single, double \in \{present, absent\}$



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## Timed automaton model of a traffic light controller

**continuous variable:**  $x(t) : \mathbb{R}$   
**inputs:** *pedestrian*: pure  
**outputs:** *sigR, sigG, sigY*: pure

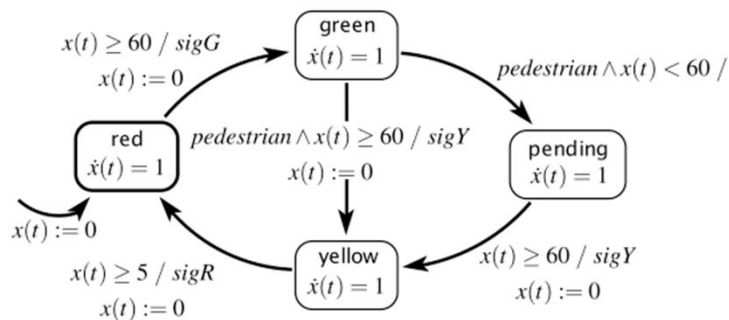


This light remains green at least 60 seconds, and then turns yellow if a pedestrian has requested a crossing. It then remains red for 60 seconds.

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## When do reactions occur in a hybrid automaton?

**continuous variable:**  $x(t) : \mathbb{R}$   
**inputs:** *pedestrian*: pure  
**outputs:** *sigR, sigG, sigY*: pure



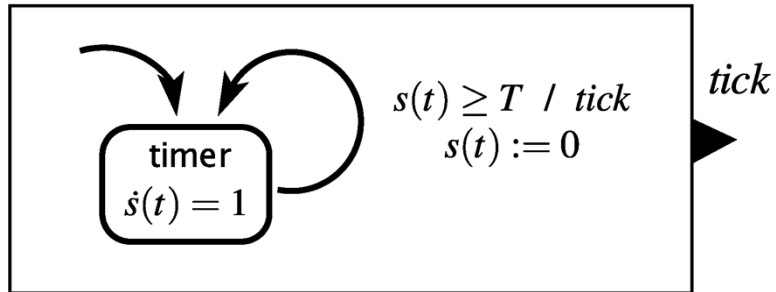
Reactions are occurring continually, with the continuous state variable  $x$  being continually updated.

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## Example: "Tick" Generator (Timer)

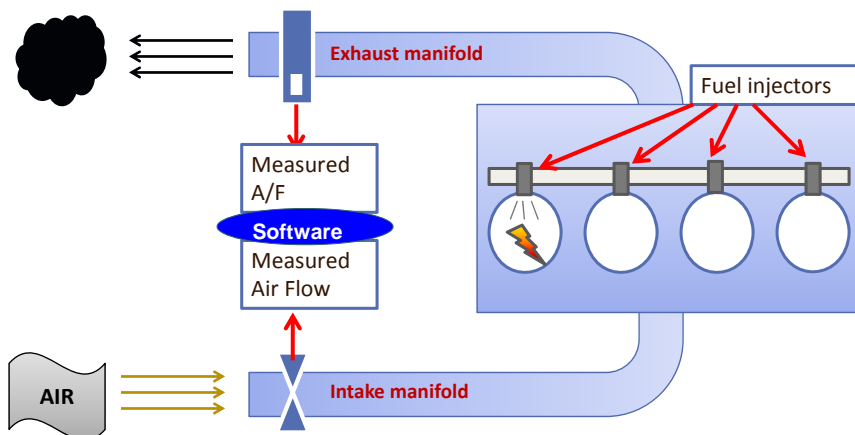
*How would you model a timer that generates a 'tick' each time  $T$  time units elapses?*



A similar timed automaton can model a generator of a timer interrupt.

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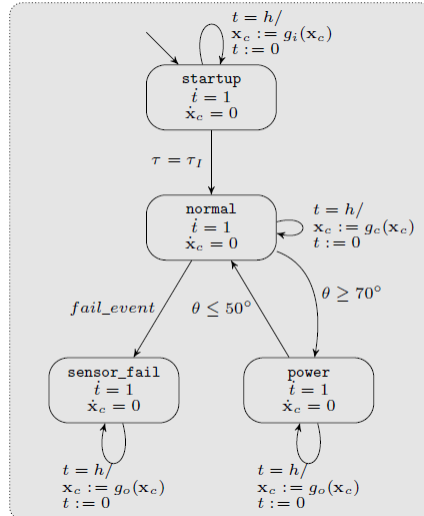
## Hybrid Automaton Model of Toyota Powertrain Control Example



[Slide due to J. Deshmukh, Toyota]

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## Hybrid Automaton Model of Toyota Powertrain Control Example



### Four Operating Modes:

1. **Startup**: Wait for O2 sensors to start giving accurate readings (temp dependent), employ open-loop control
2. **Normal**: Use combination of feedback PI control and feedforward control to regulate A/F ratio
3. **Power**: Driver depresses gas pedal more (higher throttle angle) – switch to feedforward
4. **Sensor Failure**: switch to feedforward control

"Powertrain Control Verification Benchmark", Jin et al., HSCC 2014

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