MOBILE MILLENNIUM

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Societal need for [traffic] information systems

Rough estimates of congestion impacts
- 4.2 billion hours extra travel in the US
- Accounts for 2.9 billion gallons of fuel
- Congestion cost of 78 billion dollars

[2007 Urban Mobility Report, September 2007, Texas Transportation Institute, David Schrank & Tim Lomax]

Traffic information systems
- Call in numbers (511)
- Changeable message signs (CMS)
- Online navigation devices
- Web-based commuter services:
  - www.511.org
  - www.traffic.com
  - Google Traffic
- Cellular phone web browsing
- Connected aftermarket devices

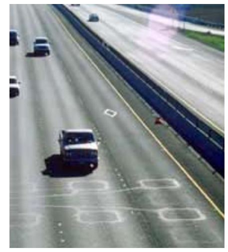
Source of today’s traffic information

Dedicated traffic monitoring infrastructure:

- Self inductive loops
- Wireless pavement sensors
- FasTrak, EZ-pass transponders
- Cameras
- Radars
- License plate readers

Issues of today’s dedicated infrastructure

- Installation costs
- Maintenance costs
- Reliability
- Coverage
- Privacy intrusion



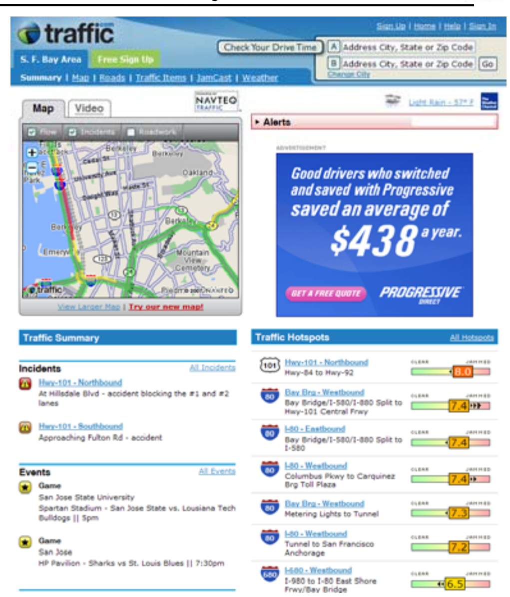
The next battle of traffic information systems

Secondary network coverage

- Expressways
- Side roads
- Arterials
- Rural roads

Scientific challenges

- Varying penetration rate
- Proper traffic models based on accurate mapping information
- Reliability of the traffic estimates

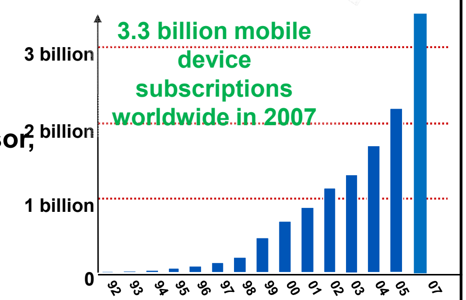


Web 2.0 on wheels

Emergence of the mobile internet
- Internet accesses from mobile devices skyrocketing
- Mobile devices outnumber PCs by 5:1
- 1. 5 million devices/day (Nokia)
- Redefining the mobile market: Google, Apple, Nokia, Microsoft, Intel, IBM, etc.
- Open source computing: Symbian Foundation, Android, Linux

Sensing and communication suite
- GSM, GPRS, WiFi, bluetooth, infrared
- GPS, accelerometer, light sensor, camera, microphone

Smartphones and Web 2.0
- Context awareness
- Sensing based user generated content



[Courtesy J. Shen, Nokia Research Center Palo Alto]

Outline

1. **Traffic information systems**
   1. Existing dedicated traffic monitoring infrastructure
   2. Web 2.0 on wheels
2. **Mobile Millennium**
   1. System
   2. Privacy aware sampling
3. **Inverse modeling and data assimiliation**
   1. A short introduction to traffic modeling
   2. The Moskowitz Hamilton-Jacobi equation
   3. Internal boundary conditions using the inf-morphism property
   4. Data assimilation in a privacy aware environment
   5. Mobile Century (February 8th, 2008)
4. **The launch of Mobile Millennium**
   1. The Bay Area
   2. New York

Ubiquitous traffic monitoring cyberinfrastructure

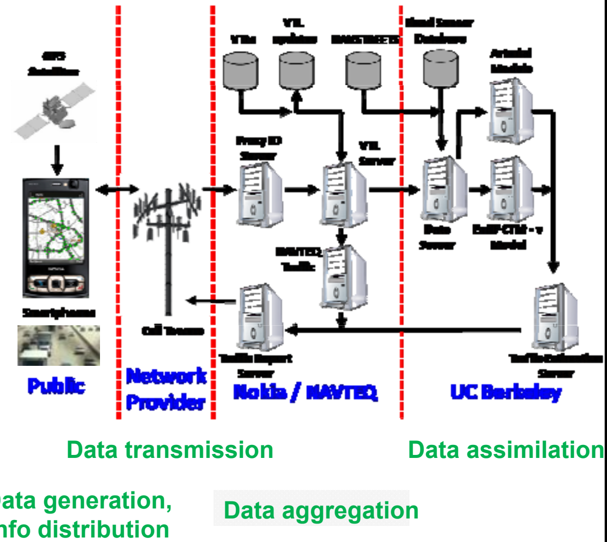
Sensing
- Millions of mobile devices as new sources for data

Communication
- Existing cell phone infrastructure to collect raw data and receive traffic information

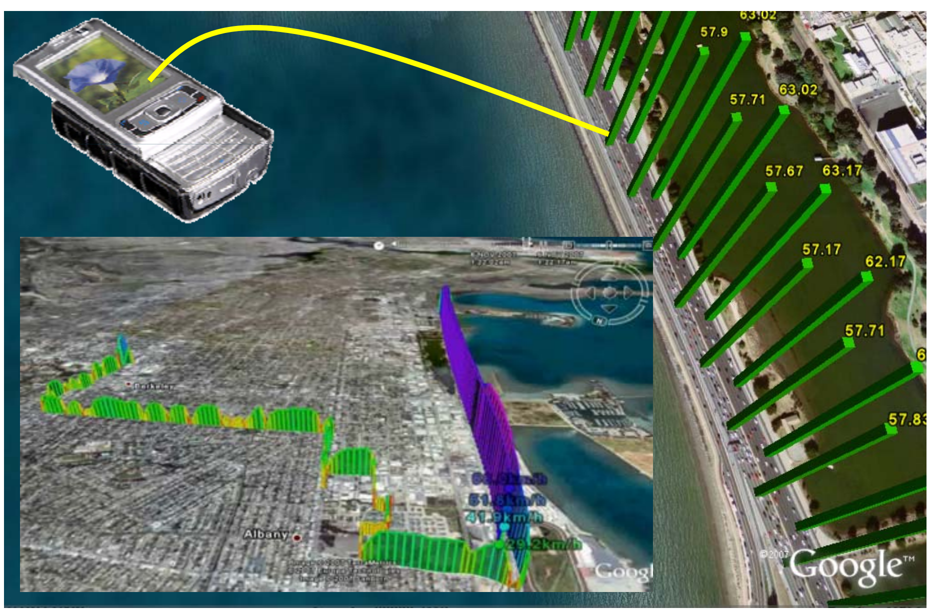
Data assimilation
- Real-time, online traffic estimation

Privacy Management
- Encrypted transactions
- Client authentication
- Data anonymization

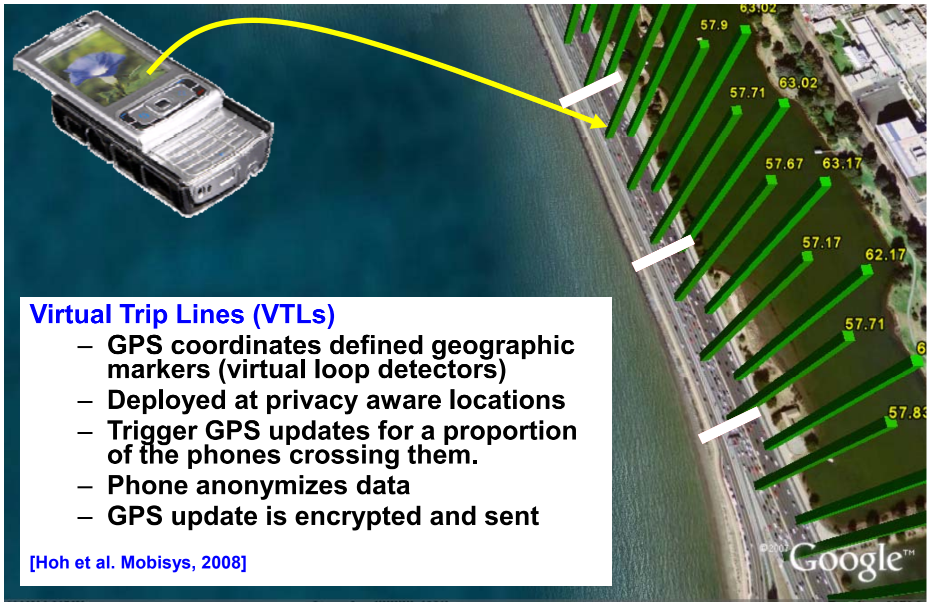
Mobile Millennium system architecture



Privacy issues for location based services



Spatially aware traffic monitoring

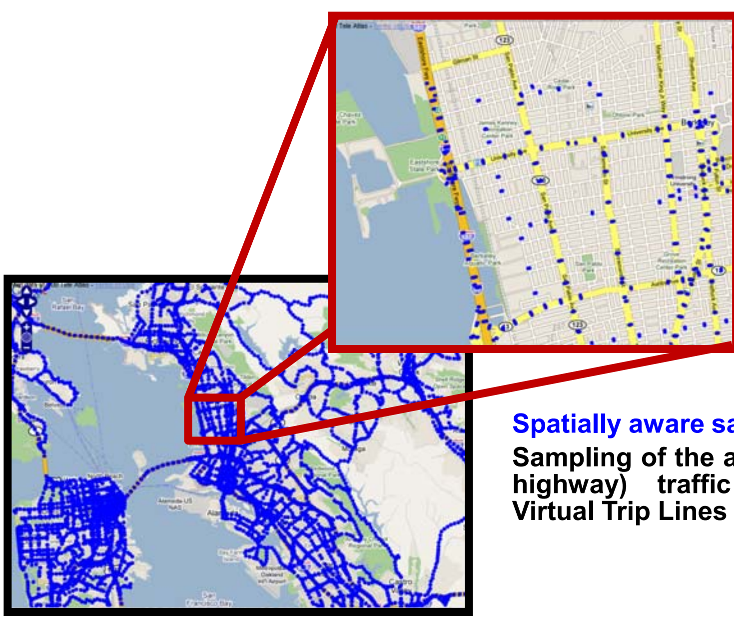


Virtual Trip Lines (VTLs)

- **GPS coordinates defined geographic markers (virtual loop detectors)**
- **Deployed at privacy aware locations**
- **Trigger GPS updates for a proportion of the phones crossing them.**
- **Phone anonymizes data**
- **GPS update is encrypted and sent**

[Hoh et al. Mobisys, 2008]

Nationwide deployment of a virtual infrastructure



Spatially aware sampling

Sampling of the arterial (and the highway) traffic done using Virtual Trip Lines (VTLs)

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A very brief introduction to traffic flow modelling

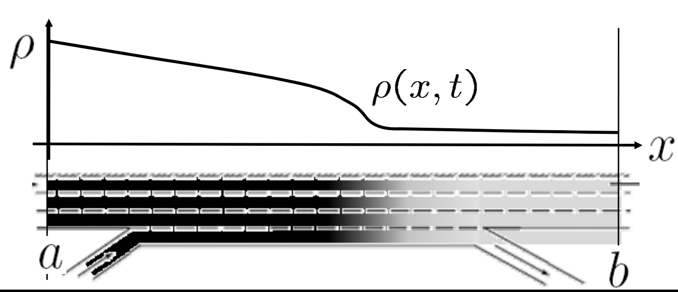
Seminal hydrodynamic model: the Lighthill-Whitham-Richards partial differential equation

- **Nonlinear first order hyperbolic scalar conservation law**
- **Concave flux function (empirical fundamental diagram)**
- **Weak boundary conditions**

[Lighthill-Whitham, 1955; Richards, 1956; Bardos Leroux Nedelec, 1979; Bayen Strub 2006]

$\rho(x,t)$ **is the vehicle density.**

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0$$



A very brief introduction to traffic flow modelling

Most basic fundamental diagram – Greenshields flux function

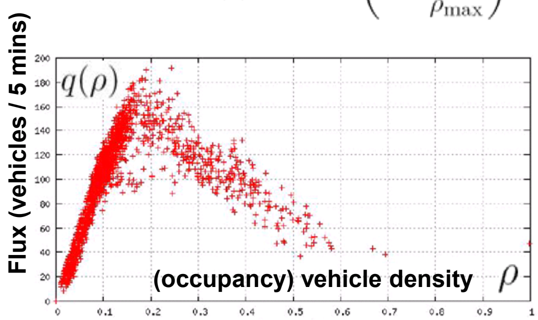
- **Velocity modelled as a linear function of density:**

$$v(\rho) = v_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right)$$

- **Corresponding flux is parabolic:**

$$q(\rho) = \rho v_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0$$



Numerous other diagrams used in this work:

- **Parabolic**
- **Triangular**
- **Trapezoidal**
- **Hybrid**
- **Tong**
- **...**

Challenges of nonlinearity

Well posed Cauchy problem

First order scalar nonlinear hyperbolic PDE (LWR PDE)

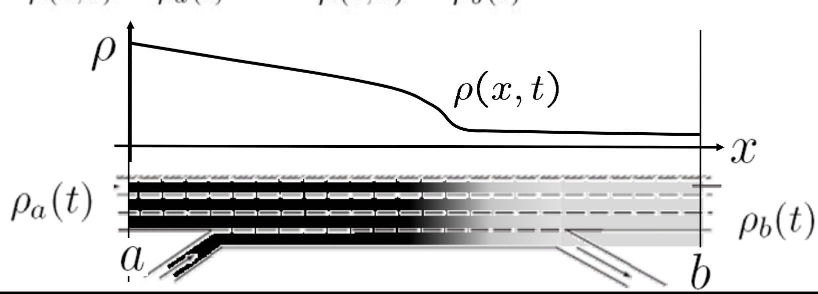
$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0$$

Initial conditions

$$\rho(x,0) = \rho_0(x)$$

Strong boundary conditions: not well posed

$$\rho(a,t) = \rho_a(t) \qquad \rho(b,t) = \rho_b(t)$$



Challenges of nonlinearity

Well posed Cauchy problem

First order hyperbolic PDE (LWR PDE)

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0$$

Initial conditions

$$\rho(x,0) = \rho_0(x)$$

Weak boundary conditions: necessary for well-posedness

$$\begin{cases} \rho(a,t) = \rho_a(t) \text{ or} \\ q'\left(\rho\left(a,t\right)\right) \leqslant 0 \text{ and } q'\left(\rho_a\left(t\right)\right) \leqslant 0 \text{ or} \\ q'\left(\rho(a,t)\right) \leqslant 0 \text{ and } q'\left(\rho_a(t)\right) \geqslant 0 \text{ and } q\left(\rho(a,t)\right) \leqslant q\left(\rho_a(t)\right) \end{cases}$$

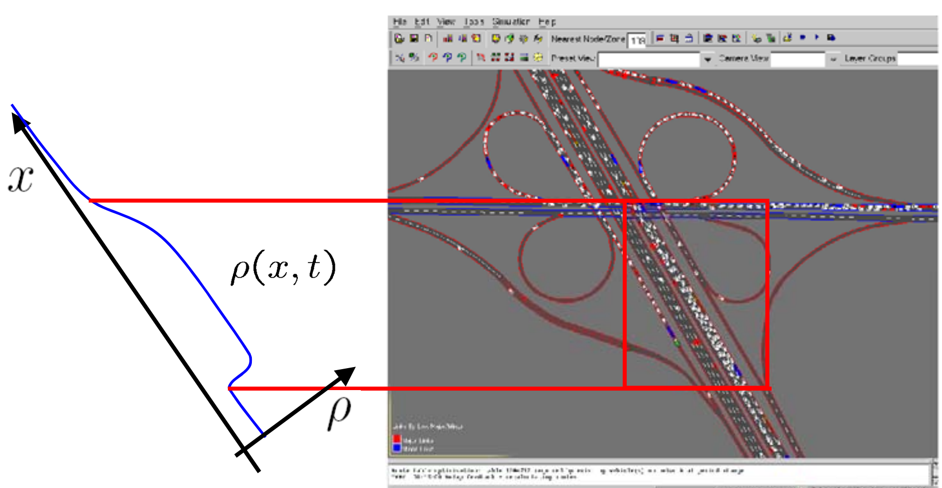
$$\begin{cases} \rho(b,t) = \rho_b(t) \text{ or} \\ q'\left(\rho\left(b,t\right)\right) \geqslant 0 \text{ and } q'\left(\rho_b\left(t\right)\right) \geqslant 0 \text{ or} \\ q'\left(\rho(b,t)\right) \geqslant 0 \text{ and } q'\left(\rho_b(t)\right) \leqslant 0 \text{ and } q\left(\rho(b,t)\right) \leqslant q\left(\rho_b(t)\right) \end{cases}$$

[Lighthill-Whitham, 1955; Richards, 1956; Bardos Leroux Nedelec, 1979; Strub, Bayen 2006]

Challenges of nonlinearity

Nonlinear features to capture for acceptable modeling accuracy

- **Spillovers, moving bottlenecks**
- **Shockwaves, expansion waves**



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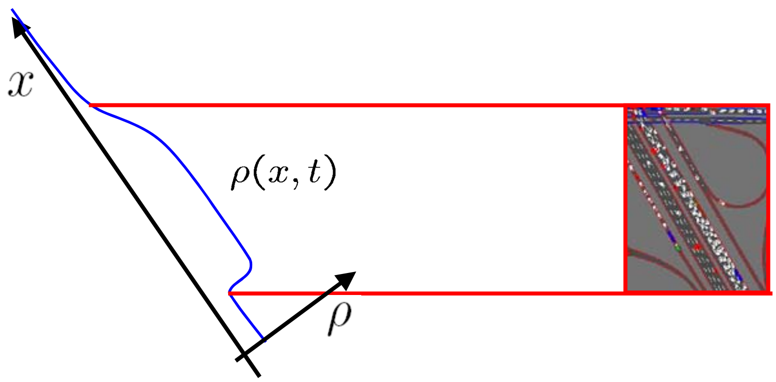


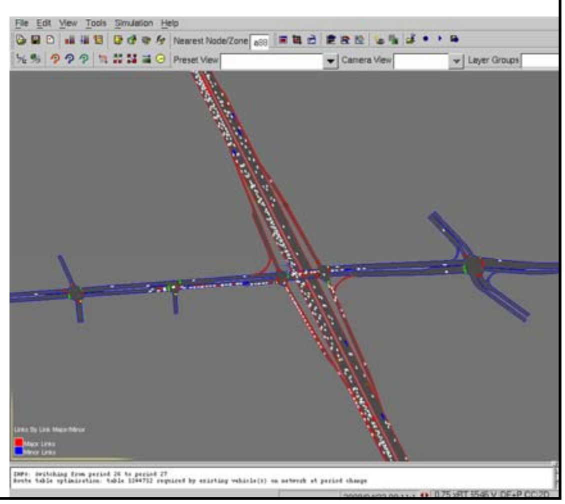
Challenges of nonlinearity

Nonlinear and nonsmooth features are necessary to capture the physics with acceptable modeling accuracy

- **Spillovers, moving bottlenecks**
- **Shockwaves, expansion waves**

Hydrodynamic models (PDEs)

- **Algebraically compact**
- **Numerically tractable**
- **Accurate abstraction of traffic**
- **Can be used for control and estimation purposes**
- **Pose challenging theoretical problems**



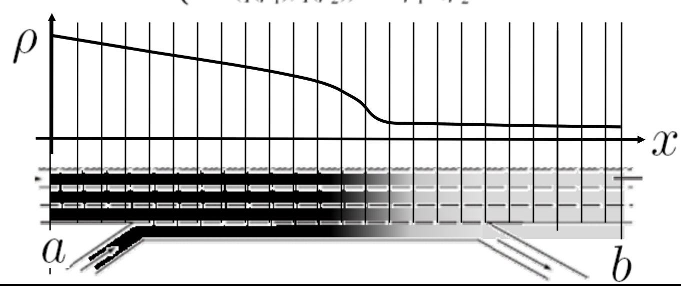
Challenges of nondifferentiability

Nonsmooth features of traffic are captured by nonsmooth discretization

- **Godunov numerical schemes (max/min operators)**
- **Boolean logic in the implementation of boundary conditions (in the form of ghost cells)**

$$\rho_i^{n+1} = \rho_i^n - r(q_G(\rho_i^n, \rho_{i+1}^n) - q_G(\rho_{i-1}^n, \rho_i^n))$$

$$q_G(\rho_1, \rho_2) = \begin{cases} q(\rho_2) & \text{if } \rho_c < \rho_2 < \rho_1 \\ q(\rho_c) & \text{if } \rho_2 < \rho_c < \rho_1 \\ q(\rho_1) & \text{if } \rho_2 < \rho_1 < \rho_c \\ \min(q(\rho_1), q(\rho_2)) & \text{if } \rho_1 \leqslant \rho_2 \end{cases}$$



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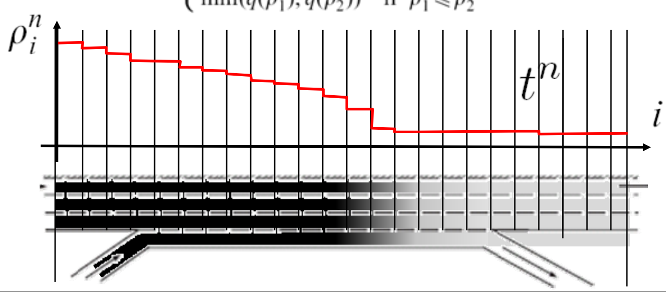


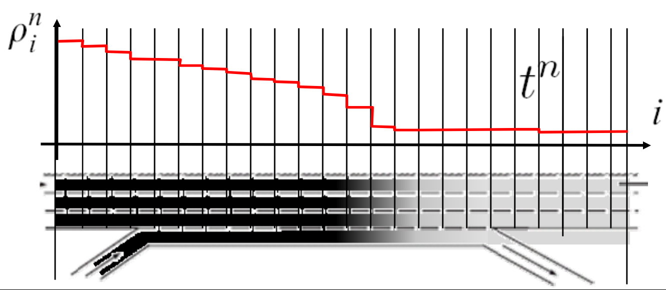
Challenges of nondifferentiability

Nonsmooth features of traffic are captured by nonsmooth discretization

- Godunov numerical schemes (max/min operators)
- Boolean logic in the implementation of boundary conditions (in the form of ghost cells)

$$\rho^{n+1} = \mathcal{F}(\rho^n) + \mathcal{G}(u^n)$$

- Dynamics is nonlinear and nonsmooth
- It is required to capture the proper entropy solution of the PDE



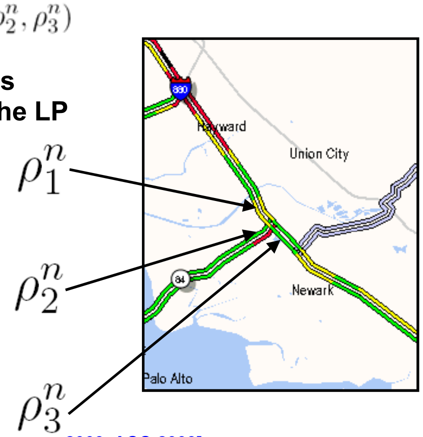
Challenges of nondifferentiability (network effect)

Proper treatment of network junctions introduces additional nonlinearity

- Network junctions are nonsmooth (weak boundary conditions)
- The solution needs to be computed iteratively by solving a linear program at each time step

$$(\rho_1^{n+1}, \rho_2^{n+1}, \rho_3^{n+1}) = \mathcal{LP}(\rho_1^n, \rho_2^n, \rho_3^n)$$

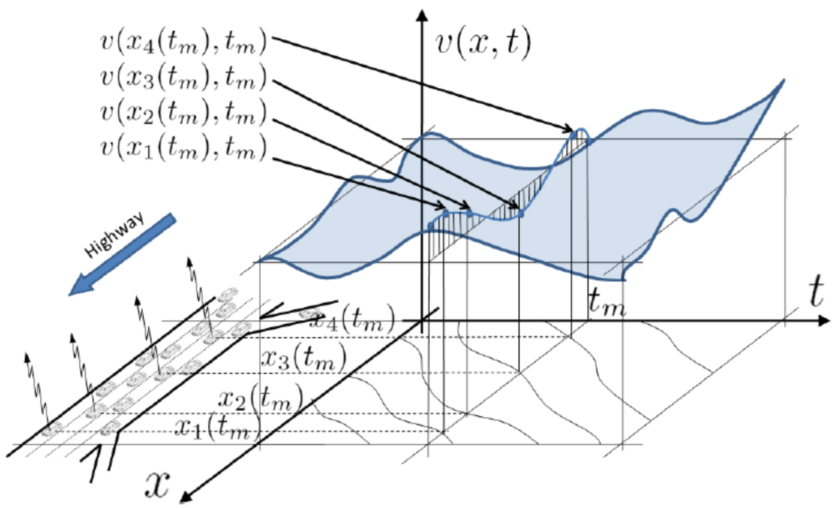
- Solution of a LP is not continuous or smooth in the coefficients of the LP



[Work, Blandin, Piccoli, Bayen, IEEE Trans. Autom. Cont. in prep, 2009, ACC 2009]

Data assimilation / inverse modeling

How to incorporate Lagrangian (trajectory based) and Eulerian (control volume based) measurements in a flow model.



Contributions

Modeling

- **Weak BC for the LWR- ρ model**
  [Strub, Bayen, IJRNC, 2006]
- **Hamilton-Jacobi equation for Cumulated vehicle number**
  [Aubin, Bayen, Saint-Pierre, SIAM SICON, 2008]
- **Second order models (PDE systems) for highway traffic**
  [Blandin, Work, Gaotin, Piccoli, Bayen, subm. SIAM SIAP, 2008]
- **First order velocity model (PDE systems) for highway traffic**
  [Work, Blandin, Piccoli, Bayen, in prep. IEEE Trans. Autom. Control. 2009]
  [Work, Tossavainen, Tracton, Bayen, CDC 2008]

Data assimilation

- **Newtonian relaxation**
  [Herrera, Bayen, subm. Transportation Research B, 2008]
- **Ensemble Kalman filtering**
  [Work, Blandin, Piccoli, Bayen, in prep. IEEE Trans. Autom. Control. 2009]
  [Work, Tossavainen, Jacobson, Bayen, ACC 2009]

Contributions

Hamilton-Jacobi framework

- **Integration of internal boundary conditions**
  [Claudel, Bayen, subm. IEEE Trans. Autom. Control, 2008]
- **Explicit solutions for piecewise affine conditions**
  [Claudel, Bayen, subm. IEEE Trans. Autom. Control, 2008]
- **Linear programming solutions of robust estimation problems**
  [Claudel, Bayen, Allerton CCC, 2009]

Systems and data work

- **Mobile Century data assimilation and analysis**
  [Herrera, Work, Bayen, subm. Transportation Research C, 2009]
- **Systems architecture**
  [Hoh, et al., Mobisys 2008]

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The Moskowitz function

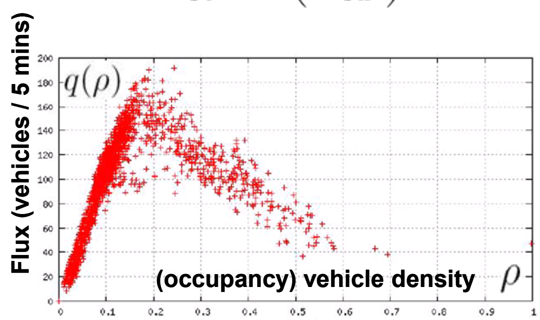
Main problem: link an Eulerian representation of traffic (conservation of vehicle based) with Lagrangian (trajectory based) sensing

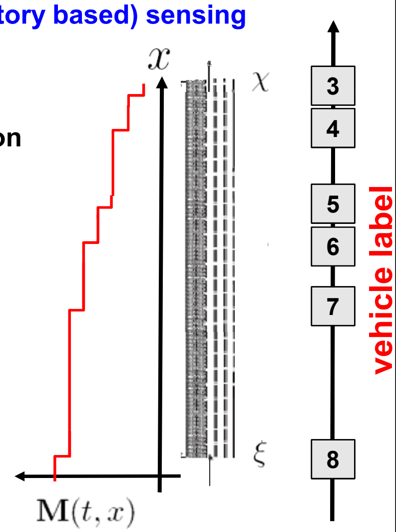
- **Cumulative vehicle number (label)**

  $\mathbf{M}(t,x)$

- **Satisfies a Hamilton-Jacobi equation**

  $$\frac{\partial \mathbf{M}}{\partial t} - q\left(-\frac{\partial \mathbf{M}}{\partial x}\right) = 0$$





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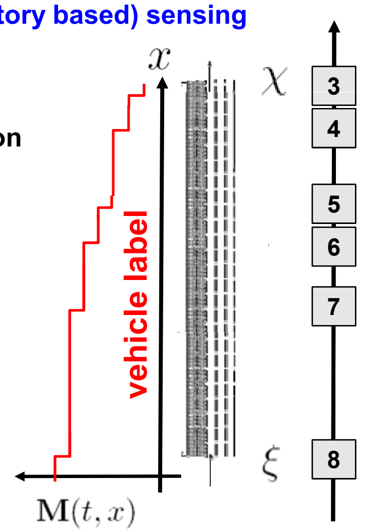
- **With boundary condition(s)**

  $\mathbf{M}(t,\chi)$

  $\mathbf{M}(t,\xi)$

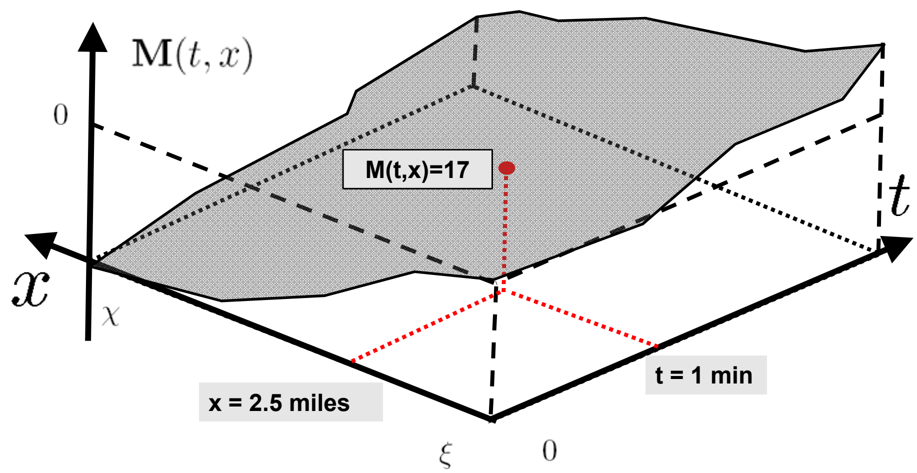
- **And initial conditions**

  $\mathbf{M}(0,x)$



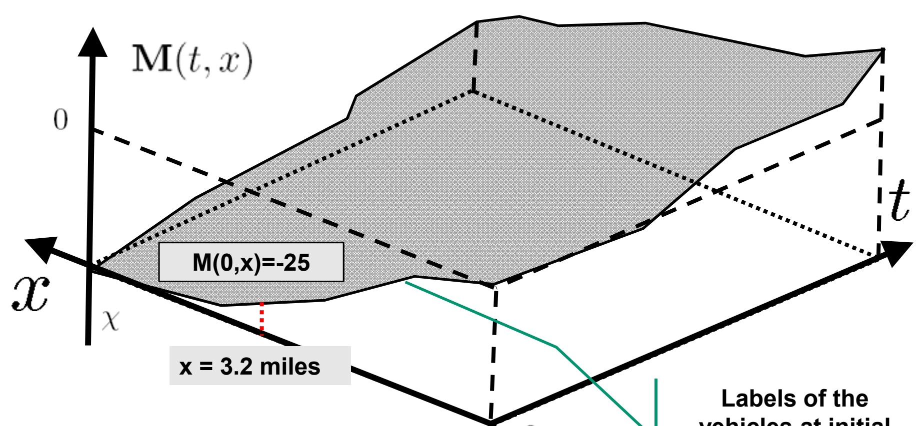
Physical interpretation of the Moskowitz function

17th vehicle which entered the link of highway has driven exactly 2.5 miles in 1 minute



Physical interpretation of the initial condition

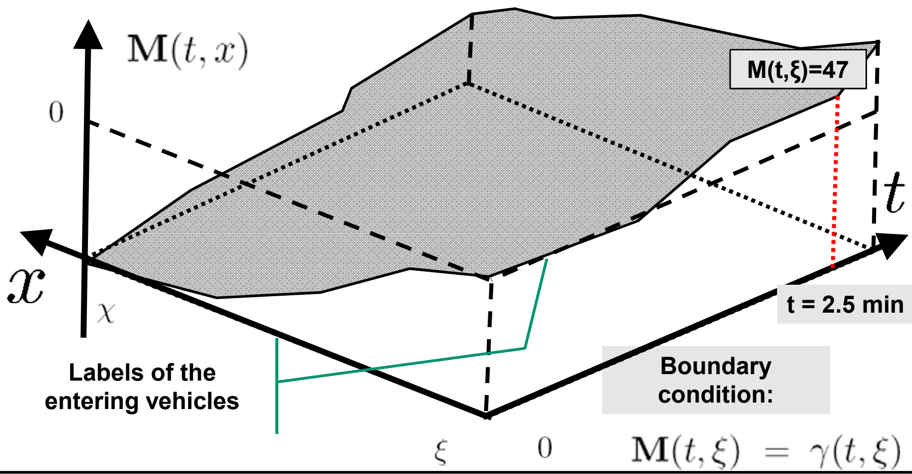
Vehicle entering at t=0 is labeled zero (arbitrary). If value at 3.2 miles at t=0 is -25, there are 25 cars between x=0 and x=3.2



$$\mathbf{M}(0,x) = \mathbf{M}_0(0,x)$$

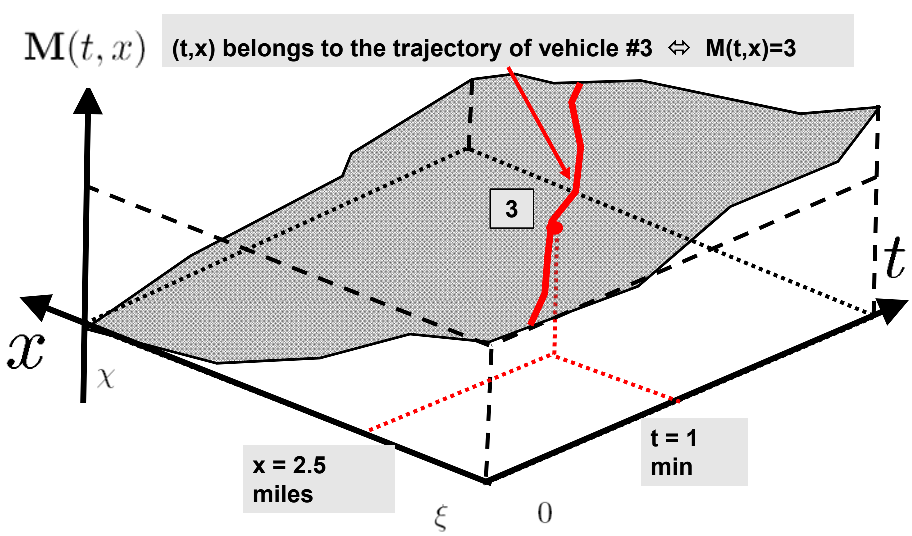
Physical interpretation of the boundary condition

47 vehicles entered between t=0 and t=2.5



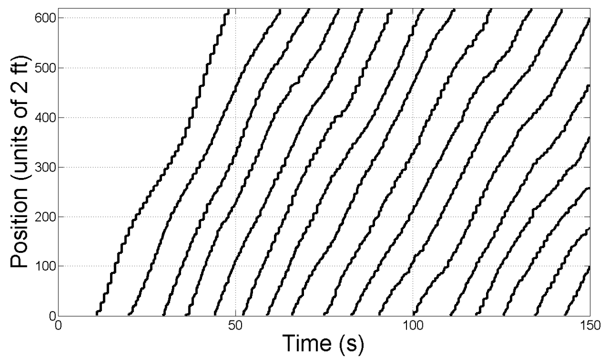
Physical interpretation of the level sets

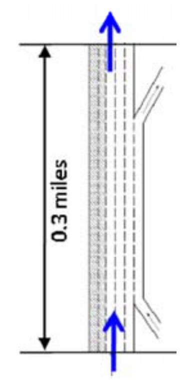
Level sets of the Moskowitz function correspond to trajectories



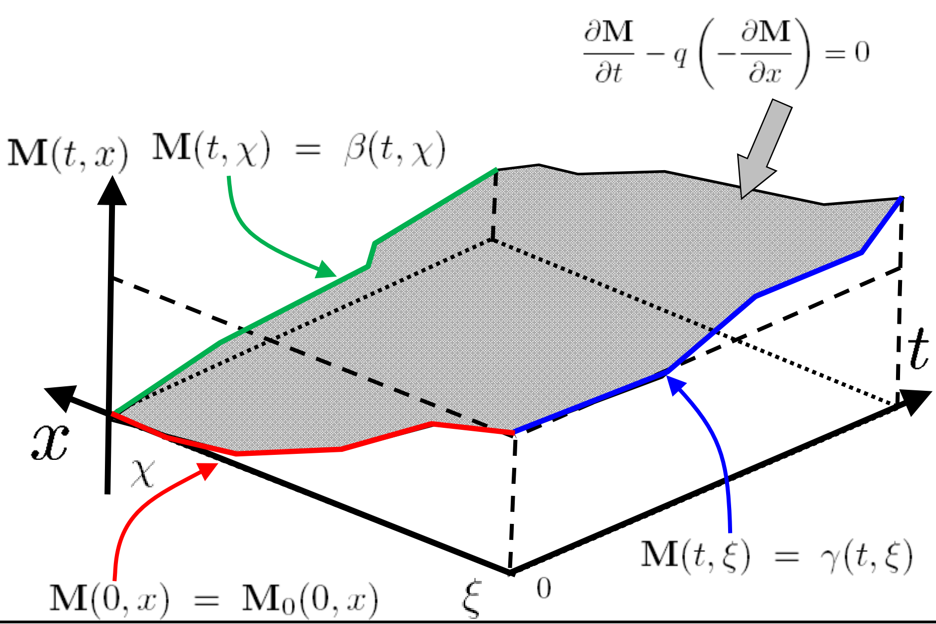
Experimentally measured Moskowitz surface

Level sets of the Moskowitz function correspond to trajectories





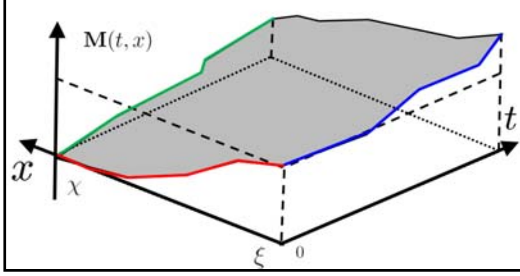
Full Cauchy problem



Full Cauchy problem

PDE, left boundary condition, right boundary condition, initial condition

$$\begin{cases} \dfrac{\partial \mathbf{M}}{\partial t} - q\left(-\dfrac{\partial \mathbf{M}}{\partial x}\right) = 0 & \\ \mathbf{M}(0,x) = \mathbf{M}_0(0,x) & \forall x \in X \\ \mathbf{M}(t,\xi) = \gamma(t,\xi) & \forall t \in \mathbb{R}_+ \\ \mathbf{M}(t,\chi) = \beta(t,\chi) & \forall t \in \mathbb{R}_+ \end{cases}$$



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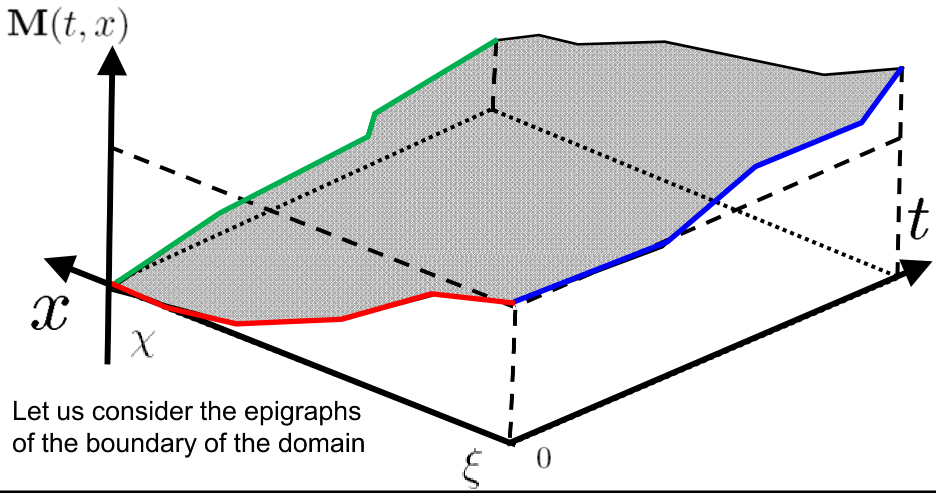
Epigraphical characterization of the solution

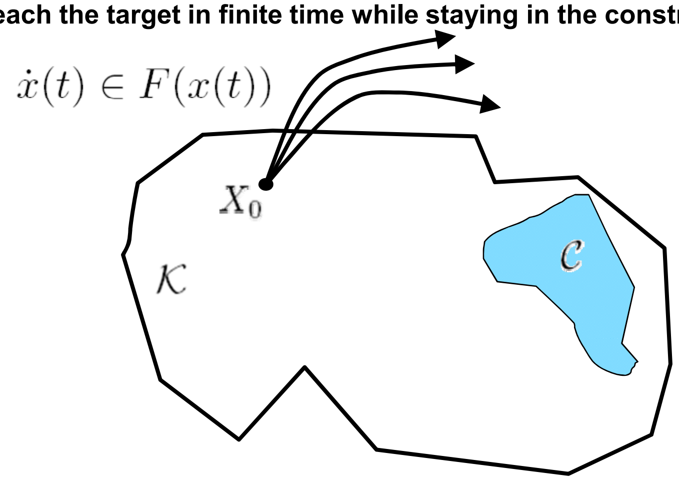
Idea: characterize the Moskowitz surface as the lower envelope of a capture basin



Let us consider the epigraphs of the boundary of the domain

Reminder: definition of the capture basin

The capture basin of a target with a constraint set is the set of points which can reach the target in finite time while staying in the constraint set



$$\text{Capt}_F(\mathcal{K},\mathcal{C}) := \{X_0 \in \mathcal{K} \mid \exists X(\cdot) \in \mathcal{S}_F(X_0) \text{ and } \exists T \geq 0 \text{ such that } X(T) \in \mathcal{C} \text{ and } \forall t \in [0,T],\ X(t) \in \mathcal{K}\}$$

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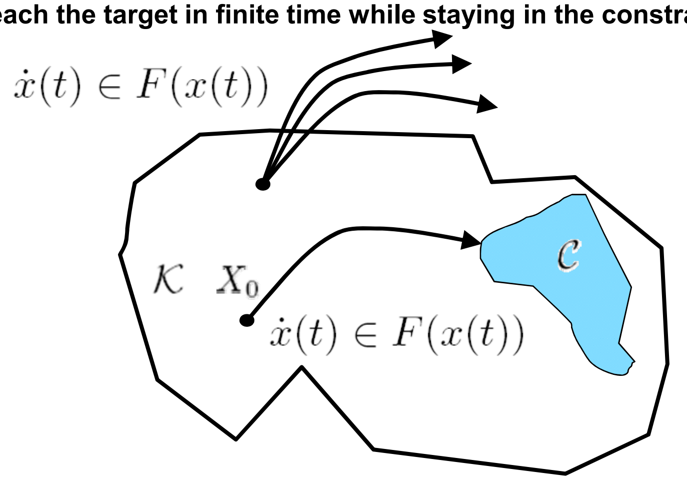
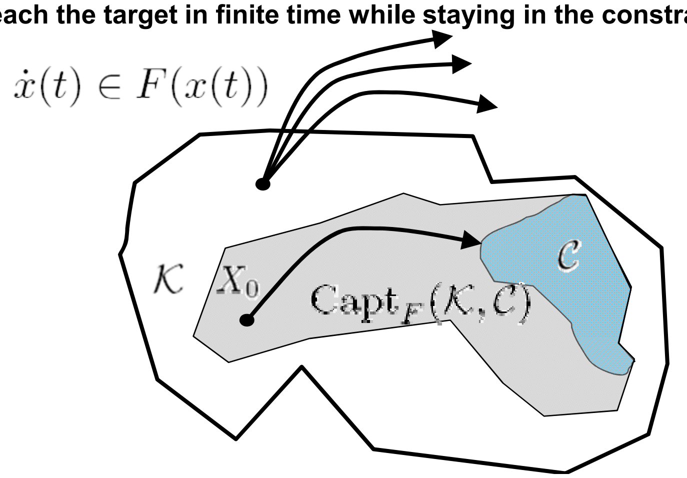


$$\text{Capt}_F(\mathcal{K},\mathcal{C}) := \{X_0 \in \mathcal{K} \,|\, \exists X(\cdot) \in \mathcal{S}_F(X_0) \text{ and } \exists T \geq 0 \text{ such that } X(T) \in \mathcal{C} \text{ and } \forall t \in [0,T],\ X(t) \in \mathcal{K}\}$$

Reminder: definition of the capture basin

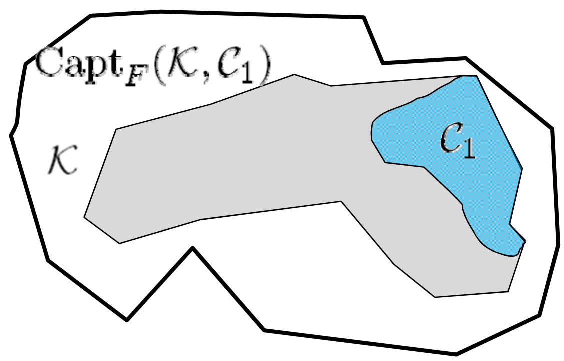
The capture basin of a target with a constraint set is the set of points which can reach the target in finite time while staying in the constraint set



$$\text{Capt}_F(\mathcal{K},\mathcal{C}) := \{X_0 \in \mathcal{K} \,|\, \exists X(\cdot) \in \mathcal{S}_F(X_0) \text{ and } \exists T \geq 0 \text{ such that } X(T) \in \mathcal{C} \text{ and } \forall t \in [0,T],\ X(t) \in \mathcal{K}\}$$

Fundamental union property of capture basins

The capture basin of a finite union of targets is the union of the capture basins of the targets



$$\mathrm{Capt}_F\left(\mathcal{K}, \bigcup_{i \in I} \mathcal{C}_i\right) = \bigcup_{i \in I} \mathrm{Capt}_F(\mathcal{K}, \mathcal{C}_i)$$

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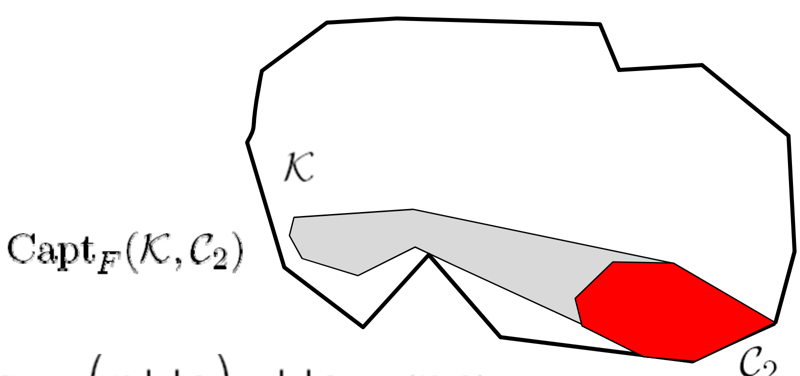

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Fundamental union property of capture basins

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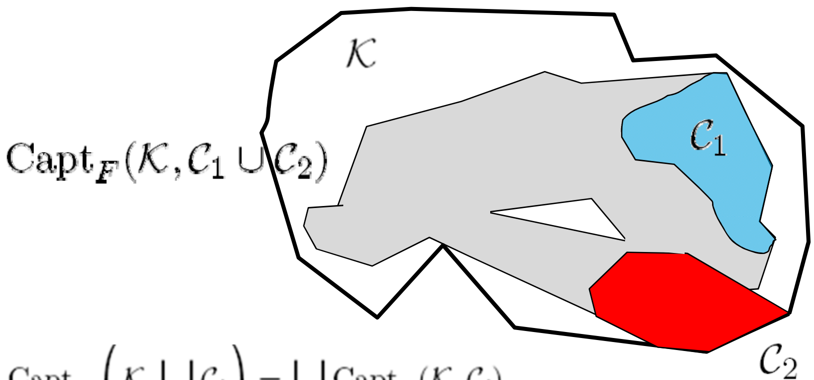
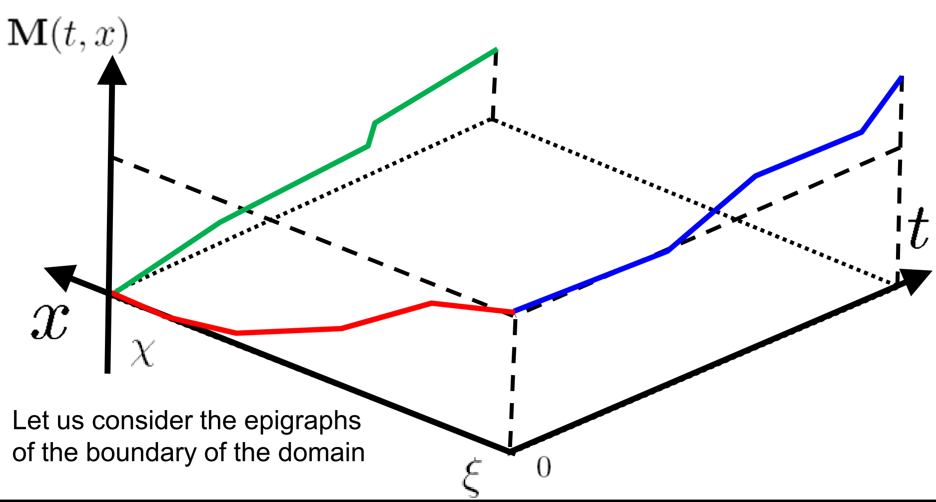


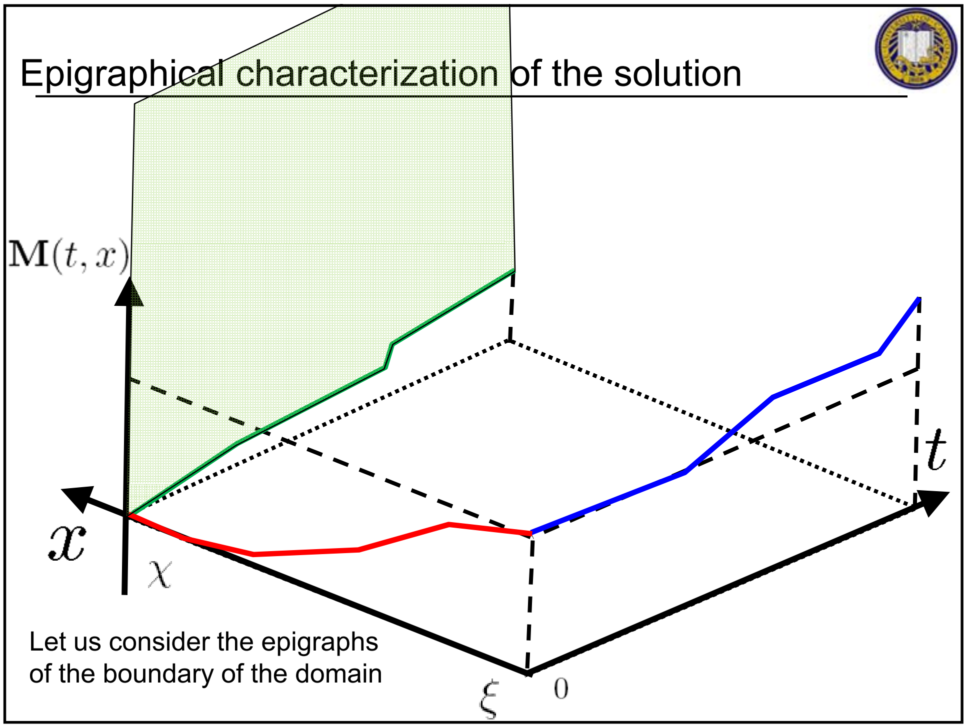
$$\mathrm{Capt}_F\left(\mathcal{K}, \bigcup_{i \in I} \mathcal{C}_i\right) = \bigcup_{i \in I} \mathrm{Capt}_F(\mathcal{K}, \mathcal{C}_i)$$

Epigraphical characterization of the solution



Let us consider the epigraphs
of the boundary of the domain

Epigraphical characterization of the solution



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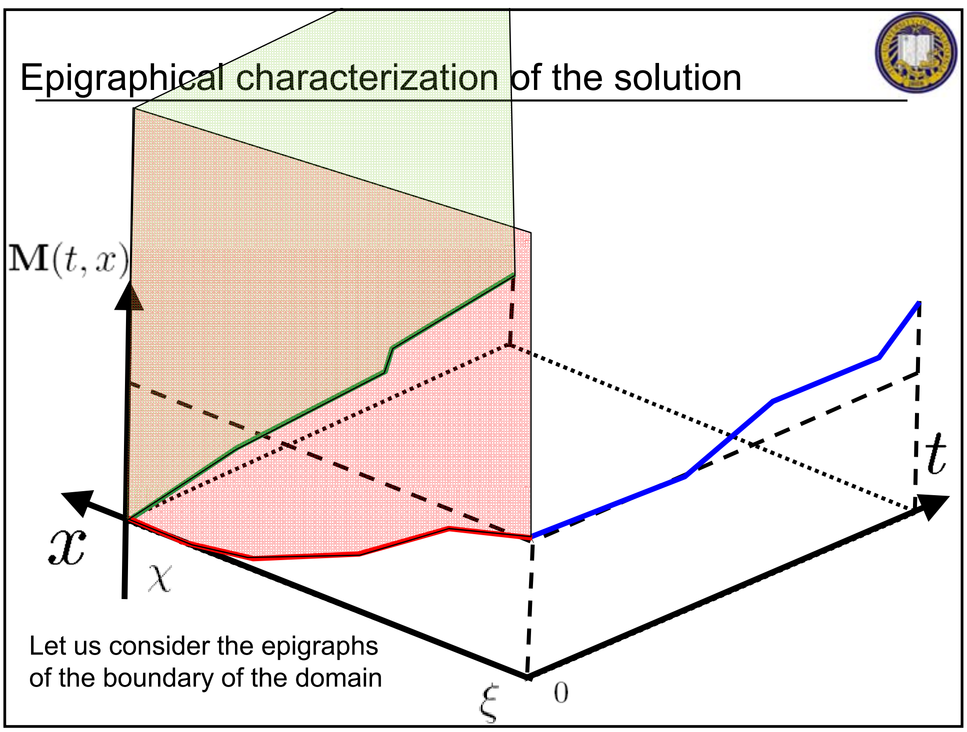
Epigraphical characterization of the solution



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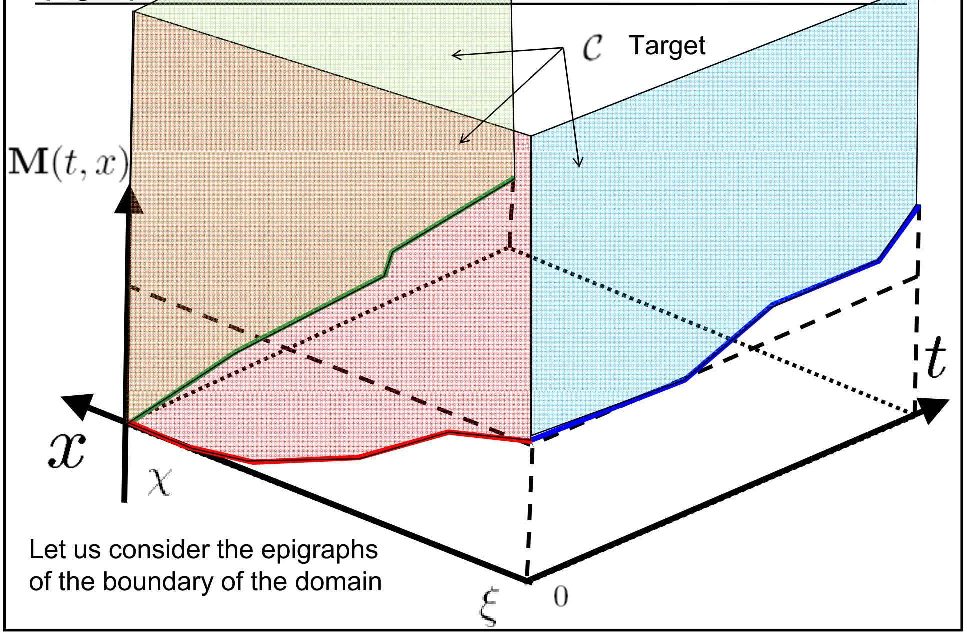
Epigraphical characterization of the solution



Let us consider the epigraphs of the boundary of the domain

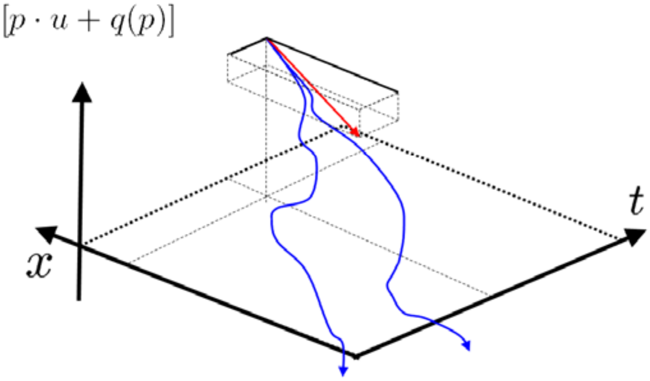
Construct an auxiliary dynamics

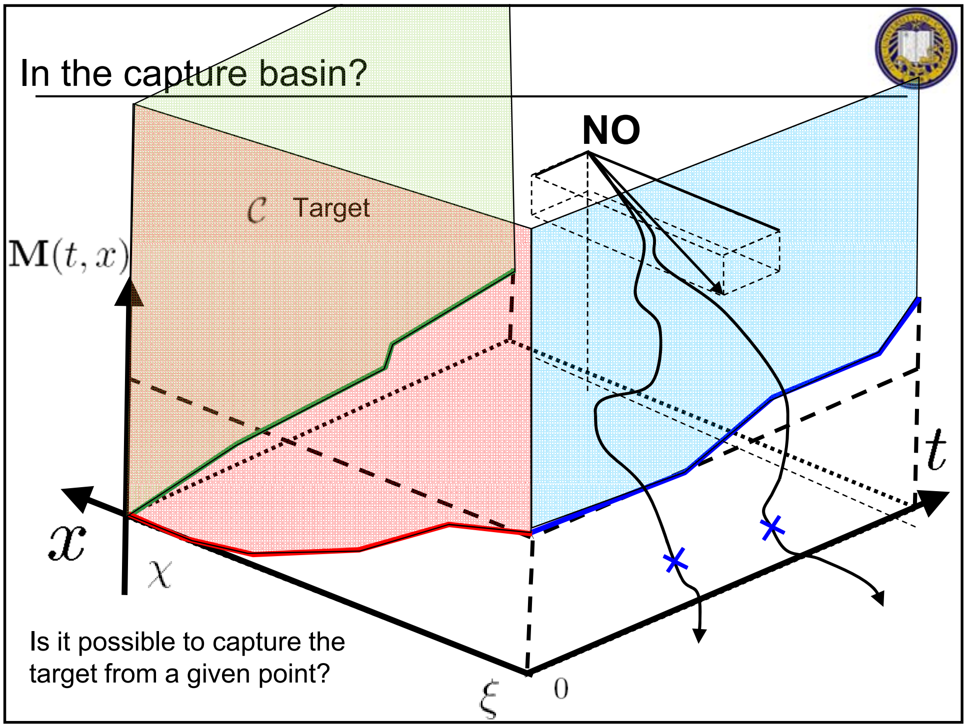
Consider the following set valued dynamics

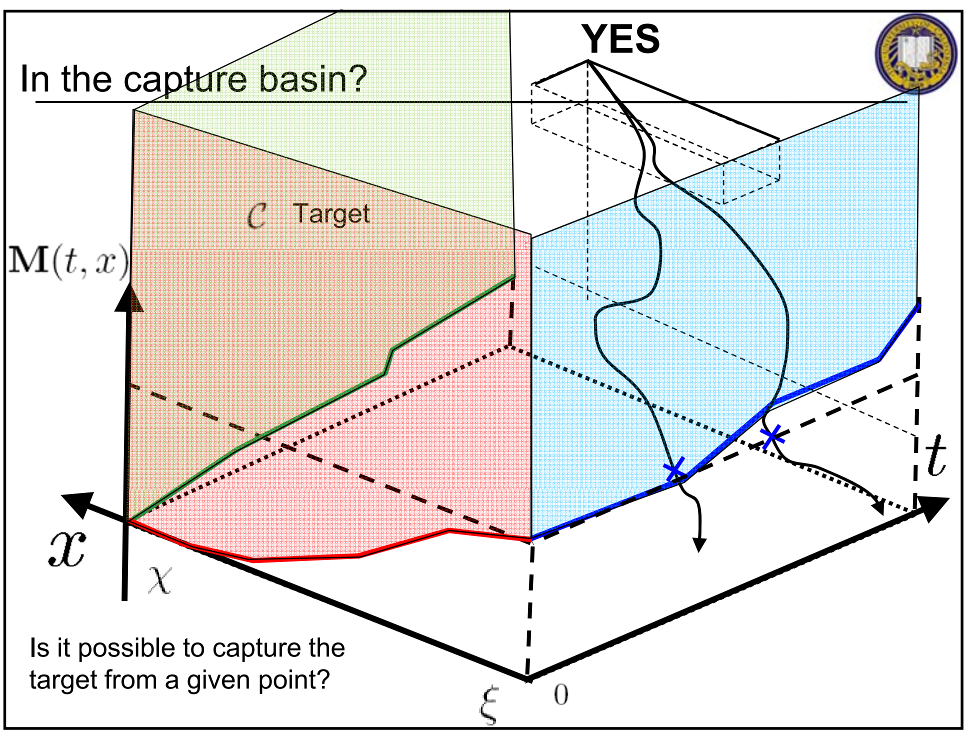
$$F := \begin{cases} \tau'(t) = -1 \\ x'(t) = u(t) \\ y'(t) = -\varphi^*(u(t)) \end{cases} \quad \text{where } u(t) \in \mathrm{Dom}(\varphi^*)$$

Where the Fenchel transform of the Hamiltonian is given by:

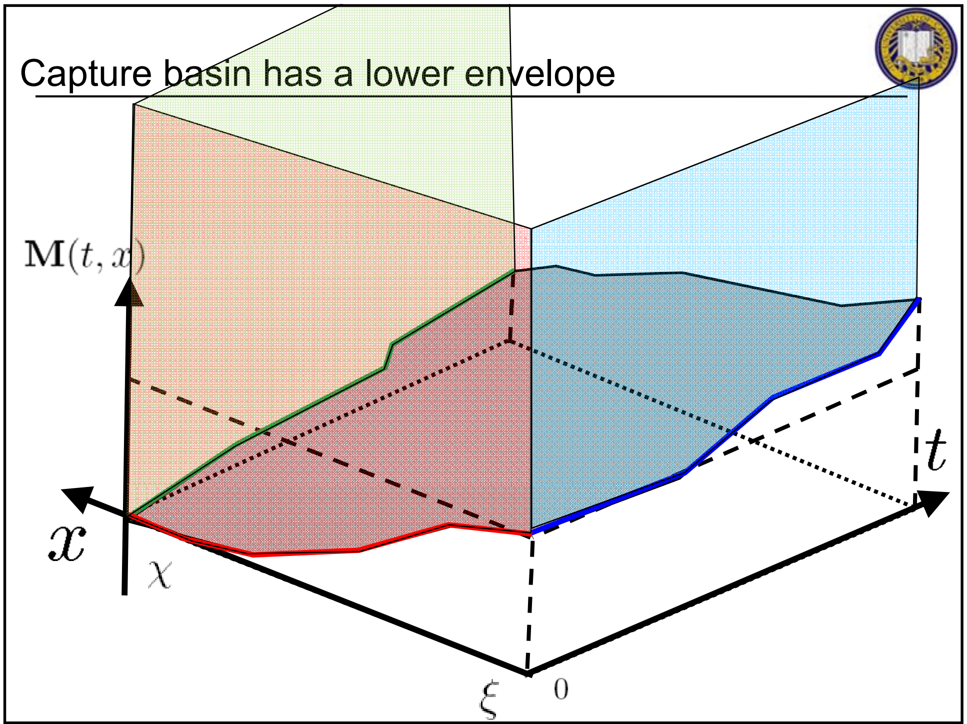
$$\varphi^*(u) = \sup_{p \in \mathrm{Dom}(\psi)} [p \cdot u + q(p)]$$



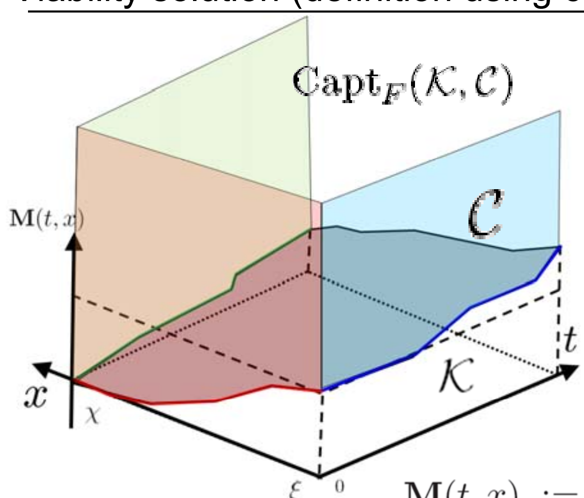

In the capture basin?
NO
$\mathcal{C}$ Target
$\mathbf{M}(t, x)$
x
χ
t
ξ
0
Is it possible to capture the target from a given point?


In the capture basin?
YES
$\mathcal{C}$ Target
$\mathbf{M}(t, x)$
x
χ
t
ξ
0
Is it possible to capture the target from a given point?

Capture basin has a lower envelope



Viability solution (definition using capture basin)



$$\mathbf{M}(t,x) := \inf_{(t,x,y)\in \mathrm{Capt}_F(\mathcal{K},\mathcal{C})} y$$

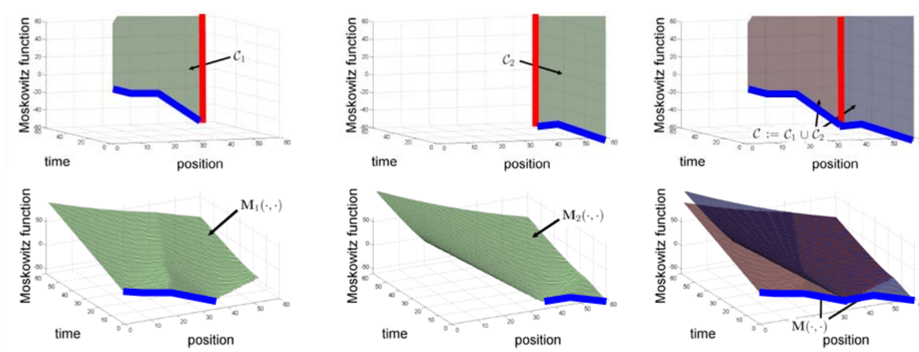
[Aubin, Bayen, Saint-Pierre, SIAM SICON 2008]

The inf-morphism property

The union property for capture basins $\mathrm{Capt}_F\left(\mathcal{K}, \bigcup_{i \in I} \mathcal{C}_i\right) = \bigcup_{i \in I} \mathrm{Capt}_F(\mathcal{K}, \mathcal{C}_i)$

translates into an inf-morphism property

$$\forall\, t \geq 0,\ x \in X,\quad \mathbf{M}_{\mathbf{c}}(t,x) \;=\; \inf_{i \in I} \mathbf{M}_{\mathbf{c}_i}(t,x)$$



Tangential property of the capture basin

This defines a new class of solutions to the HJ PDE:

$$\mathbf{M}(t,x) \;:=\; \inf_{(t,x,y) \in \mathrm{Capt}_F(\mathcal{K},\mathcal{C})} y$$

The solution provided by this formula is a lower semicontinuous function. It is the solution to the HJ PDE considered before, in a weaker sense than the viscosity solution. This solution is called the Barron/Jensen-Frankowska (B/J-F) solution.

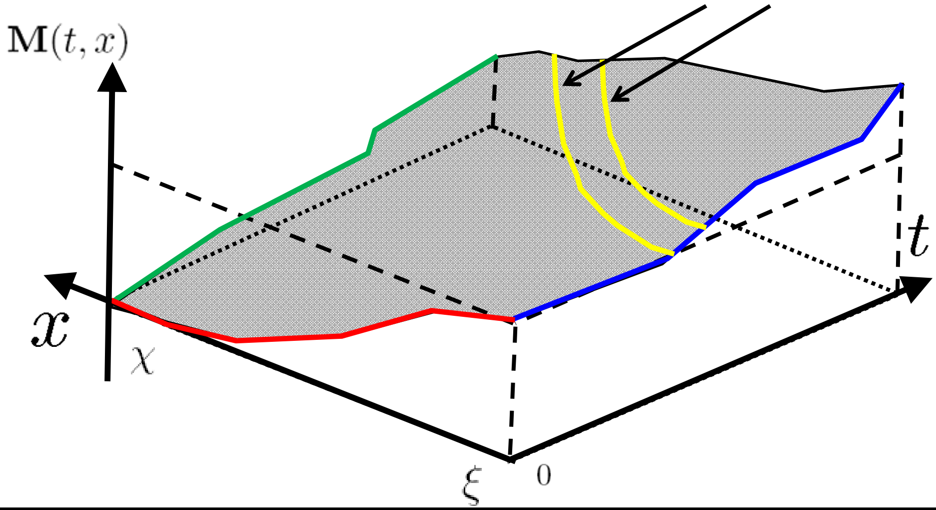
B/J-F solutions require only the lower semicontinuity of the solution.

In particular: whenever M is differentiable the tangential properties of the capture basin imply:

$$\forall (t,x) \in \mathrm{Dom}(\mathbf{M}_{\mathbf{c}}) \backslash \mathrm{Dom}(\mathbf{c}) \quad \frac{\partial \mathbf{M}_{\mathbf{c}}(t,x)}{\partial t} - \psi\left(-\frac{\partial \mathbf{M}_{\mathbf{c}}(t,x)}{\partial x}\right) \;=\; 0$$

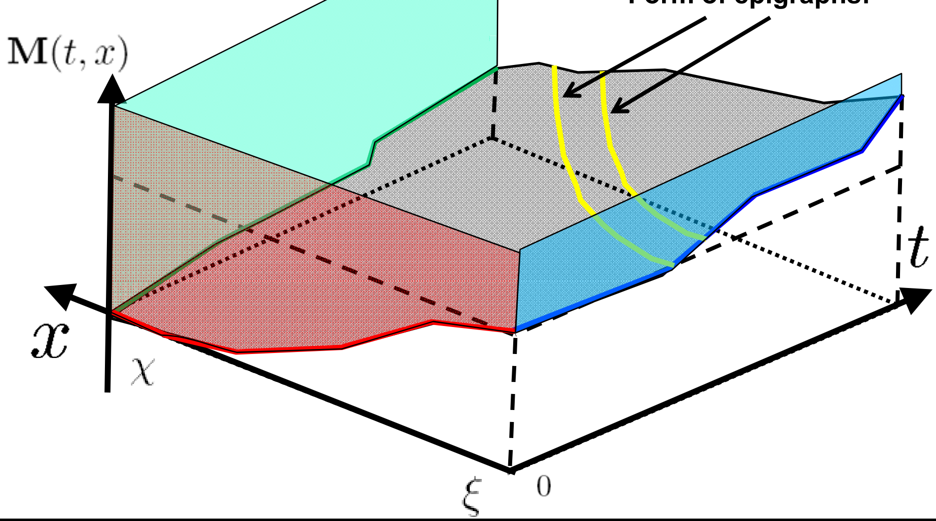
Adding trajectories is equivalent to adding epigraphs

Very often, drivers drive in violation of the LWR HJ PDE model (because of external disturbances not included in the model: accidents, distraction, excessive speed, etc.). This can be measured:

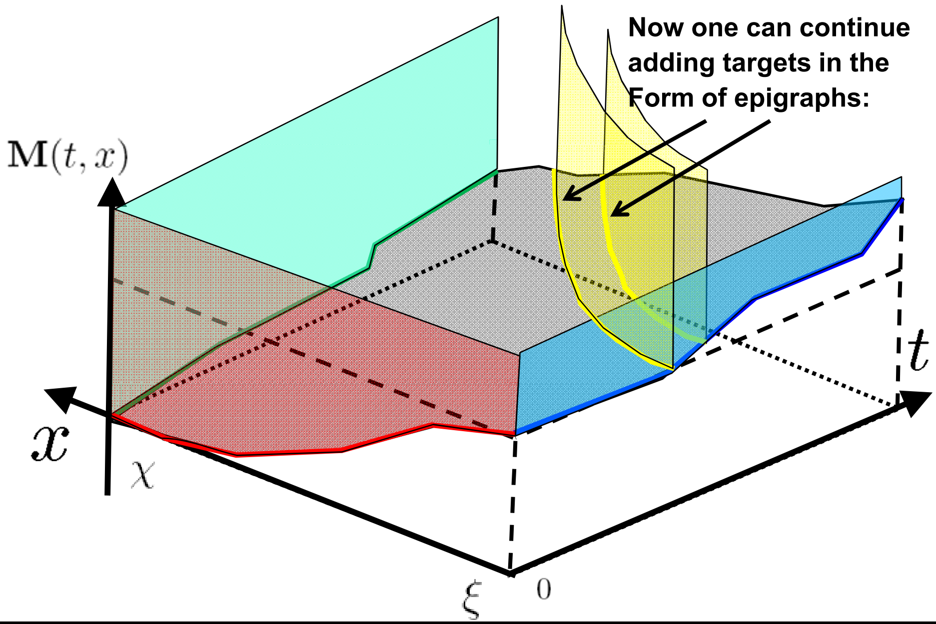


Adding trajectories is equivalent to adding epigraphs

Now one can continue adding targets in the Form of epigraphs:



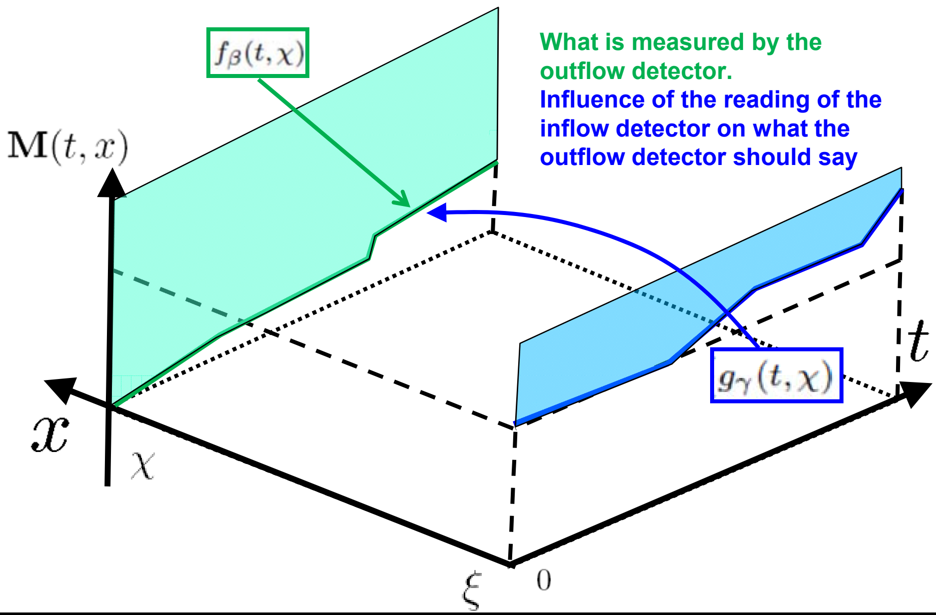
Adding trajectories is equivalent to adding epigraphs



Outline

1. **Traffic information systems**
    1. **Existing dedicated traffic monitoring infrastructure**
    2. **Web 2.0 on wheels**
2. **Mobile Millennium**
    1. **System**
    2. **Privacy aware sampling**
3. **Inverse modeling and data assimiliation**
    1. **A short introduction to traffic modeling**
    2. **The Moskowitz Hamilton-Jacobi equation**
    3. **Internal boundary conditions using the inf-morphism property**
    4. **Data assimilation in a privacy aware environment**
    5. **Mobile Century (February 8th, 2008)**
4. **The launch of Mobile Millennium**
    1. **The Bay Area**
    2. **New York**

Comparing information



Data assimilation using linear programming

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels M_i unknown

$$\begin{cases} (i) \quad \inf_{t \in \mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi) \right) \geq \Delta \end{cases}$$

Data assimilation using linear programming

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels M_i unknown

Condition on the outflow due to the inflow measurement

$$(i) \quad \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta$$

Data assimilation using linear programming

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Reading of our outflow sensor

$$(i) \quad \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta$$

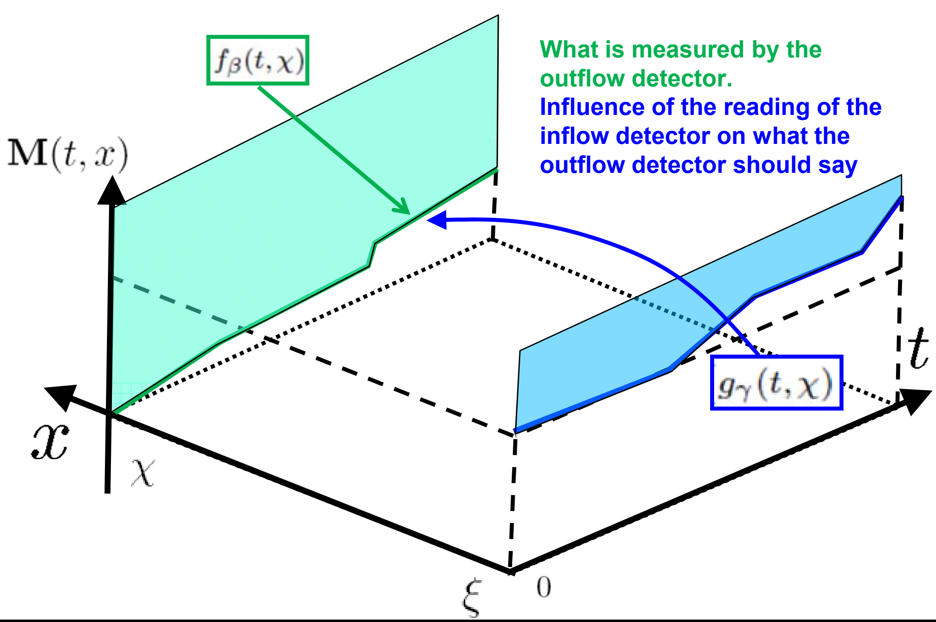
Data assimilation using linear programming

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels M_i **unknown**

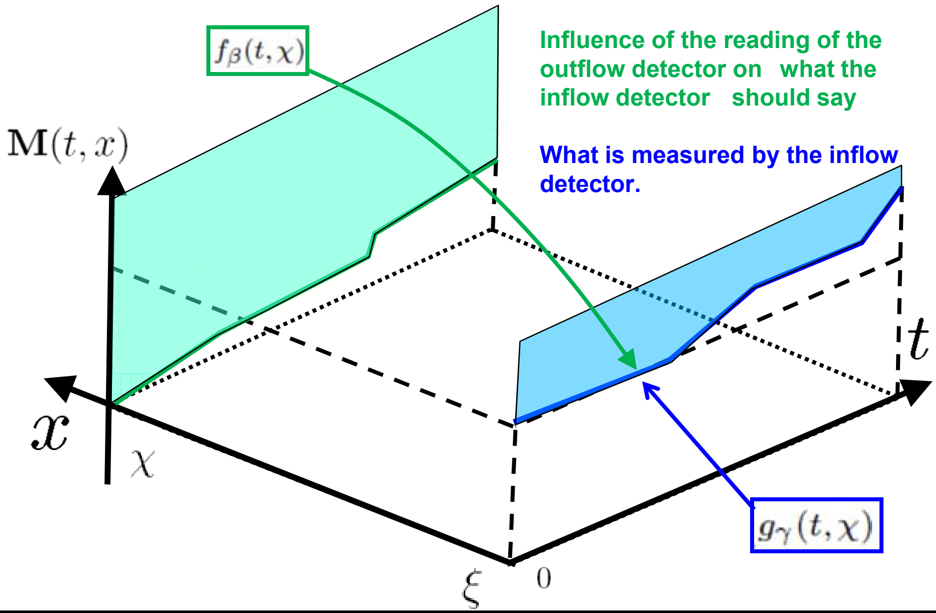
Initial number of vehicles (to be estimated)

$$\begin{cases} (i) \quad \inf_{t \in \mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi) \right) \geq \Delta \end{cases}$$

Comparing information



Comparing information



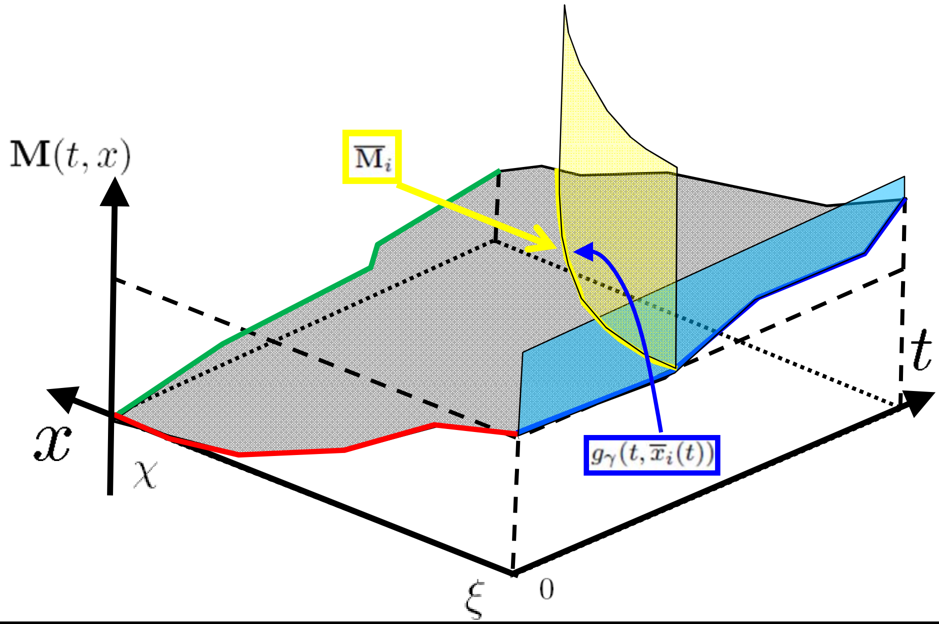
Data assimilation using linear programming

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels M_i **unknown**

Symmetric condition

$$\begin{cases} (i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta \\ (ii) & \Delta \geq \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) \end{cases}$$

Adding trajectories is equivalent to adding epigraphs



Data assimilation using linear programming

Initial condition unknown -Δ

Left and right boundary conditions known

Internal conditions known, but labels M_i **unknown**

Influence of the inflow measurement on the label of the trajectory

$$
\begin{cases}
(i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta \\
(ii) & \Delta \geq \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) \\
(iii) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\gamma(t,\overline{x}_i(t))\right) \geq \overline{\mathbf{M}}_i \qquad \forall i \in I
\end{cases}
$$

Data assimilation using linear programming

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels M_i unknown

Label of vehicle i
(to be estimated)

$$\begin{cases} (i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta & \\ (ii) & \Delta \geq \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) & \\ (iii) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\gamma(t,\overline{x}_i(t))\right) \geq \overline{\mathrm{M}}_i & \forall i \in I \end{cases}$$

Data assimilation using linear programming

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels M_i unknown

Similar condition between outflow and label estimated by the trajectory measurement

$$\begin{cases} (i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta & \\ (ii) & \Delta \geq \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) & \\ (iii) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\gamma(t,\overline{x}_i(t))\right) \geq \overline{\mathrm{M}}_i & \forall i \in I \\ (iv) & \overline{\mathrm{M}}_i \geq \sup_{t\in\mathbb{R}_+} \left(f_\gamma(t,\xi) - g_{\mu_i}(t,\xi)\right) & \forall i \in I \end{cases}$$

Data assimilation using linear programming

Initial condition unknown -Δ

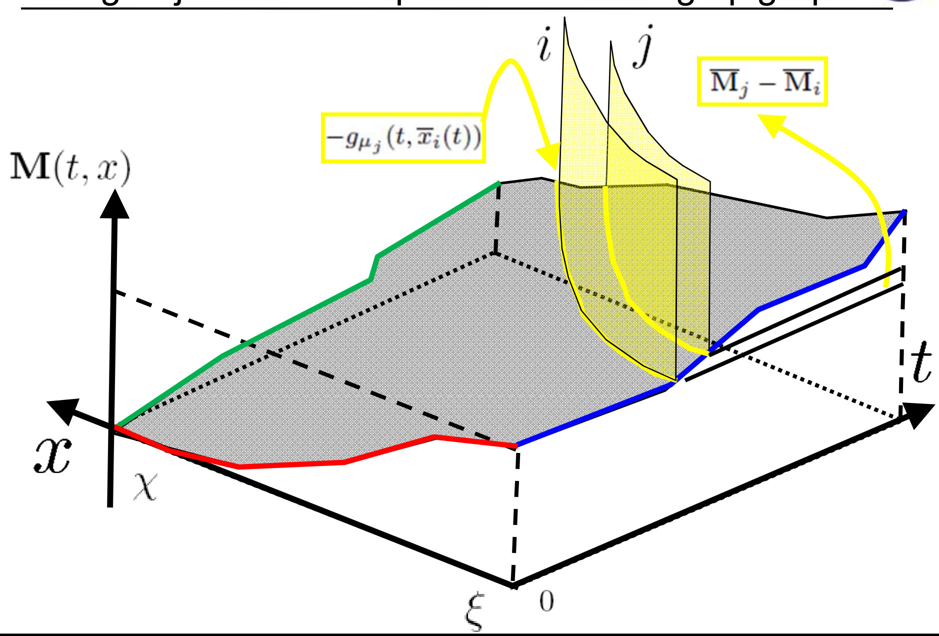
Left and right boundary conditions known

Internal conditions known, but labels M_i **unknown**

Similar conditions

$$
\begin{cases}
(i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta & \\
(ii) & \Delta \geq \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) & \\
(iii) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\gamma(t,\overline{x}_i(t))\right) \geq \overline{\mathbf{M}}_i & \forall i \in I \\
(iv) & \overline{\mathbf{M}}_i \geq \sup_{t\in\mathbb{R}_+} \left(f_\gamma(t,\xi) - g_{\mu_i}(t,\xi)\right) & \forall i \in I \\
(v) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\beta(t,\overline{x}_i(t))\right) \geq -\Delta + \overline{\mathbf{M}}_i & \forall i \in I \\
(vi) & \overline{\mathbf{M}}_i - \Delta \geq \sup_{t\in\mathbb{R}_+} \left(f_\beta(t,\chi) - g_{\mu_i}(t,\chi)\right) & \forall i \in I
\end{cases}
$$

Adding trajectories is equivalent to adding epigraphs



Data assimilation using linear programming

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels M_i unknown

Difference in possible labels of vehicle i and j (unknown)

$$\begin{cases}
(i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta \\
(ii) & \Delta \geq \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) \\
(iii) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\gamma(t,\overline{x}_i(t))\right) \geq \overline{\mathrm{M}}_i & \forall i \in I \\
(iv) & \overline{\mathrm{M}}_i \geq \sup_{t\in\mathbb{R}_+} \left(f_\gamma(t,\xi) - g_{\mu_i}(t,\xi)\right) & \forall i \in I \\
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(vi) & \overline{\mathrm{M}}_i - \Delta \geq \sup_{t\in\mathbb{R}_+} \left(f_\beta(t,\chi) - g_{\mu_i}(t,\chi)\right) & \forall i \in I \\
(vii) & \overline{\mathrm{M}}_j - \overline{\mathrm{M}}_i \geq \sup_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(-g_{\mu_j}(t,\overline{x}_i(t))\right) \\
& \forall i \in I, \quad \forall j \in I\backslash\{i\}
\end{cases}$$

Data assimilation using linear programming

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels M_i unknown

Constraint on the label of vehicle i based on the fact that vehicle j has a measured trajectory

$$\begin{cases}
(i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta \\
(ii) & \Delta \geq \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) \\
(iii) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\gamma(t,\overline{x}_i(t))\right) \geq \overline{\mathrm{M}}_i & \forall i \in I \\
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& \forall i \in I, \quad \forall j \in I\backslash\{i\}
\end{cases}$$

Data assimilation using linear programming

Initial condition unknown -Δ

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Grey: non linear analytical solution of the Hamilton Jacobi equation. Can be computed explicitly for piecewise affine functions, and semi-explicitly for general nonlinear functions

$$
\begin{cases}
(i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \ge \Delta & \\
(ii) & \Delta \ge \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) & \\
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\end{cases}
$$

Data assimilation using linear programming

Initial condition unknown -Δ

Left and right boundary conditions known

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$$
\begin{cases}
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(vii) & \mathrm{M}_j - \mathrm{M}_i \geq \sup_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(-g_{\mu_j}(t,\overline{x}_i(t))\right) \\
& \forall i \in I, \quad \forall j \in I\backslash\{i\}
\end{cases}
$$

Outline

1. **Traffic information systems**
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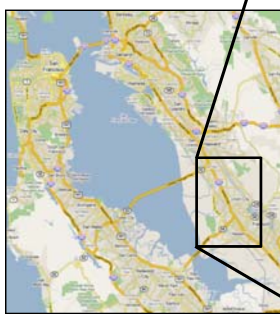
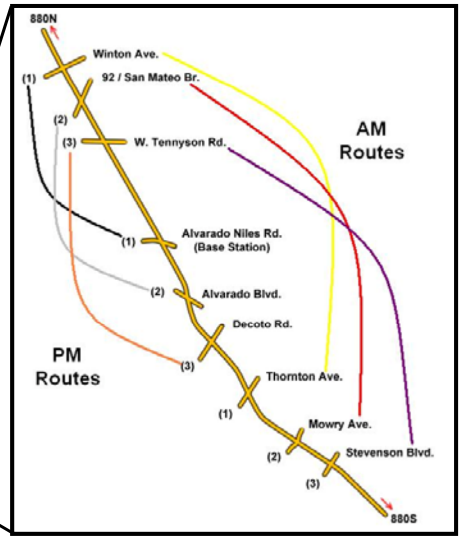
   2. New York

Prototype experiment: *Mobile Century*

Experimental proof of concept: the *Mobile Century* field test

- **February 8th 2008**
- **I80, Union City, CA**
- **Field test, 100 cars**
- **165 Berkeley students drivers**
- **10 hours deployment,**
- **About 10 miles**
- **2% - 5% penetration rate**

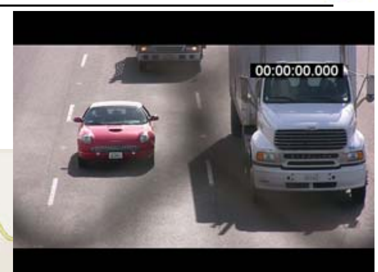


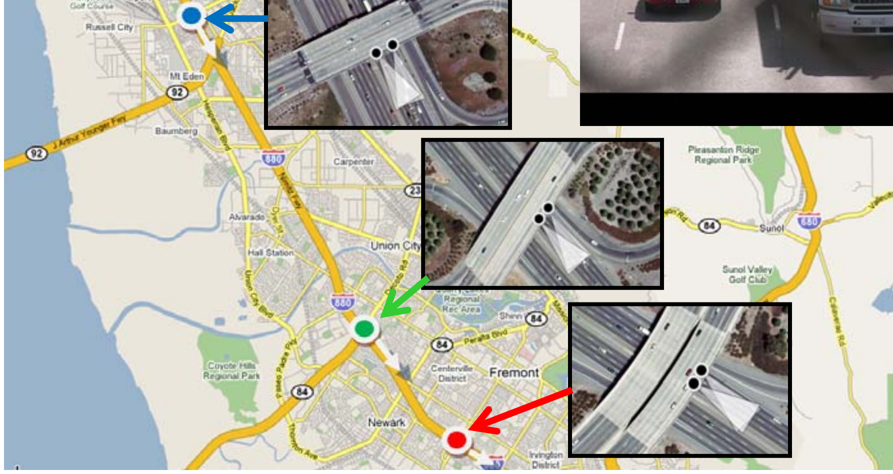


Mobile Century validation video data collection

Video data:

- **Vehicles counts**
- **Travel time validation**





A glimpse of *Mobile Century* (February 8th, 2008)

A glimpse of *Mobile Century* (February 8th, 2008)

Mobile Century experimental data

Data assimilation using linear programming

Example: Computation of the upper and lower bounds on the total number of vehicles at initial time:

Minimize (respectively Maximize): Δ
Subject to:

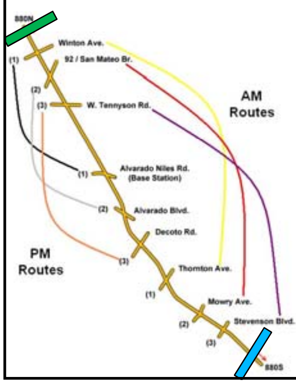
$$\begin{cases} \inf_{t \in \mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi) \right) \geq \Delta & \\ \Delta \geq \sup_{t \in \mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi) \right) & \\ \inf_{t \in [\overline{t}_{\min_i}, \overline{t}_{\max_1}]} \left(g_\gamma(t, \overline{x}_i(t)) \right) \geq \overline{\mathbf{M}}_i & \forall i \in I \\ \overline{\mathbf{M}}_i \geq \sup_{t \in \mathbb{R}_+} \left(f_\gamma(t,\xi) - g_{\mu_i}(t,\xi) \right) & \forall i \in I \\ \inf_{t \in [\overline{t}_{\min_i}, \overline{t}_{\max_1}]} \left(g_\beta(t, \overline{x}_i(t)) \right) \geq -\Delta + \overline{\mathbf{M}}_i & \forall i \in I \\ \overline{\mathbf{M}}_i - \Delta \geq \sup_{t \in \mathbb{R}_+} \left(f_\beta(t,\chi) - g_{\mu_i}(t,\chi) \right) & \forall i \in I \\ \overline{\mathbf{M}}_j - \overline{\mathbf{M}}_i \geq \sup_{t \in [\overline{t}_{\min_i}, \overline{t}_{\max_i}]} \left(-g_{\mu_j}(t, \overline{x}_i(t)) \right) & \forall i \in I, \ \forall j \in I \setminus \{i\} \end{cases}$$

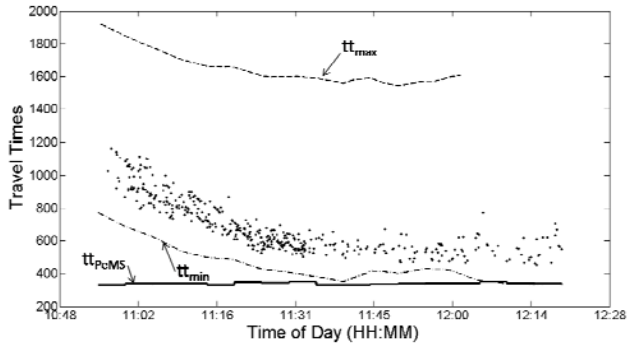
[Claudel, Bayen, subm. IEEE Trans. Autom. Contr. 2008, Part 1: general solution]
[Claudel, Bayen, subm. IEEE Trans. Autom. Contr. 2008, Part 2: explicit solution for PWA functions]
[Claudel, Bayen, in prep. 2009, SIAM SICON, data assimiliation]

Bounds on travel time (PeMS)

Outflow loop

Inflow loop

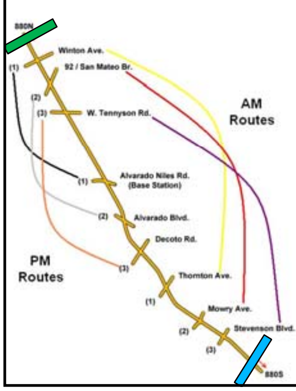


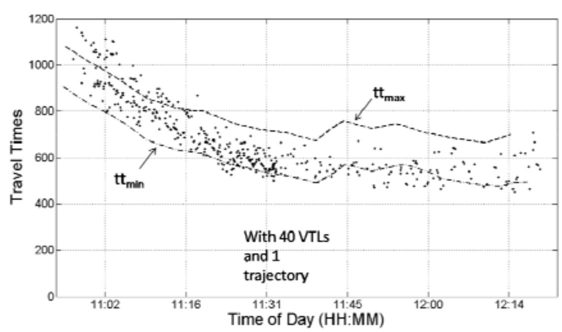


Bounds on travel time (PeMS and phones)

Outflow loop

Inflow loop

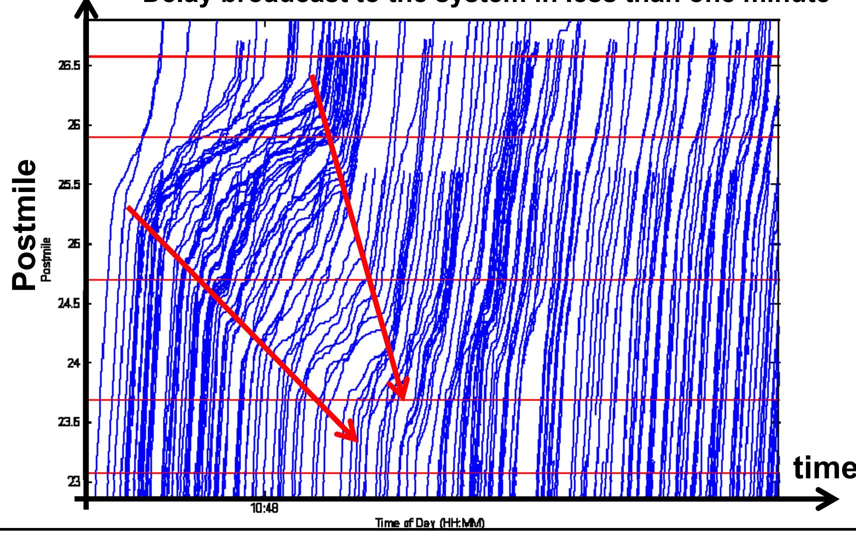




Revealing the previously unobservable

5 car pile up accident (not Mobile Century vehicles)

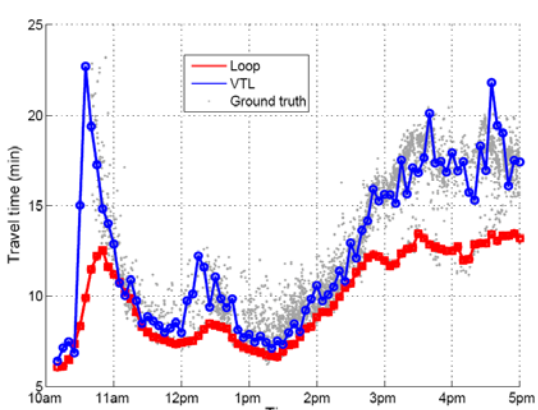
- Captured in real time
- Delay broadcast to the system in less than one minute

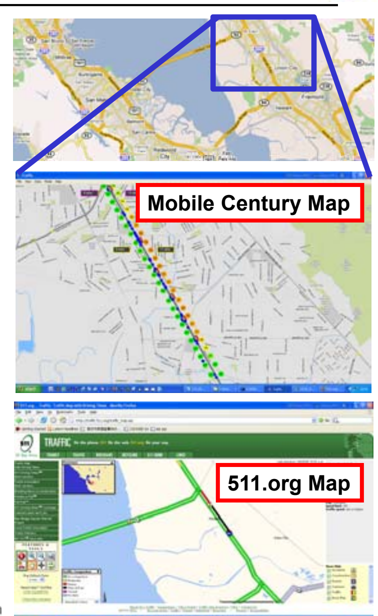


Validation of the data (video)

Travel time predictions

- Can be done in real time at a 2% penetration rate of traffic
- Proved accurate against data from www.511.org, with higher degree of granularity





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Mobile Millennium: a pilot project

Mission statement

The goal of *Mobile Millennium* is to establish the design of a system that collects data from GPS-enabled mobile phones, fuses it with data from existing sensors and turn it into relevant traffic information.

***Mobile Millennium* is a field operational test**

- **Deployment of thousands of cars on a network including arterials**
- **Participating users agree to share position and speed**
- **Phones receive live information on map application**
- **Project duration at least 6 months**
- **Mobile Millennium is a pilot**

Launch

***Mobile Millennium* was launched on November 10th, at 8:30am from the UC Berkeley campus**

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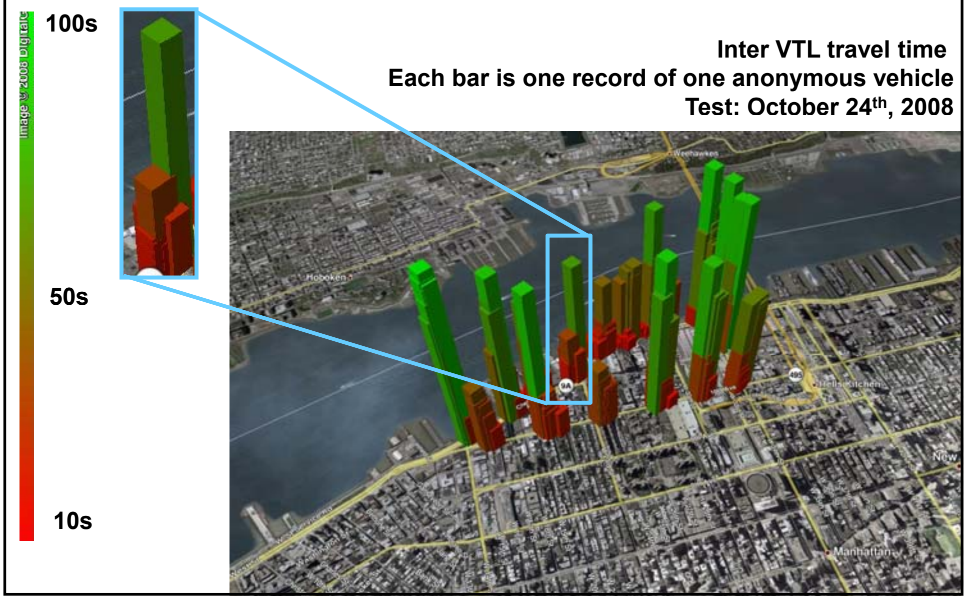
Testing the *Mobile Millennium* in Manhattan

Testing the *Mobile Millennium* in Manhattan

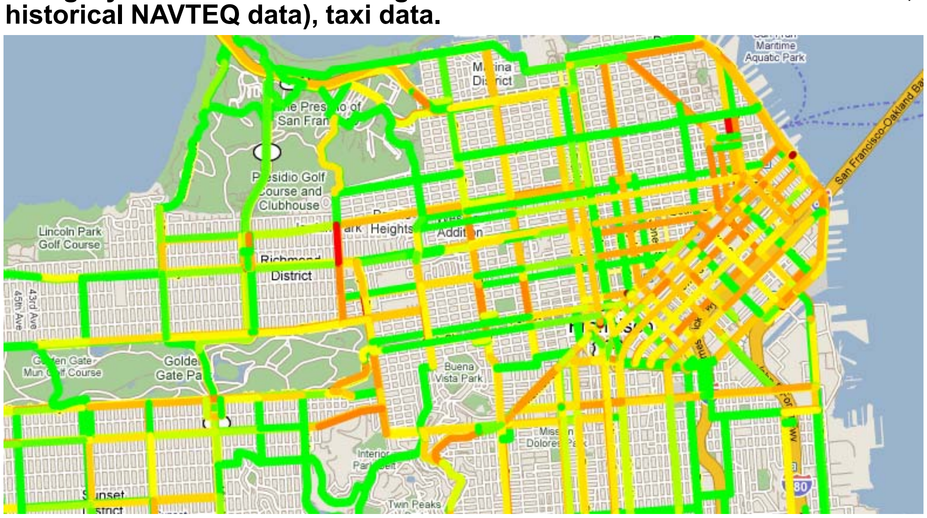
Inter VTL travel time
Each bar is one record of one anonymous vehicle
Test: October 24th, 2008



Arterial network covered by Mobile Millennium

Arterial model

Category 1 to 4 roads running in Mobile Millennium. Fuse VTL data, historical NAVTEQ data), taxi data.



MOBILE MILLENNIUM

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