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Center For Hybrid & Embedded Software Systems

Hybrid Reduction of a Bipedal Walker from Three to Two Dimensions

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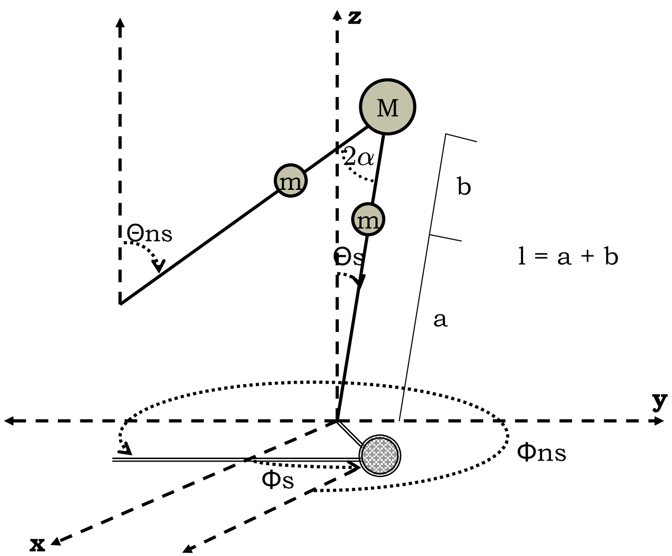
Abstract

Because the complexity of bipedal walking robots doubles when increasing a model from 2D to 3D, many previously established analytical techniques are computably impractical for 3D models. This project offers a systematic approach to reducing a 3D hybrid model into two dimensions, on which 2D analytical methods can be used, such as numerical analysis to find the limit cycles that result in asymptotically stable walking.



Motivation

The Scaling Complexity Problem: From 2D to 3D

- Increasing the model's dimensions from two to three results in a two-fold increase of state dependency
- Thus, in three dimensions, numerical analysis requires a phase space search of *eight* dimensions
- Analysis is computably impractical!

❖Solution: Hybrid Reduction on the 3D Model

Results and 3D Reconstruction

- Continuous Equations of Motion (passive):

$$M(\Theta)\,\Theta'' + F(\Theta,\Theta')\,\Theta' + g(\Theta) + aug_c(\Theta) = 0$$

- The reduced model (now 2D) is equivalent to the original 2D model with an augmented term
- Generality: If *c* is zero => original 2D model
- Uniqueness: Bring back to unique 3D model

$$\Phi` = M_2^{-1}(\Theta)\,c,$$
$$\Phi = \Phi_0 + \int M_2^{-1}(\Theta)\,c$$

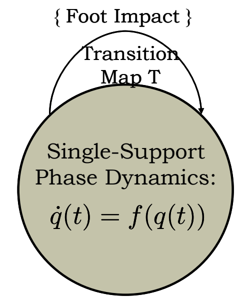
- Reconstruction: Solutions for the reduced model are also solutions for the higher-order 3D model
- Below is a periodic limit cycle for the 3D model

Future Work

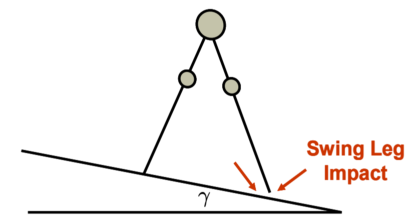
- Investigate natural 3D walking through animation
- Application to actuated robots:
  M + F + g + a = B (actuation matrix)
- Generalization of reduction: Different models and reduction without simplification constraints

Methodology

Hybridization



Discrete Foot Impact



$$\theta_{ns}(t) + \theta_s(t) = -2\gamma$$

Lagrangian Continuous Dynamics

- The Lagrangian accounts for all energy in the system
  Lagrangian = Kinetic Energy – Potential Energy
  $L(\Theta) = \frac{1}{2}\,\Theta'^T M(\Theta)\,\Theta' - \int g(\Theta)$
- Derive the continuous Equations of Motion (passive):
  $M(\Theta)\,\Theta'' + F(\Theta,\Theta')\,\Theta' + g(\Theta) = 0$

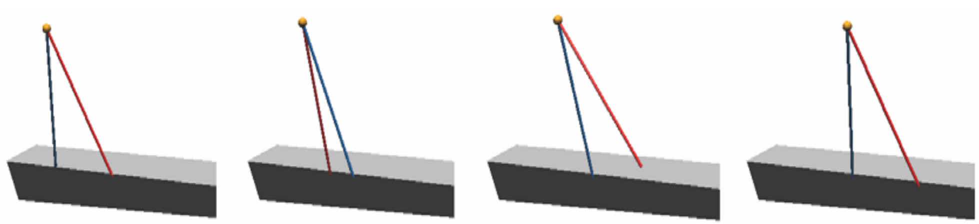
Dependency Simplification

- Find independence for Φ states
- Fixing Φs(t) = Φns(t) (x-y plane)

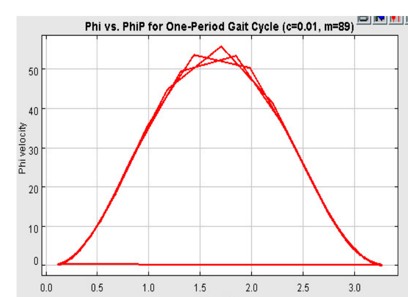
$$M_s(\theta) = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

Routhian Reduction to 2D

- Eliminate Φ` variables using reduction!
  $M_2(\Theta)\,\Phi` = c$ (Routhian constant)
- Solve: $\Phi` = c/m(\Theta)$ (scalar term)
  Routhian = $[\, L(\Theta, \Theta`, \Phi`) - c\,\Phi` \,]_{\Phi`=c/m(\Theta)}$
  $R(\Theta, \Theta`) = \frac{1}{2}\,\Theta`^T M_1(\Theta)\,\Theta` - \int q(\Theta) - \frac{1}{2}\,c^2/m(\Theta)$



Periodic 2D Walking: Reduced Model (M = 178kg, m = 89kg, a=b=0.5m, and c = 0.01)



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