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An Efficient Game for Coordinating Electric Vehicle Charging

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***Abstract*—A novel class of auction-based games is formulated to study coordination problems arising from charging a population of electric vehicles (EVs) over a finite horizon. To compete for energy allocation over the horizon, each individual EV submits a multidimensional bid, with the dimension equal to two times the number of time-steps in the horizon. Use of the progressive second price (PSP) auction mechanism ensures that incentive compatibility holds for the auction games. However, due to the cross elasticity of EVs over the charging horizon, the marginal valuation of an individual EV at a particular time is determined by both the demand at that time and the total demand over the entire horizon. This difficulty is addressed by partitioning the allowable set of bid profiles based on the total desired energy over the entire horizon. It is shown that the efficient bid profile over the charging horizon is a Nash equilibrium of the underlying auction game. An update mechanism for the auction game is designed. A numerical example demonstrates that the auction process converges to an efficient Nash equilibrium. The auction-based charging coordination scheme is adapted to a receding horizon formulation to account for disturbances and forecast uncertainty.**

***Index Terms*—Electric vehicles; progressive second price auction; incentive compatibility; cross elasticity; game theory; Nash equilibrium.**

I. Introduction

Vehicles that connect to the electricity grid to recharge, referred to generically as electric vehicles (EVs), offer a range of potential benefits, including reductions in reliance on liquid fuels and in pollutant emissions, and increased energy efficiency [1]–[3]. It is therefore anticipated that EV sales will substantially increase over the next few years [4]. If such growth does eventuate, it will become necessary to account for EV charging patterns in grid operation, as argued in [4]–[7] and references therein.

Accommodating large numbers of vehicles on the grid will require coordination of EV charging so that their power and energy requirements can be optimally and robustly satisfied. This is a challenging control problem. Work on analyzing EV charging schedules, and their effect on utilities, began in the 1980s [8]. Recent work is extensive and includes [9]–[11] which formulate EV charging control as constrained optimization problems, and [12] which develops an EV dispatch algorithm in the context of a day-ahead electricity market.

Centralized coordination faces numerous difficulties, from computational complexity to the loss of EV decision-making autonomy. Many distributed coordination methods have been proposed to address those challenges, including [11], [13]–[22] and references therein. In a general sense, that work is structured around individual players determining their optimal charging strategy over the charging horizon with respect to either the total demand of the other players, or the system (clearing) price, which is based on the total system demand. More specifically, a hierarchical structure is considered in [14], [15], [20], [21] for scheduling EV charging. Each EV determines its preliminary charging (load) profile by solving an individual optimization problem with respect to the latest forecast of the system clearing price. The clearing price is then updated to take into account the latest charging profiles of the individual EVs. For the update process designed in [14], the resulting strategies asymptotically approach a Nash equilibrium (NE) as the EV population increases to infinity. The resulting NE is nearly socially optimal. The market design in [21] considers both energy and reserve capacity[1], and also incorporates distribution network losses. It is shown that in such a market setting, the NE may not coincide with the socially optimal solution. To establish a tractable formulation, only energy markets will be considered in the remainder of the paper. Also, network losses are not modelled.

Most of the distributed methods cited above are quite distinct from the economic generation dispatch that underpins deregulated day-ahead electricity markets [23]. To economically dispatch generation, auction mechanisms, such as uniform market-clearing-price [24] and pay-as-bid [25], have been widely adopted in electricity markets around the world [26]. Each generating unit submits to the ISO their bids over the forward market period (typically 24 hours), with bids consisting of pairings of minimal selling price and maximum supplied electricity for each market subinterval. The ISO dispatches the generation requirements among units based on their submitted bid profiles. However these auction mechanisms do not achieve incentive compatibility and usually cannot attain the efficient solution [27]. In contrast, this paper studies EV charging coordination over multiple time intervals under an incentive compatibility mechanism [28], [29]. The paper utilizes a progressive second price (PSP) auction mechanism, designed by Lazar and Semret [30], [31] and initially applied in the allocation of network resources.

In a single divisible resource allocation problem under the PSP auction mechanism, each player only reports a two-dimensional bid. This bid is composed of a maximum amount of demand and an associated buying price, and is used to replace the player's complete (private) utility function. Under the PSP mechanism, the money transfer (or payment) of a player measures the externality that they impose on the system through their participation. This concept will be formalized in Section III-B. As analyzed in [30], [31], the PSP auction mechanism is a VCG-style auction [32]–[34]. Therefore incentive compatibility holds, ensuring that all players submit truth-telling bids, and resources are allocated efficiently. Under this mechanism, as verified in [31], [35] in the context of single-unit network resource allocations, the efficient bid profile is a NE.

In formulating their bids, players must consider tradeoffs between energy costs that vary over the charging horizon, the benefit derived from the total acquired energy, and battery degradation. Individual EVs are therefore inter-temporal cross-elastic[2] loads, as defined in [36]. This results in an auction-based allocation of a collection of divisible resources, where electric energy at each time-step of the horizon is a separate divisible resource. Consequently, each EV must submit a bid that has dimension double the number of divisible resources to be shared (equivalently double the number of time-steps in the charging horizon). Such auctions have received limited

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[1]Reserves refer to generator and/or load capacity that is available to compensate for sudden changes in energy supply or demand.

[2]Cross elasticity refers to the ability of EVs to move charging demand from one time interval to another.

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attention in the literature. It will be shown that a player's marginal valuation for electric energy at a particular time is dependent upon both the amount of energy requested at that time and the total energy request over the entire charging horizon. A key contribution of the paper is to show that the efficient set of EV bids over the charging horizon is a NE of the underlying auction game. However, due to the cross elasticity of a bid over the multi-step time horizon, it is infeasible to directly verify this NE property using analysis that is applicable for a single-resource auction game [31], [35]. An alternative approach is proposed in this paper.

In order to address cross elasticity, the set of bids of a player is partitioned into a collection of subsets, each of which is composed of bids that possess the same total desired electric energy over the horizon. Consequently, cross elasticity is eliminated for bids within each subset. For such bids, the marginal valuation at each time-step includes a *variable part* determined by the amount of energy requested at that time and a *fixed part* that is identical for all bids in that subset. With this construction, the NE property of the efficient solution can be established by verifying for each subset whether any player can benefit by unilaterally deviating from their efficient bid profile.

The paper is organized as follows. Section II establishes the problem structure by formulating a class of EV charging coordination problems over a multiple time-step horizon. Distributed charging under the PSP auction mechanism is introduced in Section III. Section IV shows that the efficient (truth-telling) bid profile is a NE of the underlying PSP auction game. In Section V, an update process is designed to implement the PSP auction, and an example illustrates that this process converges to the efficient NE. Section VI concludes the paper and provides a discussion of future work.

II. Electric Vehicle Charging Coordination Formulation

A. Charging and cost models

This study focuses on coordinating the charging of a population of EVs, denoted by $\mathcal{N}$, finite charging horizon $\mathcal{T} \triangleq \{t_0, \cdots, t_0 + T - 1\}$, with t_0 representing the initial time. For each EV, $n \in \mathcal{N}$, the energy delivered [3] over the t-th time interval is denoted by x_{nt}, and the battery state of charge (SoC) evolves according to:

$$s_{n,t+1} = s_{nt} + \frac{1}{\Theta_n} x_{nt}, \tag{1}$$

where Θ_n is the battery capacity and s_{nt} is the normalized SoC for the n-th EV at time t. An *admissible charging strategy*, $\boldsymbol{x}_n \equiv (x_{nt}, t \in \mathcal{T})$, satisfies the constraints:

$$x_{nt} \begin{cases} \geq 0, & \text{when } t \in \mathcal{T}_n \\ = 0, & \text{otherwise} \end{cases}, \quad \text{with} \sum_{t \in \mathcal{T}} x_{nt} \leq \Gamma_n, \tag{2}$$

where $\mathcal{T}_n \subset \mathcal{T}$ denotes the charging horizon of the n-th EV, $\Gamma_n = \Theta_n(\overline{s}_n - s_{n0})$ gives the maximum energy that it can receive, and $0 \leq s_{n0} \leq \overline{s}_n \leq 1$ give the normalized minimum (initial) and maximum SoC, respectively. The values for $\mathcal{T}_n$, Γ_n and s_{n0} follow from the driving style and vehicle battery capacity, see for example [37]. The set of all possible admissible charging strategies is denoted by $\mathcal{X}_n$. Also, define the collection of admissible charging strategies for all EVs by $\boldsymbol{x} \equiv (\boldsymbol{x}_n; n \in \mathcal{N})$, with its corresponding set being $\mathcal{X}$.

As specified in (2), the charging demand of each EV, $n \in \mathcal{N}$, at each time interval is elastic, but charging over the time horizon is coupled through the maximum energy Γ_n that can be delivered during that time. As a consequence, EV coordination is inherently a problem of scheduling demand that exhibits inter-temporal cross-elasticity [36].

The utility function of the n-th EV, for a charging strategy $\boldsymbol{x}_n$, is given by:

$$w_n(\boldsymbol{x}_n) = -\sum_{t \in \mathcal{T}} f_n(x_{nt}) - \delta_n \Big(\sum_{t \in \mathcal{T}} x_{nt} - \Gamma_n \Big)^2, \tag{3}$$

where $\delta_n > 0$ is a fixed parameter, and $f_n(\cdot)$ denotes the battery degradation cost of the n-th EV.

Remarks:

- This work considers LiFePO4 lithium-ion batteries which have been widely used in a variety of EVs. Key characteristics of this type of battery, including state of health, growth of resistance, and cycle life, are effected by charging behavior over many cycles. A degradation cost model for this type of battery cell is formulated in [38], based on the evolution of battery cell characteristics developed in [39], [40]. This degradation model expresses the energy capacity loss per second of a cell with respect to the charging current and voltage. Using this model, the relationship between degradation cost and charging rate x_{nt} can be developed.
- The second term in (3) captures the penalty cost due to not fully charging the EV over the time horizon, with δ_n weighting the relative importance of delivering the maximum energy during charging [41].
- The utility function $w_n(\boldsymbol{x}_n)$ therefore establishes the tradeoff between the battery degradation cost and the benefit derived from delivering the full charge.

For a collection of admissible charging strategies $\boldsymbol{x}$, the system cost is given by:

$$J_s(\boldsymbol{x}) = \sum_{t \in \mathcal{T}} c_t(D_t + \sum_{n \in \mathcal{N}} x_{nt}) - \sum_{n \in \mathcal{N}} w_n(\boldsymbol{x}_n), \tag{4}$$

where $c_t(\cdot)$, D_t and $D_t + \sum_{n \in \mathcal{N}} x_{nt}$ denote the generation cost, the aggregate inelastic background demand and the total demand at time t respectively. It is assumed that a forecast for D_t is available over the charging horizon. For the example presented in Section V-C, the demand profile $\boldsymbol{D} \equiv (D_t; t \in \mathcal{T})$ is given by the aggregate demand for a typical summer day in the Midwest ISO region of North America.

The system cost function (4) considers tradeoffs between total generation cost, aggregate battery costs and the penalty for deviating from full charging. This contrasts with the general literature, for example [14], [15], [18], where the objective is to achieve valley-filling. While valley filling minimizes the total generation cost, it may result in high battery degradation costs across the EV population.

B. Efficient charging

It is desirable to determine the collection of *efficient* (socially optimal) charging strategies $\boldsymbol{x}^{**}$ that minimizes the system cost (4). This centralized EV charging coordination problem can be formulated as the following optimization problem:

Problem 1:

$$\min_{\boldsymbol{x} \in \mathcal{X}} \; J_s(\boldsymbol{x}),$$

such that $\boldsymbol{x}$ satisfies constraints (2) for all $n \in \mathcal{N}$. ■

The efficient charging strategy $\boldsymbol{x}^{**}$ of Problem 1 can be characterized by its associated KKT conditions. Firstly, the Lagrangian can be written:

$$L(\boldsymbol{x}, \boldsymbol{\lambda}) = J_s(\boldsymbol{x}) + \sum_{n \in \mathcal{N}} \lambda_n \Big(\sum_{t \in \mathcal{T}} x_{nt} - \Gamma_n \Big),$$

[3] It is assumed that the energy is delivered at a constant rate (power) over each time interval, and that the time intervals are of unit length. Therefore the charging rate is also given by x_{nt}.

where λ_n is the Lagrangian multiplier associated with the constraint $\sum_{t\in\mathcal{T}} x_{nt} \leq \Gamma_n$ from (2). The KKT conditions for Problem 1 are therefore given by:

$$\frac{\partial}{\partial x_{nt}} L(\boldsymbol{x},\boldsymbol{\lambda}) \geq 0, \quad x_{nt} \geq 0, \qquad \frac{\partial}{\partial x_{nt}} L(\boldsymbol{x},\boldsymbol{\lambda}) x_{nt} = 0, \tag{5a}$$

$$\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n \leq 0, \quad \lambda_n \geq 0, \quad \lambda_n \Big(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n \Big) = 0, \tag{5b}$$

for all $t \in \mathcal{T}$ and $n \in \mathcal{N}$, where:

$$\frac{\partial}{\partial x_{nt}} L(\boldsymbol{x},\boldsymbol{\lambda}) = c_t'(D_t + \sum_{n\in\mathcal{N}} x_{nt}) - \frac{\partial}{\partial x_{nt}} w_n(\boldsymbol{x}_n) + \lambda_n. \tag{5c}$$

Assumptions: The following conditions apply throughout the remainder of the paper:

(A1) $c_t(y)$ is monotonically increasing, strictly convex and differentiable on y;

(A2) $f_n(x)$, for all $n \in \mathcal{N}$, is monotonically increasing, strictly convex and differentiable on x.

Remarks:

- The generation cost $c_t(\cdot)$ is widely assumed to be a convex function of total generation, see for example [42]–[44].
- The battery degradation cost $f_n(\cdot)$ is governed by the chemical processes inherent in charging. It is shown in Fig. 7 of [39] that growth of battery resistance, hence the fade of battery energy capacity, is generally increasing and convex with respect to charging rate. This provides some justification for (A2) since f_n measures the cost related to the fade of battery capacity with respect to charging rate.

Lemma 2.1: The collection of efficient charging strategies $\boldsymbol{x}^{**}$ for Problem 1 is unique.

Proof. Under Assumptions (A1, A2), the cost function $J_s(\boldsymbol{x})$ is strictly convex and differentiable. Also, the constraints (2) determine a convex domain. Therefore Problem 1 is a strictly convex optimization problem. Thus there exists a unique solution. ■

This *centralized* charging coordination strategy can only be effectively implemented when the system has complete information and can directly schedule the behavior of all EVs. In practice, however, individuals are often unwilling to share their private information with others. Furthermore, transmission of complete information may incur excessive communications, and centralized control might be computationally infeasible. Thus this paper focuses on the development of a distributed control method that is based on the *progressive second price (PSP) auction mechanism*, which has been applied in [30], [31], [35] for efficient allocation of single-unit network resources. However, because EVs are scheduled over a multiple time-step horizon, charging coordination is a multi-unit resource allocation problem.

III. Distributed EV Charging Coordination under a PSP Auction Mechanism

A. Bid profiles of individual players

Each EV, $n \in \mathcal{N}$, submits a $2T$-dimensional bid, $\boldsymbol{b}_n \equiv (b_{nt}, t \in \mathcal{T})$, where:

$$b_{nt} = (\beta_{nt}, d_{nt}), \text{ with}$$
$$d_{nt} \begin{cases} \geq 0, & t \in \mathcal{T}_n \\ = 0, & \text{otherwise} \end{cases}, \text{ and } \sum_{t\in\mathcal{T}} d_{nt} \leq \Gamma_n,$$

specifies the price β_{nt} that player n is willing to pay for energy at time t and the maximum electrical energy d_{nt} that is desired at that time. The corresponding feasible allocation $\boldsymbol{x}_n \equiv (x_{nt}, t \in \mathcal{T})$ with respect to $\boldsymbol{b}_n$ must satisfy:

$$0 \leq x_{nt} \leq d_{nt}, \quad \forall t \in \mathcal{T}. \tag{6}$$

Let $\mathcal{B}_n$ denote the allowable set of bids for player n, so that $\boldsymbol{b}_n \in \mathcal{B}_n$.

Each player's revealed utility function, denoted by $\widehat{w}_n(\boldsymbol{x}_n(\boldsymbol{b}_n); \boldsymbol{b}_n)$, is defined as:

$$\widehat{w}_n(\boldsymbol{x}_n(\boldsymbol{b}_n); \boldsymbol{b}_n) \triangleq \sum_{t\in\mathcal{T}} \beta_{nt} x_{nt}.$$

The revealed system cost with respect to a bid profile $\boldsymbol{b} \equiv (\boldsymbol{b}_n, n \in \mathcal{N})$ is given by:

$$J(\boldsymbol{x}(\boldsymbol{b}); \boldsymbol{b}) = \sum_{t\in\mathcal{T}} c_t(D_t + \sum_{n\in\mathcal{N}} x_{nt}) - \sum_{n\in\mathcal{N}} \widehat{w}_n(\boldsymbol{x}_n(\boldsymbol{b}_n); \boldsymbol{b}_n). \tag{7}$$

Auction-based EV charging allocation can be written as the following optimization problem:

Problem 2:

$$J^*(\boldsymbol{b}) = \min_{\text{Constraint (6)}} J(\boldsymbol{x}(\boldsymbol{b}); \boldsymbol{b}).$$

The objective of the auctioneer is to assign an optimal allocation $\boldsymbol{x}^*(\boldsymbol{b})$ with respect to a bid profile $\boldsymbol{b}$ to minimize the revealed system cost given by J. ■

Unlike the single-sided auctions considered in [30], [31], the cross elasticity inherent in Problem 2 suggests that the optimal allocation $\boldsymbol{x}^*(\boldsymbol{b})$ over the charging horizon $\mathcal{T}$ depends upon both the bid profile $\boldsymbol{b}$ over the entire horizon and the generation cost $c_t(\cdot)$. However, the following lemma shows that the optimal allocation $\boldsymbol{x}_t^*$ at time t is completely determined by the bid profile $\boldsymbol{b}_t$ and $c_t(\cdot)$ at only that time.

Lemma 3.1: Suppose $\boldsymbol{x}^*(\boldsymbol{b}) \equiv (\boldsymbol{x}_t^*, t \in \mathcal{T})$ is the optimal allocation subject to bid profile $\boldsymbol{b}$. Then:

$$\boldsymbol{x}_t^*(\boldsymbol{b}) \equiv \boldsymbol{x}_t^*(\boldsymbol{b}_t), \qquad \text{for all } t \in \mathcal{T}.$$

Proof. For notational simplicity, consider:

$$\mathcal{S}(\boldsymbol{b}_t) \equiv \{\boldsymbol{x}_t; \text{ s.t. } 0 \leq x_{nt} \leq d_{nt}, \text{ for all } n \in \mathcal{N}\},$$
$$h_t(\boldsymbol{x}_t; \boldsymbol{b}_t) \equiv c_t(D_t + \sum_{n\in\mathcal{N}} x_{nt}) - \sum_{n\in\mathcal{N}} \beta_{nt} x_{nt}.$$

It follows from (7) that:

$$\min_{\text{Constraint (6)}} J(\boldsymbol{x}(\boldsymbol{b}); \boldsymbol{b}) = \min_{\boldsymbol{x}_t \in \mathcal{S}(\boldsymbol{b}_t),\, t\in\mathcal{T}} \sum_{t\in\mathcal{T}} h_t(\boldsymbol{x}_t; \boldsymbol{b}_t)$$
$$= \sum_{t\in\mathcal{T}} \min_{\boldsymbol{x}_t \in \mathcal{S}(\boldsymbol{b}_t)} h_t(\boldsymbol{x}_t; \boldsymbol{b}_t),$$

where the last equality holds because the summation is separable in terms of $\boldsymbol{x}_t$. The desired conclusion follows. ■

The optimal charging allocation $\boldsymbol{x}^*(\boldsymbol{b})$ of Problem 2 can be characterized by the associated KKT conditions. Firstly, the Lagrangian can be written:

$$L^a(\boldsymbol{x}, \boldsymbol{\sigma}; \boldsymbol{b}) = J(\boldsymbol{x}(\boldsymbol{b}); \boldsymbol{b}) + \sum_{n\in\mathcal{N}} \sum_{t\in\mathcal{T}} \sigma_{nt}(x_{nt} - d_{nt}),$$

where σ_{nt} is the Lagrangian multiplier associated with each constraint in (6). The KKT conditions for Problem 2 are given by:

$$\frac{\partial}{\partial x_{nt}} L^a(\boldsymbol{x}, \boldsymbol{\sigma}; \boldsymbol{b}) \geq 0, \quad x_{nt} \geq 0, \quad \frac{\partial}{\partial x_{nt}} L^a(\boldsymbol{x}, \boldsymbol{\sigma}; \boldsymbol{b}) x_{nt} = 0, \tag{8a}$$

$$x_{nt} - d_{nt} \leq 0, \quad \sigma_{nt} \geq 0, \qquad (x_{nt} - d_{nt})\sigma_{nt} = 0, \tag{8b}$$

for all $t \in \mathcal{T}$ and $n \in \mathcal{N}$, where:

$$\frac{\partial}{\partial x_{nt}} L^a(\boldsymbol{x}, \boldsymbol{\sigma}; \boldsymbol{b}) = c_t'(D_t + \sum_{k\in\mathcal{N}} x_{kt}) - \beta_{nt} + \sigma_{nt}. \tag{8c}$$

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It is now possible to establish a connection between the optimal charging strategies given by Problems 1 and 2.

Lemma 3.2: Consider a collection of bids,

$$b_{nt}^* = (\beta_{nt}^*, d_{nt}^*) = \left(\frac{\partial}{\partial x_{nt}} w_n(\boldsymbol{x}_n^{**}), x_{nt}^{**} \right), \tag{9}$$

for all $n \in \mathcal{N}$ and $t \in \mathcal{T}$. Then, under Assumptions (A1, A2), $\boldsymbol{x}^*(\boldsymbol{b}^*) = \boldsymbol{x}^{**}$. Also,

$$\beta_{nt}^* \begin{cases} = c_t'(D_t + \sum_{k\in\mathcal{N}} d_{kt}^*), & \text{if } x_{nt}^* > 0 \\ \leq c_t'(D_t + \sum_{k\in\mathcal{N}} d_{kt}^*), & \text{if } x_{nt}^* = 0, \end{cases} \tag{10}$$

for all $n \in \mathcal{N}$ and $t \in \mathcal{T}$.

This lemma is essentially the so-called *fundamental theorem of welfare economics* [45]. Verification is provided in Appendix A. ■

Remark: Lemma 3.2 establishes two important properties:

(i) It specifies a bid profile $\boldsymbol{b}^*$, given by (9), under which the optimal charging allocation $\boldsymbol{x}^*$ of Problem 2 is *efficient*.

(ii) At bid profile $\boldsymbol{b}^*$, EVs with an allocation larger than zero share the same marginal price as generation, which is larger than or equal to the price of EVs with zero allocation.

Incentive compatibility holds under the PSP auction mechanism [30], [31]. Therefore, a bid with price satisfying $\beta_{nt} = \frac{\partial}{\partial d_{nt}} w_n(\boldsymbol{d}_n)$, for all $t \in \mathcal{T}$, as is the case in (9), is the best choice among all possible bids. It follows from (3) that the truth-telling bid of the n-th EV is given by:

$$\beta_{nt}(d_{nt}; \sum_{s\in\mathcal{T}} d_{ns}) = -f_n'(d_{nt}) + 2\delta_n \Big(\Gamma_n - \sum_{s\in\mathcal{T}} d_{ns} \Big). \tag{11}$$

This implies that an EV's marginal valuation at each time-step is determined by both its electrical energy request d_{nt} at that time and its total energy request $\sum_{s\in\mathcal{T}} d_{ns}$ over the entire multi-period charging horizon.

B. Calculation of EV payment and payoff

The payment incurred by each EV will be specified with respect to the allocation law defined by Problem 2. Each EV's payment is exactly the externality imposed on the system through its participation in the auction. For the n-th EV, this is given by the system-wide utility when the n-th EV does not join the auction process, minus the system-wide utility (but excluding the contribution of the n-th EV itself) when the n-th EV joins the auction.

To express this payment, it is convenient to introduce a slight abuse of notation by writing the collection of bids as $\boldsymbol{b} \equiv (\boldsymbol{b}_n, \boldsymbol{b}_{-n})$, where $\boldsymbol{b}_{-n} \equiv (\boldsymbol{b}_k, k \in \mathcal{N} \setminus \{n\})$. The payment of the n-th EV, for a bid profile $\boldsymbol{b}$, is then given by:

$$\tau_n(\boldsymbol{b}) = -J^*(\boldsymbol{0}_n, \boldsymbol{b}_{-n}) - \left(-J^*(\boldsymbol{b}) - \sum_{t\in\mathcal{T}} \beta_{nt} x_{nt}^*(\boldsymbol{b}) \right), \tag{12}$$

where $(\boldsymbol{0}_n, \boldsymbol{b}_{-n})$ denotes the bid profile without the n-th EV's participation, i.e., with the bid d_{nt} replaced by $d_{nt} = 0$ for all $t \in \mathcal{T}$, and $\boldsymbol{x}^*(\boldsymbol{b})$ is the optimal charging allocation given by Problem 2, with respect to $\boldsymbol{b}$. Thus by (7), (12) and Lemma 3.1, the payment of player n at time t can be expressed as:

$$\begin{aligned} \tau_{nt}(\boldsymbol{b}_t) = &- c_t(D_t + \sum_{m\neq n} x_{mt}^{*,-n}) + c_t(D_t + \sum_{k\in\mathcal{N}} x_{kt}^*(\boldsymbol{b}_t)) \\ &+ \sum_{m\neq n} \beta_{mt} \left(x_{mt}^{*,-n} - x_{mt}^*(\boldsymbol{b}_t) \right), \end{aligned}$$

where $x_{mt}^{*,-n}$ denotes the optimal charging allocation of EVs $m \neq n$ given by Problem 2 with respect to $(\boldsymbol{0}_n, \boldsymbol{b}_{-n})_t$. (Recall from Lemma 3.1 that allocations at time t are unrelated to other times.) The total payment of player n is given by:

$$\tau_n(\boldsymbol{b}) = \sum_{t\in\mathcal{T}} \tau_{nt}(\boldsymbol{b}_t). \tag{13}$$

The *payoff function* of the n-th EV is given by the difference between the EV's utility and its payment:

$$u_n(\boldsymbol{b}) = w_n(\boldsymbol{x}_n^*(\boldsymbol{b})) - \tau_n(\boldsymbol{b}). \tag{14}$$

This payoff function provides the basis for defining a Nash equilibrium for the PSP auction game.

Definition 1: A collection of bid profiles $\boldsymbol{b}^0$ is a *Nash equilibrium* for Problem 2 if:

$$u_n(\boldsymbol{b}_n^0, \boldsymbol{b}_{-n}^0) \geq u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}^0),$$

for all $\boldsymbol{b}_n \in \mathcal{B}_n$ and for all $n \in \mathcal{N}$. That is, no EV can benefit by unilaterally deviating from its bid profile $\boldsymbol{b}_n^0$. ■

C. Related work on EV charging games

1) Hierarchical EV charging games: Many of the distributed schemes that have been proposed for scheduling EV charging have adopted a hierarchical structure, see for example [14], [15], [20], [21]. Each EV first determines its optimal charging schedule with respect to the expected energy price that is broadcast by the ISO. The price is then updated based on the latest charging schedules attained by the population of EVs. This process repeats until price updates become negligible. Typically, the outcome of such a scheme approaches a NE asymptotically as the size of the EV population approaches infinity [14]. In contrast, the auction game formulated in this paper achieves a NE for small collections of EVs.

2) Efficiency of NE for auction games: The tradeoff between efficiency and risk is considered in [22] in the context of scheduling load over a finite time horizon in oligopoly electricity markets. In non-cooperative schemes, the system may suffer a loss, known as the price of anarchy, which relates to the difference between the efficient solution and the NE. The PSP auction proposed in this paper circumvents that loss by ensuring the NE is efficient.

Efficient games have been studied in a variety of related resource allocation problems. The energy consumption scheduling game formulated in [18] does not employ user utility and considers a linear payment rule that differs from VCG. It achieves a globally optimal NE that minimizes the total generation cost. The efficiency of single-unit network resource allocation is studied in [30], [31], [35], though EV charging coordination is essentially a multi-unit resource allocation problem.

IV. Efficiency of the Charging Coordination PSP Auction Game

This section establishes that the efficient solution given by Problem 1 is a NE of the PSP auction game. Budget balance under the PSP auction is considered in Section IV-D. Finally, it is shown in Section IV-E that for a single-interval auction game, where $T = 1$, the efficient NE is unique and so there in no efficiency loss.

Suppose $\boldsymbol{b}^*$ is the bid profile specified in Lemma 3.2, such that the corresponding optimal charging allocation is efficient. It will be shown in this section that $\boldsymbol{b}^*$ is a NE for the underlying auction game. By Definition 1, this implies:

$$u_n(\boldsymbol{b}_n^*, \boldsymbol{b}_{-n}^*) \geq u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}^*), \tag{15}$$

for all $\boldsymbol{b}_n \in \mathcal{B}_n$ and for all $n \in \mathcal{N}$.

Due to the cross elasticity arising from the second term in (11), directly verifying that the efficient bid profile is the best for every player is infeasible. Accordingly, the approach developed in [35] for auction games of a single divisible resource is not applicable. In order to overcome this difficulty, an alternative approach will be developed. This involves partitioning the set of bid profiles $\mathcal{B}_n$ into a collection of subsets:

$$\mathcal{B}_n(A) \triangleq \Big\{\boldsymbol{b}_n \in \mathcal{B}_n; \quad \text{s.t.} \sum_{t\in\mathcal{T}} d_{nt} = A\Big\}, \tag{16}$$

each of which is composed of admissible bid profiles that possess a common total A for the desired energy over the charging horizon $\mathcal{T}$. The set of bids is then given by:

$$\mathcal{B}_n = \bigcup_{A\in[0,\Gamma_n]} \mathcal{B}_n(A),$$

noting that $\mathcal{B}_n(\widehat{A}) \bigcap \mathcal{B}_n(\widetilde{A}) = \emptyset$ whenever $\widehat{A} \neq \widetilde{A}$. For all bids in a particular subset $\mathcal{B}_n(A)$, it follows from (11) that the marginal valuation price β_{nt} at any time t includes a variable part $-f_n'(d_{nt})$ that is dependent upon the request d_{nt} at that time, and a fixed part $2\delta_n(\Gamma_n - A)$ that is identical for all bid profiles in $\mathcal{B}_n(A)$. Therefore, the cross elasticity over the time horizon is avoided.

By Definition 1 and the specification of $\mathcal{B}_n(A)$, it is sufficient to show that $\boldsymbol{b}^*$ is a NE, if for every fixed $A \in [0, \Gamma_n]$:

$$u_n(\boldsymbol{b}_n^*, \boldsymbol{b}_{-n}^*) \geq u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n}^*), \quad \text{for all } \widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A), \tag{17}$$

and for all $n \in \mathcal{N}$. This is easier to verify than (15), since the difficulty associated with cross elasticity is avoided when considering each specific subset $\mathcal{B}_n(A)$ with fixed $A \in [0, \Gamma_n]$.

It will be shown in Section IV-A that (17) holds when $A \geq \sum_{t\in\mathcal{T}} d_{nt}^*$, and Section IV-B considers the case where $A < \sum_{t\in\mathcal{T}} d_{nt}^*$. These results build on the following lemma.

Lemma 4.1: Suppose $\beta_{nt}(d_{nt}; A)$ is the bidding price given by (11) for player n at time t, but with the summation $\sum_t d_{nt}$ in the second term replaced by A. Then $\beta_{nt}(d_{nt}; A)$ satisfies the properties:

$$\beta_{nt}(d_{nt}^1; A) > \beta_{nt}(d_{nt}^2; A) > 0, \ \text{ with } d_{nt}^1 < d_{nt}^2, \text{ for all } A, \tag{18a}$$

$$\beta_{nt}(d_{nt}; A_1) > \beta_{nt}(d_{nt}; A_2), \qquad \text{with } A_1 < A_2, \ \text{ for all } d_{nt}. \tag{18b}$$

Proof. By (11), $\beta_{nt}(d_{nt}; A) = 2\delta_n(\Gamma_n - A) - f_n'(d_{nt})$. It is straightforward to verify (18) under Assumption (A2). ∎

Lemma 4.1 implies that $\beta_{nt}(d_{nt}, A)$ decreases with increasing d_{nt} and A, as illustrated in Fig. 1.

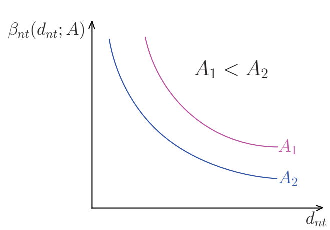

Fig. 1. Illustration of $\beta_{nt}(d_{nt}; A)$ with respect to d_{nt} and A.

A. Verification of (17) when $A \geq \sum_{t\in\mathcal{T}} d_{nt}^*$

The first step in verifying (17) is to show that all EVs $m \in \mathcal{N} \setminus \{n\}$ are fully allocated when $A \geq \sum_{t\in\mathcal{T}} d_{nt}^*$. This is established by Lemma 4.2. The main result then follows as Theorem 4.1.

For notational simplicity, let $\boldsymbol{x}^*$ and $\widehat{\boldsymbol{x}}$ denote the optimal allocations with respect to $\boldsymbol{b}^*$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n}^*)$ respectively.

Lemma 4.2: If $A \geq \sum_{t\in\mathcal{T}} d_{nt}^*$, then

$$\widehat{x}_{mt} = d_{mt}^*, \qquad \text{for all } m \in \mathcal{N} \setminus \{n\}.$$

Proof. If $d_{mt}^* = 0$ for any $m \in \mathcal{N} \setminus \{n\}$, then $\widehat{x}_{mt} = x_{mt}^* = 0$ and the desired result is obtained trivially for that m at time t. It will be assumed that $d_{mt}^* > 0$ in subsequent analysis.

Case I, when $\widehat{d}_{nt} \geq d_{nt}^*$. From Lemma 4.1, since $\widehat{d}_{nt} \geq d_{nt}^*$, (18a) gives:

$$\beta_{nt}(\widehat{d}_{nt}; \sum_{t\in\mathcal{T}} d_{nt}^*) \leq \beta_{nt}(d_{nt}^*; \sum_{t\in\mathcal{T}} d_{nt}^*) = \beta_{nt}^*,$$

and because $A \geq \sum_{t\in\mathcal{T}} d_{nt}^*$, (18b) gives:

$$\widehat{\beta}_{nt} \equiv \beta_{nt}(\widehat{d}_{nt}; A) \leq \beta_{nt}(\widehat{d}_{nt}; \sum_{t\in\mathcal{T}} d_{nt}^*).$$

This implies $\widehat{\beta}_{nt} \leq \beta_{nt}^*$, as illustrated in Fig. 2, with equality holding only when $\widehat{d}_{nt} = d_{nt}^*$ and $A = \sum_{t\in\mathcal{T}} d_{nt}^*$. In that special case, the bids $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n}^*)_t$ and $\boldsymbol{b}_t^*$ coincide, so $\widehat{x}_{mt} = x_{mt}^* = d_{mt}^*$ as desired. The following analysis considers $\widehat{\beta}_{nt} < \beta_{nt}^*$. Given the earlier assumption that $d_{mt}^* > 0$, Lemma 3.2 indicates that $\beta_{nt}^* \leq \beta_{mt}^*$. Hence, $\widehat{\beta}_{nt} < \beta_{nt}^* \leq \beta_{mt}^*$.

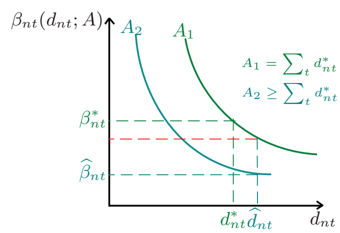

Fig. 2. Illustration of $\widehat{\beta}_{nt}$ and β_{nt}^* when $A \geq \sum_{t\in\mathcal{T}} d_{nt}^*$.

If $\widehat{x}_{nt} > 0$ then by (8a), $\frac{\partial}{\partial x_{nt}} L^a(\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{\sigma}}; (\widehat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n}^*)) = 0$. Because $\widehat{\sigma}_{nt} \geq 0$, (8c) implies $c_t'(D_t + \sum_{k\in\mathcal{N}} \widehat{x}_{kt}) \leq \widehat{\beta}_{nt}$. It follows that,

$$c_t'(D_t + \sum_{k\in\mathcal{N}} \widehat{x}_{kt}) \leq \widehat{\beta}_{nt} < \beta_{mt}^* = c_t'(D_t + \sum_{k\in\mathcal{N}} x_{kt}^*), \tag{19}$$

and hence that $\sum_{k\in\mathcal{N}} \widehat{x}_{kt} < \sum_{k\in\mathcal{N}} x_{kt}^*$. Therefore the total energy allocated to all the EVs at time t decreases under the bid profile $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n}^*)$. Furthermore, (19) gives $c_t'(D_t + \sum_{k\in\mathcal{N}} \widehat{x}_{kt}) - \beta_{mt}^* < 0$, so (8c) implies that $\widehat{\sigma}_{mt} > 0$. The complementarity condition (8b) then ensures $\widehat{x}_{mt} = d_{mt}^*$ for all $m \in \mathcal{N} \setminus \{n\}$, which is the desired result.

If $\widehat{x}_{nt} = 0$, then $\widehat{x}_{kt} \leq d_{kt}^* = x_{kt}^*$ for all $k \in \mathcal{N}$. Assume $\widehat{x}_{mt} < d_{mt}^*$ for some $m \in \mathcal{N} \setminus \{n\}$. Then $\sum_{k\in\mathcal{N}} \widehat{x}_{kt} < \sum_{k\in\mathcal{N}} x_{kt}^*$, and so

$$c_t'(D_t + \sum_{k\in\mathcal{N}} \widehat{x}_{kt}) < c_t'(D_t + \sum_{k\in\mathcal{N}} x_{kt}^*) = \beta_{mt}^*. \tag{20}$$

Again, (8c) implies $\widehat{\sigma}_{mt} > 0$, which is inconsistent with the assumption $\widehat{x}_{mt} < d_{mt}^*$ because of complementarity (8b). Therefore, $\widehat{x}_{mt} = d_{mt}^*$ for all $m \in \mathcal{N} \setminus \{n\}$, as desired.

Case (II), when $\widehat{d}_{nt} < d_{nt}^*$. Because $\widehat{x}_{nt} \leq \widehat{d}_{nt} < d_{nt}^* = x_{nt}^*$, then $\sum_{k\in\mathcal{N}} \widehat{x}_{kt} < \sum_{k\in\mathcal{N}} x_{kt}^*$, and so (20) holds. It again follows from (8c) that $\widehat{\sigma}_{mt} > 0$, and hence the complementarity condition (8b) ensures that $\widehat{x}_{mt} = d_{mt}^*$, as desired. ∎

The main result for this section can now be established.

Theorem 4.1: Under Assumptions (A1, A2), (17) holds when $A \geq \sum_{t\in\mathcal{T}} d_{nt}^*$.

Proof. Considering the n-th EV, the first step is to compute the difference between the payoff given by the optimal strategy $u_n(\boldsymbol{b}^*)$

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and that obtained from an alternative strategy $u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$. Using (14), these payoffs are given by,

$$u_n(\boldsymbol{b}^*) = w_n(\boldsymbol{d}^*_n) - \tau_n(\boldsymbol{b}^*), \tag{21a}$$

$$u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) = w_n(\widehat{\boldsymbol{x}}_n) - \tau_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}). \tag{21b}$$

Hence, the difference is:

$$\begin{aligned}
\Delta u_n &\triangleq u_n(\boldsymbol{b}^*) - u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) \\
&= w_n(\boldsymbol{d}^*_n) - \tau_n(\boldsymbol{b}^*) - \Big(w_n(\widehat{\boldsymbol{x}}_n) - \tau_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) \Big) \\
&= w_n(\boldsymbol{d}^*_n) - w_n(\widehat{\boldsymbol{x}}_n) + \Big(J^*(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) + \sum_{t\in\mathcal{T}} \widehat{\beta}_{nt}\widehat{x}_{nt} \\
&\qquad - J^*(\boldsymbol{b}^*) - \sum_{t\in\mathcal{T}} \beta^*_{nt} d^*_{nt} \Big),
\end{aligned}$$

where the final equality follows from (12). Straightforward analysis, using the result from Lemma 4.2 that $\widehat{x}_{mt} = d^*_{mt}$ for all $m \in \mathcal{N} \setminus \{n\}$, gives:

$$\begin{aligned}
\Delta u_n = w_n(\boldsymbol{d}^*_n) - w_n(\widehat{\boldsymbol{x}}_n) + \sum_{t\in\mathcal{T}} \Big\{ c_t(D_t + \sum_{m\neq n} d^*_{mt} + \widehat{x}_{nt}) \\
- c_t(D_t + \sum_{k\in\mathcal{N}} d^*_{kt}) \Big\}.
\end{aligned}$$

Also, using Lemma 4.2,

$$\begin{aligned}
J_s(\boldsymbol{x}^*) - J_s(\widehat{\boldsymbol{x}}) &= J_s(\boldsymbol{d}^*) - J_s(\widehat{\boldsymbol{x}}_n, \boldsymbol{d}^*_{-n}) \\
&= \sum_{t\in\mathcal{T}} \Big\{ c_t(D_t + \sum_{k\in\mathcal{N}} d^*_{kt}) - c_t(D_t + \sum_{m\neq n} d^*_{mt} + \widehat{x}_{nt}) \Big\} \\
&\qquad - \sum_{k\in\mathcal{N}} w_k(\boldsymbol{d}^*_k) + \sum_{m\neq n} w_m(\boldsymbol{d}^*_m) + w_n(\widehat{\boldsymbol{x}}_n) \\
&= \sum_{t\in\mathcal{T}} \Big\{ c_t(D_t + \sum_{k\in\mathcal{N}} d^*_{kt}) - c_t(D_t + \sum_{m\neq n} d^*_{mt} + \widehat{x}_{nt}) \Big\} \\
&\qquad - w_n(\boldsymbol{d}^*_n) + w_n(\widehat{\boldsymbol{x}}_n) \\
&= -\Delta u_n.
\end{aligned}$$

Since $\boldsymbol{x}^*$ is the efficient allocation, $J_s(\boldsymbol{x}^*) \leq J_s(\widehat{\boldsymbol{x}})$, so

$$\Delta u_n = J_s(\widehat{\boldsymbol{x}}) - J_s(\boldsymbol{x}^*) \geq 0.$$

This implies that player n cannot benefit by unilaterally changing its bid $\boldsymbol{b}^*_n$ to any other bid $\widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ with $A \geq \sum_{t\in\mathcal{T}} d^*_{nt}$. ■

B. Verification of (17) when $0 \leq A < \sum_{t\in\mathcal{T}} d^*_{nt}$

This section considers a fixed A satisfying $0 \leq A < \sum_{t\in\mathcal{T}} d^*_{nt}$. Firstly, Lemma 4.3 establishes the n-th EV's optimal bid in the set $\mathcal{B}_n(A)$ and under the constraint,

$$0 \leq \widehat{d}_{nt} \begin{cases} < d^*_{nt}, & \text{when } d^*_{nt} > 0, \\ = 0, & \text{otherwise}, \end{cases} \quad \text{for all } t \in \mathcal{T}. \tag{22}$$

Theorem 4.2 then shows that the bid of Lemma 4.3 remains optimal when the constraint (22) is relaxed. Using this result, Theorem 4.3 finally establishes that (17) holds when $0 \leq A < \sum_{t\in\mathcal{T}} d^*_{nt}$.

Lemma 4.3: Consider a bid $\widehat{\boldsymbol{b}}^*_n \equiv \widehat{\boldsymbol{b}}^*_n(A) \equiv \big((\widehat{\beta}^*_{nt}, \widehat{d}^*_{nt}), t \in \mathcal{T} \big)$, with $A \in [0, \sum_{t\in\mathcal{T}} d^*_{nt})$, such that

$$\widehat{\boldsymbol{b}}^*_n = \underset{\substack{\widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A) \\ \text{Constraint (22)}}}{\operatorname{argmax}} \; u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}). \tag{23}$$

Let $\widehat{\boldsymbol{x}}^* \equiv (\widehat{x}^*_{kt}, k \in \mathcal{N}, t \in \mathcal{T})$ denote the optimal allocations with respect to $(\widehat{\boldsymbol{b}}^*_n, \boldsymbol{b}^*_{-n})$. Then,

$$\widehat{\boldsymbol{x}}^*_n = \widehat{\boldsymbol{d}}^*_n, \qquad \widehat{\boldsymbol{x}}^*_m = \boldsymbol{d}^*_m \text{ for all } m \in \mathcal{N} \setminus \{n\}. \tag{24}$$

Furthermore, define the function,

$$g_{nt}(d) \triangleq c_t(D_t + \sum_{m\neq n} d^*_{mt} + d) + f_n(d). \tag{25}$$

Then, under Assumptions (A1, A2), $\widehat{\boldsymbol{b}}^*_n$ satisfies the property:

$$g'_{nt}(\widehat{d}^*_{nt}) \begin{cases} = \mu, & \text{when } \widehat{d}^*_{nt} > 0, \\ \geq \mu, & \text{when } \widehat{d}^*_{nt} = 0, \end{cases} \quad \text{for all } t \in \mathcal{T}, \tag{26}$$

where μ is a constant. ■

The proof of Lemma 4.3 is given in Appendix B. It will now be shown that the bid $\widehat{\boldsymbol{b}}^*_n$ established in this lemma remains optimal when the constraint (22) is relaxed.

Theorem 4.2: Suppose that $\widehat{\boldsymbol{b}}^*_n$ is the optimal bid from Lemma 4.3. Then, under Assumptions (A1, A2) and with $A \in [0, \sum_{t\in\mathcal{T}} d^*_{nt})$, $\widehat{\boldsymbol{b}}^*_n$ satisfies:

$$\widehat{\boldsymbol{b}}^*_n = \underset{\widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A)}{\operatorname{argmax}} \; u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}).$$

Proof. Assume that $A \in [0, \sum_{t\in\mathcal{T}} d^*_{nt})$. Consider a bid $\widehat{\boldsymbol{b}}_n \equiv \big((\widehat{\beta}_{nt}, \widehat{d}_{nt}), t \in \mathcal{T} \big) \in \mathcal{B}_n(A)$ such that $\widehat{d}_{nt} \geq d^*_{nt}$ for some $t \in \mathcal{T}$. Then the desired result follows if it can be proven that $\widehat{\boldsymbol{b}}_n$ cannot be the optimal bid profile in the subset $\mathcal{B}_n(A)$.

Firstly, the following points hold:

(i) $\beta_{nt}(\widehat{d}_{nt}; A) > \beta_{nt}(\widehat{d}_{nt}; \sum_{t\in\mathcal{T}} d^*_{nt})$ because $A < \sum_{t\in\mathcal{T}} d^*_{nt}$, by (18b).

(ii) $c'_t(D_t + \sum_{m\neq n} d^*_{mt} + d_{nt})$ increases with d_{nt} under Assumption (A1).

(iii) (β^*_{nt}, d^*_{nt}) is the point at which $\beta_{nt}(d_{nt}; \sum_{t\in\mathcal{T}} d^*_{nt})$ and $c'_t(D_t + \sum_{m\neq n} d^*_{mt} + d_{nt})$ coincide, by Lemma 3.2.

Using (i)–(iii) together with (18a) gives,

$$0 < d^*_{nt} < \widehat{d}^1_{nt}, \qquad \beta^*_{nt} < \widehat{\beta}^1_{nt},$$

where $(\widehat{\beta}^1_{nt}, \widehat{d}^1_{nt})$ denotes the point of intersection between $\beta_{nt}(d_{nt}; A)$ and $c'_t(D_t + \sum_{m\neq n} d^*_{mt} + d_{nt})$. This relationship is illustrated in Fig. 3.

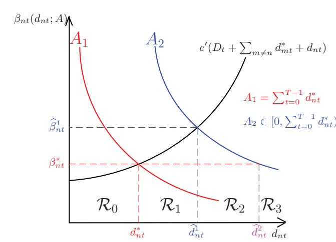

Fig. 3. An illustration of partitioned subsets $\mathcal{R}_i, i = 0, \cdots, 3$.

Let $(\widehat{\beta}^2_{nt}, \widehat{d}^2_{nt})$ denote the bid with respect to A such that $\widehat{\beta}^2_{nt} = \beta^*_{nt}$, in other words $\widehat{\beta}^2_{nt} = \beta^*_{nt} = \beta_{nt}(\widehat{d}^2_{nt}, A)$. It follows from (18a) that $\widehat{d}^1_{nt} < \widehat{d}^2_{nt}$. Based on the ordering $0 < d^*_{nt} < \widehat{d}^1_{nt} < \widehat{d}^2_{nt}$, the set $[0, \infty)$ can be partitioned into four disjoint regions,

$$\mathcal{R}_0 \triangleq [0, d^*_{nt}), \quad \mathcal{R}_1 \triangleq (d^*_{nt}, \widehat{d}^1_{nt}],$$
$$\mathcal{R}_2 \triangleq (\widehat{d}^1_{nt}, \widehat{d}^2_{nt}], \quad \mathcal{R}_3 \triangleq (\widehat{d}^2_{nt}, +\infty),$$

together with the point d^*_{nt}. This partitioning is shown in Fig. 3, and is used in Appendix C.

Suppose there exists a $\widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ such that at time t_2, $\widehat{d}_{nt_2} = d^*_{nt_2}$. Then, because $\sum_{t\in\mathcal{T}} \widehat{d}_{nt} = \sum_{t\in\mathcal{T}} \widehat{d}^*_{nt} = A < \sum_{t=0}^{T-1} d^*_{nt}$, and $\widehat{d}^*_{nt} < d^*_{nt}$ for all $t \in \mathcal{T}$ due to (22), there must exist another time t_1 such that $\widehat{d}_{nt_1} < \widehat{d}^*_{nt_1}$. Consider two bid profiles $\widehat{\boldsymbol{b}}_n, \widetilde{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ such that,

$$\widehat{d}_{nt_1} < \widetilde{d}_{nt_1} < \widehat{d}^*_{nt_1}, \tag{27a}$$

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$$\widehat{d}^*_{nt_2} < \widetilde{d}_{nt_2} < \widehat{d}_{nt_2} = d^*_{nt_2}, \tag{27b}$$

$$\widehat{d}_{nt_1} + \widehat{d}_{nt_2} = \widetilde{d}_{nt_1} + \widetilde{d}_{nt_2}, \tag{27c}$$

$$\widehat{d}_{nt} = \widetilde{d}_{nt}, \quad \text{for all } t \neq t_1, t_2. \tag{27d}$$

Then, it is shown in Appendix C that,

$$u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) < u_n(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}). \tag{28}$$

Hence, such a $\widehat{\boldsymbol{b}}_n$ cannot be the optimal bid with respect to $\boldsymbol{b}^*_{-n}$.

Similarly, suppose there exists a $\widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ such that at time t_2, $\widehat{d}_{nt_2} \in \mathcal{R}_1 \cup \mathcal{R}_2 \cup \mathcal{R}_3$. Following the same argument as previously, there must exist a time t_1 such that $\widehat{d}_{nt_1} < \widehat{d}^*_{nt_1}$. Again, consider two bid profiles $\widehat{\boldsymbol{b}}_n, \widetilde{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ such that (27a),(27c),(27d) are satisfied, but with (27b) replaced by,

$$d^*_{nt_2} < \widetilde{d}_{nt_2} < \widehat{d}_{nt_2}. \tag{29}$$

Appendix C shows that (28) again holds, so $\widehat{\boldsymbol{b}}_n$ cannot be the optimal bid with respect to $\boldsymbol{b}^*_{-n}$. This is the desired result.

Moreover, because Assumptions (A1, A2) ensure differentiability of $c_t(\cdot)$ and $f_n(.)$ for all n, the payoff function of every EV is continuous in the bid profiles. Therefore, it follows from (28) that the optimal bid for the n-th EV must lie in region $\mathcal{R}_0$ for all $t \in \mathcal{T}$. ■

Theorem 4.3: Under Assumptions (A1, A2), (17) holds when $0 \leq A < \sum_{t\in\mathcal{T}} d^*_{nt}$.

Proof. By Theorem 4.2, $\widehat{\boldsymbol{b}}^*_n$ is the optimal bid with respect to $\boldsymbol{b}^*_{-n}$ in $\mathcal{B}_n(A)$, when $A \in [0, \sum_{t\in\mathcal{T}} d^*_{nt})$. Therefore, the best possible payoff for the n-th EV is,

$$u_n(\widehat{\boldsymbol{b}}^*_n, \boldsymbol{b}^*_{-n}) = w_n(\widehat{\boldsymbol{d}}^*_n) - \tau_n(\widehat{\boldsymbol{b}}^*_n, \boldsymbol{b}^*_{-n}). \tag{30}$$

Using (24) and recalling that $\boldsymbol{x}^* = \boldsymbol{d}^*$ gives,

$$\begin{aligned}
J_s(\boldsymbol{x}^*) - J_s(\widehat{\boldsymbol{x}}^*) &= J_s(\boldsymbol{d}^*) - J_s(\widehat{\boldsymbol{d}}^*_n, \boldsymbol{d}^*_{-n}) \\
&= \sum_{t\in\mathcal{T}} \Big\{ c_t(D_t + \sum_{k\in\mathcal{N}} d^*_{kt}) - c_t(D_t + \sum_{m\neq n} d^*_{mt} + \widehat{d}^*_{nt}) \Big\} \\
&\qquad - \sum_{k\in\mathcal{N}} w_k(\boldsymbol{d}^*_k) + \sum_{m\neq n} w_m(\boldsymbol{d}^*_m) + w_n(\widehat{\boldsymbol{d}}^*_n) \\
&= \sum_{t\in\mathcal{T}} \Big\{ c_t(D_t + \sum_{k\in\mathcal{N}} d^*_{kt}) - c_t(D_t + \sum_{m\neq n} d^*_{mt} + \widehat{d}^*_{nt}) \Big\} \\
&\qquad - w_n(\boldsymbol{d}^*_n) + w_n(\widehat{\boldsymbol{d}}^*_n) \\
&= -\Big(u_n(\boldsymbol{b}^*) - u_n(\widehat{\boldsymbol{b}}^*_n, \boldsymbol{b}^*_{-n}) \Big) \equiv -\Delta u_n,
\end{aligned}$$

where the final equality makes use of (21a) and (30). By Lemma 3.2, $\boldsymbol{x}^* \equiv \boldsymbol{x}^*(\boldsymbol{b}^*)$ is the efficient allocation solution, so $J_s(\boldsymbol{x}^*) \leq J_s(\widehat{\boldsymbol{x}}^*)$. Therefore,

$$\Delta u_n = J_s(\widehat{\boldsymbol{x}}^*) - J_s(\boldsymbol{x}^*) \geq 0,$$

which implies that player n cannot benefit by unilaterally changing its bid $\boldsymbol{b}^*_n$ to any other bid $\widehat{\boldsymbol{b}}^*_n \in \mathcal{B}_n(A)$ with $A \in [0, \sum_{t\in\mathcal{T}} d^*_{nt})$. ■

C. Existence of efficient Nash equilibrium

Applying the analysis and results from Sections IV-A and IV-B, and under Assumptions (A1, A2), it can now be shown that the efficient bid profile $\boldsymbol{b}^*$ specified in (9) is a Nash equilibrium of the EV charging coordination game under the PSP auction mechanism.

Corollary 4.1: Under Assumptions (A1, A2), the efficient bid profile $\boldsymbol{b}^* \equiv (\boldsymbol{b}^*_n; n \in \mathcal{N})$ specified in (9) satisfies the property:

$$u_n(\boldsymbol{b}^*_n, \boldsymbol{b}^*_{-n}) \geq u_n(\boldsymbol{b}_n, \boldsymbol{b}^*_{-n}), \qquad \text{for all } \boldsymbol{b}_n \in \mathcal{B}_n. \tag{31}$$

Proof. It was shown in Theorems 4.1 and 4.3, under Assumptions (A1, A2), that

$$u_n(\boldsymbol{b}^*_n, \boldsymbol{b}^*_{-n}) \geq u_n(\boldsymbol{b}_n, \boldsymbol{b}^*_{-n}), \qquad \text{for all } \boldsymbol{b}_n \in \mathcal{B}_n(A),$$

holds when $A \geq \sum_{t\in\mathcal{T}} d^*_{nt}$ and when $0 \leq A < \sum_{t\in\mathcal{T}} d^*_{nt}$, respectively. The desired result (31) holds since $\mathcal{B}_n = \bigcup_{A\in[0,\Gamma_n]} \mathcal{B}_n(A)$. ■

D. Analysis of budget balance at the efficient NE

As studied in [35], [46], [47], budget balance may not hold under VCG-style auction mechanisms. Define the surplus (net income) for a bid profile $\boldsymbol{b}$ as:

$$\psi(\boldsymbol{b}) \triangleq \sum_{n\in\mathcal{N}} \Big\{ \tau_n(\boldsymbol{b}) \Big\} - \sum_{t\in\mathcal{T}} \Big\{ c_t\big(D_t + \sum_{k\in\mathcal{N}} x^*_{kt}(\boldsymbol{b})\big) - c_t(D_t) \Big\},$$

which captures the aggregated EV payments minus the extra generation costs arising as a consequence of the EV population. Therefore, $\psi(\boldsymbol{b}^*)$ may not equal zero at the efficient NE $\boldsymbol{b}^*$. The sign of $\psi(\boldsymbol{b}^*)$ is established by the following lemma:

Lemma 4.4:

$$\psi(\boldsymbol{b}^*) > 0.$$

The proof is given in Appendix D. ■

Lemma 4.4 establishes that the budget balance for $\boldsymbol{b}^*$ will always be in surplus (positive). This surplus $\psi(\boldsymbol{b}^*)$ can be used to reimburse the services provided by the system operator.

E. Efficiency loss of single-interval auction games

Given that the efficient bid profile $\boldsymbol{b}^*$ is a NE, efficiency loss can only occur if there exist multiple NE. The following theorem shows that for single-interval auction games, i.e., an auction game with $T = 1$, the efficient NE is unique. Hence, for such games, there is no possibility of efficiency loss. In this case, the efficient bid profile degenerates to $\boldsymbol{b}^* = ((\beta^*_n, d^*_n), n \in \mathcal{N})$.

Theorem 4.4: $\boldsymbol{b}^* = ((\beta^*_n, d^*_n), n \in \mathcal{N})$ is the unique NE for PSP auction games where $T = 1$.

Proof. Corollary 4.1 establishes that $\boldsymbol{b}^*$ is a NE. Verifying Theorem 4.4 is equivalent to showing that every inefficient bid profile $\boldsymbol{b}$ cannot be a NE. This is undertaken in Appendix E. ■

Ongoing research is considering efficiency loss for EV charging auction games over a multiple time-step horizon.

V. PSP Auction Process for EV Charging

Section IV established the existence of efficient Nash equilibria, under appropriate conditions, for the underlying EV charging coordination game. This section develops a bid-profile update process which motivates an algorithm for determining efficient NE.

A process for determining an EV's best bid, given the collection of bids for the other EVs, is presented in Section V-A. It will be shown that this process can be formulated as a dynamic programming problem. Section V-B then provides an algorithmic description of the update mechanism that governs the underlying auction game. A numerical example is provided in Section V-C to demonstrate this update mechanism and show that it achieves an efficient NE.

A. An EV's best bid with respect to other EVs

Recall that $\boldsymbol{b}^*_n(\boldsymbol{b}_{-n})$ denotes the best bid of the n-th EV with respect to the bid profiles $\boldsymbol{b}_{-n}$ of all the other EVs,

$$\boldsymbol{b}^*_n(\boldsymbol{b}_{-n}) = \underset{\boldsymbol{b}_n\in\mathcal{B}_n}{\operatorname{argmax}}\ u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}),$$

where $u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n})$ is the individual payoff of the n-th EV, as established in (14). However, due to the cross-temporal coupling arising from the summation term $\sum_{t\in\mathcal{T}} d_{nt}$ in truth-telling bids, as identified in (11), it is impractical to directly determine the best response $\boldsymbol{b}_n^*(\boldsymbol{b}_{-n})$ that is incentive compatible. This can be addressed by finding the best response when bids are constrained to possess a common total desired demand $A = \sum_{t\in\mathcal{T}} d_{nt}$, and then optimizing over $A \in [0, \Gamma_n]$. The resulting optimization is given by,

$$u_n(\boldsymbol{b}_n^*, \boldsymbol{b}_{-n}) = \max_{A\in[0,\Gamma_n]} \max_{\boldsymbol{b}_n\in\mathcal{B}_n(A)} u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}).$$

The remainder of this section describes a dynamic programming approach to solving the inner optimal bidding problem that arises for each fixed total demand request $A \in [0, \Gamma_n]$.

The dynamics associated with the physical charging process (1) can be rewritten,

$$s_{n,t+1}(\boldsymbol{b}_n, \boldsymbol{b}_{-n}) = s_{nt}(\boldsymbol{b}_n, \boldsymbol{b}_{-n}) + \frac{1}{\Theta_n} x_{nt}(\boldsymbol{b}_n, \boldsymbol{b}_{-n}), \tag{32}$$

with $t \in \mathcal{T}$, and where $x_{nt}(\boldsymbol{b}_n, \boldsymbol{b}_{-n})$ denotes the allocated charging rate of the n-th EV at time t, with respect to the bid profile $(\boldsymbol{b}_n, \boldsymbol{b}_{-n})$.

Lemma 5.1:

Consider a bid $\boldsymbol{b}_n \in \mathcal{B}_n(A)$, for any fixed $A \in [0, \Gamma_n]$. Then the payoff function of the n-th EV has the summation form:

$$u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}) = \sum_{t\in\mathcal{T}} a_n(\boldsymbol{b}_t) - \delta_n \Theta_n^2 \left(\bar{s}_n - s_n^T\right)^2, \tag{33}$$

where $a_n(\boldsymbol{b}_t) \equiv -f_n(x_{nt}(\boldsymbol{b}_t)) - \tau_{nt}(\boldsymbol{b}_t)$ and $\tau_{nt}(\boldsymbol{b}_t)$ is defined in (13).

Proof. From (14),

$$\begin{aligned} u_n(\boldsymbol{b}) &= w_n(\boldsymbol{x}_n^*(\boldsymbol{b}_n)) - \tau_n(\boldsymbol{b}) \\ &= \sum_{t\in\mathcal{T}} \Big\{ - f_n(x_{nt}(\boldsymbol{b}_t)) - \tau_{nt}(\boldsymbol{b}_t) \Big\} \\ &\quad - \delta_n \Big(\sum_{t\in\mathcal{T}} x_{nt}(\boldsymbol{b}_t) - \Gamma_n \Big)^2, \end{aligned}$$

where equality holds by (3), Lemma 3.1, and (13). Then (33) follows directly from (32). ■

Let $\mathcal{T}_t \equiv \{t, ..., T-1\}$, and define the value function $v_n(t, s_n^t) \equiv v_n(t, s_n^t; A, \boldsymbol{b}_{-n})$, for all $t \in \mathcal{T}$ as,

$$v_n(t, s_n^t) \triangleq \max_{\substack{\boldsymbol{b}_n(\mathcal{T}_t)\in \\ \mathcal{B}_n(\mathcal{T}_t, s_n^t; A)}} \left\{ \sum_{k=t}^{T-1} a_n(\boldsymbol{b}_k) - \delta_n \Theta_n^2 \left(\bar{s}_n - s_n^T\right)^2 \right\}, \tag{34}$$

where $a_n(\boldsymbol{b}_s)$ is given in Lemma 5.1, and

$$\mathcal{B}_n(\mathcal{T}_t, s_n^t; A) \triangleq \begin{cases} \Big\{ \big((\beta_{nk}, d_{nk}), k \in \mathcal{T}_t\big) \text{ s.t. } \beta_{nk} = -f_n'(d_{nk}) + 2\delta_n(\Gamma_n - A) \\ \qquad \text{and } \sum_{k=t}^{T-1} d_{nk} \le \min\left\{A, \Theta_n(\bar{s}_n - s_n^t)\right\} \Big\}, \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{when } t > 0 \\ \mathcal{B}_n(A), \qquad\qquad\qquad\qquad\qquad \text{when } t = 0, \end{cases} \tag{35}$$

with $\mathcal{B}_n(A)$ defined in (16). The terminal value function is defined as,

$$v_n(T, s_n^T) \triangleq -\delta_n \Theta_n^2 \left(\bar{s}_n - s_n^T\right)^2. \tag{36}$$

Note that the set of bids of the n-th EV over the interval $\mathcal{T}_t$ specified in (35) is defined in such a way that the total bidding demand over the whole interval $\mathcal{T}$ is guaranteed to equal A.

The value function definition implies:

$$v_n(0, s_n^0; A, \boldsymbol{b}_{-n}) = \max_{\boldsymbol{b}_n\in\mathcal{B}_n(A)} u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}), \tag{37}$$

and therefore,

$$u_n(\boldsymbol{b}_n^*, \boldsymbol{b}_{-n}) = \max_{A\in[0,\Gamma_n]} v_n(0, s_n^0; A, \boldsymbol{b}_{-n}). \tag{38}$$

Let $\boldsymbol{b}_n^*(\mathcal{T}_t) \equiv \big((\beta_{nk}^*, d_{nk}^*), k \in \mathcal{T}_t\big) \equiv \boldsymbol{b}_n^*(\mathcal{T}_t, s_n^t; A, \boldsymbol{b}_{-n})$ denote the best bid of the n-th EV solving the optimization problem (34) over the interval $\mathcal{T}_t$ with respect to A and $\boldsymbol{b}_{-n}$, and let $\Pi_n^*(t, s_n^t; A, \boldsymbol{b}_{-n})$ denote the total bidding demand over the interval $\mathcal{T}_t$ of the n-th EV subject to the best bid $\boldsymbol{b}_n^*(\mathcal{T}_t, s_n^t; A, \boldsymbol{b}_{-n})$,

$$\Pi_n^*(t, s_n^t; A, \boldsymbol{b}_{-n}) \triangleq \sum_{k=t}^{T-1} d_{nk}^*(\mathcal{T}_t, s_n^t; A, \boldsymbol{b}_{-n}).$$

Define $\mathcal{B}_n(t, s_n^t) \equiv \mathcal{B}_n(t, s_n^t; A, \boldsymbol{b}_{-n})$, for any $t \in \mathcal{T}$, as the set of bids at time t, such that

$$\mathcal{B}_n(t, s_n^t; A, \boldsymbol{b}_{-n}) \triangleq \begin{cases} \Big\{ (\beta_{nt}, d_{nt}) \text{ s.t. } \beta_{nt} = -f_n'(d_{nt}) + 2\delta_n\left(\Gamma_n - A\right), \\ \qquad \text{and } d_{nt} + \Pi_n^*(t+1, s_n^{t+1}; A, \boldsymbol{b}_{-n}) \\ \qquad\qquad \le \min\left\{A, \Theta_n(\bar{s}_n - s_n^t)\right\} \Big\}, \qquad \text{when } t > 0 \\ \Big\{ (\beta_{nt}, d_{nt}) \text{ s.t. } \beta_{nt} = -f_n'(d_{nt}) + 2\delta_n\left(\Gamma_n - A\right), \\ \qquad \text{and } d_{nt} + \Pi_n^*(t+1, s_n^{t+1}; A, \boldsymbol{b}_{-n}) = A \Big\}, \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{when } t = 0, \end{cases} \tag{39}$$

where s_n^{t+1} is given by (32). As with the set of bids of the n-th EV over the interval $\mathcal{T}_t$ specified in (35), the set of bids of at each time $t \in \mathcal{T}$ specified in (39) is defined such that the total bidding demand over the whole interval $\mathcal{T}$ is guaranteed to equal A.

Theorem 5.1: The value function $v_n(t, s_n^t; A, \boldsymbol{b}_{-n})$ of the n-th EV, with respect to a fixed $A \in [0, \Gamma_n]$ and a collection of bids $\boldsymbol{b}_{-n}$ of the other EVs, can be implemented by solving the Bellman equation,

$$v_n(t, s_n^t; A, \boldsymbol{b}_{-n}) = \max_{b_{nt}\in\mathcal{B}_n(t, s_n^t; A, \boldsymbol{b}_{-n})} \Big\{ a_n(\boldsymbol{b}_t) + v_n(t+1, s_n^{t+1}; A, \boldsymbol{b}_{-n}) \Big\},$$

for $t \in \mathcal{T}$, where $\mathcal{B}_n(t, s_n^t; A, \boldsymbol{b}_{-n})$ is defined in (39), $a_n(\boldsymbol{b}_t)$ is specified in Lemma 5.1, and $v_n(T, s_n^T)$ is given by (36).

Proof. It is straightforward to verify Theorem 5.1 by applying the *optimality principle* for the underlying optimization problem defined in (37) for the n-th EV with respect to A and $\boldsymbol{b}_{-n}$. ■

Finally, the n-th EV's overall optimal bid, with respect to the bid profiles $\boldsymbol{b}_{-n}$ of the other EVs, is given by (38).

B. Update mechanism for EVs

The bid-profile update process of Section V-A motivates a heuristic algorithm whereby each EV successively updates its optimal bidding strategy with respect to the latest available bidding strategies of all the other EVs. If this iterative process converges, the resulting bid profile will be a NE, as established by Definition 1.

Algorithm 1: (Nash equilibrium implementation algorithm)

- Provide an initial bid profile $\boldsymbol{b}^{(0)}$.
- Set the iterative step $k = 0$.
- Set the iteration termination criterion $\epsilon > \epsilon_0$ for some $\epsilon_0 > 0$.
- While $\epsilon > \epsilon_0$
  - For $n = 1 : N$
    Determine the best response for the n-th EV, $\boldsymbol{b}_n^{(k+1)}$, with respect to $\boldsymbol{b}_1^{(k+1)}, \cdots, \boldsymbol{b}_{n-1}^{(k+1)}, \boldsymbol{b}_{n+1}^{(k)}, \cdots, \boldsymbol{b}_N^{(k)}$, by maximizing the payoff function,

$$\boldsymbol{b}_n^{(k+1)} = \operatorname*{argmax}_{\boldsymbol{b}_n\in\mathcal{B}_n} u_n(\boldsymbol{b}_n; \boldsymbol{b}_1^{(k+1)}, \cdots, \boldsymbol{b}_{n-1}^{(k+1)}, \boldsymbol{b}_{n+1}^{(k)}, \cdots, \boldsymbol{b}_N^{(k)}),$$

    which can be achieved by applying the method developed in Section V-A.

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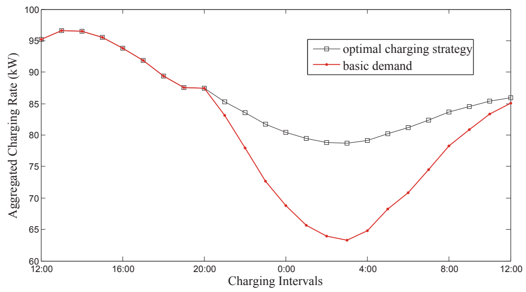

Fig. 4. Background demand and the aggregate optimal charging strategies.

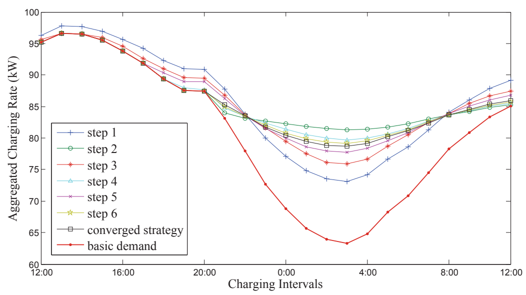

Fig. 5. Convergent updates of Algorithm 1.

– Update $\epsilon := ||\boldsymbol{b}^{(k+1)} - \boldsymbol{b}^{(k)}||_1$.
– Update $k := k + 1$.

End of Algorithm

Whilst convergence of Algorithm 1 cannot currently be guaranteed, such guarantees have been established for related auction games. Various static iterative processes have been designed to determine the efficient NE for PSP auction games with a single type of resource, see for example [30], [31], [48]. Also, the so-called quantized-PSP auction mechanism was developed in [49]–[51] for electricity sharing games in a single time interval, with that process converging to a nearly efficient solution. Accordingly, ongoing research is studying the convergence properties of Algorithm 1 and the performance of the resulting NE for auction games where multiple resources are coupled.

C. Numerical illustration

To illustrate the auction-based coordination process, a numerical example will consider EV charging over a common time horizon $T = 24$ hours, from 12:00 on one day to 12:00 the next day, with a time-step of $\Delta T = 1$ h. The background demand for the example is shown in Fig. 4. For the purpose of demonstration, a small population of 5 vehicles will be considered. Each EV has a common battery capacity of 30 kWh and a common maximum SoC value $\overline{s}_n = 0.9$. Heterogeneity is introduced by letting the initial SoC values s_n^0 for the five EVs, prior to the charging interval, take the values $\boldsymbol{s}_0 = [0.1 \ \ 0.15 \ \ 0.23 \ \ 0.14 \ \ 0.08]^\intercal$.

The generation cost is given by $c_t(\boldsymbol{x}_t, D_t) = 0.005\left(\sum_{n\in\mathcal{N}} x_{nt} + D_t\right)^{1.7}$ and the battery degradation cost by $f_n(x_{nt}) = 0.002x_{nt}^2$. Both these functions are strictly convex. The weighting factor for the quadratic charging deviation cost of each EV is set to $\delta_n = 10$ for all $n \in \mathcal{N}$.

The efficient EV charging trajectory, given by Problem 1, is shown in aggregation ($\sum_{n\in\mathcal{N}} x_{nt}^{**}$, $t \in \mathcal{T}$) in Fig. 4. In contrast, the distributed approach to charging coordination, described by Algorithm 1, gives the update evolution shown in Fig. 5. At each iteration, all EVs determine their optimal bid profile by solving the dynamic programming problem formalized by Theorem 5.1. Fig. 5 shows the aggregate allocation ($\sum_{n\in\mathcal{N}} x_{nt}^{*}(\boldsymbol{b}^{(k)})$, $t \in \mathcal{T}$) obtained by the auctioneer solving Problem 2 with respect to the bid profile $\boldsymbol{b}^{(k)}$ at the k-th iteration. Comparing Figs. 4 and 5, it is clear that the auction game converges to the efficient charging solution.

At the efficient strategy $\boldsymbol{b}^*$, the aggregated payments of the EV population amount to $\sum_{n\in\mathcal{N}} \tau_n(\boldsymbol{b}^*) = 23.44$, and the increased generation cost is $\sum_{t\in\mathcal{T}} \left\{c_t\left(D_t + \sum_{k\in\mathcal{N}} x_{kt}^*(\boldsymbol{b}^*)\right) - c_t(D_t)\right\} = 22.48$. The resulting surplus of 0.96, which is 4.1% of the total payments, can be used to reimburse the services provided by the system operator.

VI. Conclusions and Ongoing Research

As sales of electric vehicles (EVs) increase, the energy demanded by EV charging will begin to impact power system operation. Accommodating large numbers of EVs on the grid will require coordination of their charging requirements. A distributed approach to coordinating EV charging, based on the progressive second price (PSP) auction mechanism, has been proposed. This auction mechanism ensures incentive compatibility. It is shown that the efficient (socially optimal) charging schedule is a Nash equilibrium (NE) of this auction.

Individual EVs are capable of shifting their charging requirements in time across the charging horizon. Such cross elasticity complicates verification that the efficient solution is a NE. This has been addressed by partitioning players' bids into subsets, each of which is composed of bids that possess the same total desired energy over the horizon. Consequently, cross elasticity is eliminated for bids within each subset. With this construction, the NE property of the efficient solution has been established by verifying for each subset that no player can benefit by unilaterally deviating from their efficient bid profile.

An iterative update mechanism, based on dynamic programming, has been developed for the underlying auction game. This update mechanism was illustrated using a numerical example which showed convergence of the auction to an efficient NE.

The developments in this paper motivate various ongoing research directions. This current work shows that for a single-step time horizon, the efficient solution is the unique NE for the underlying game. It remains to verify whether efficiency loss may occur for multi-step time horizons due to non-uniqueness of the NE. Work is also required to establish convergence properties of the auction update algorithm.

The model underpinning the charging coordination scheme assumes that the EV population and the background demand are known with certainty prior to the charging period. The resulting auction game is static and the auction process can be undertaken off-line ahead of actual charging. In reality, the required information may be difficult to predict accurately, for example EVs may come and go without any advanced warning. Consequently, the efficient NE determined ahead of the charging interval may be suboptimal relative to a solution that considers disturbances. A practical approach to addressing this deficiency is to adopt a receding horizon strategy [52]. This is the focus of ongoing research.

Appendix

A. Proof of Lemma 3.2

The bid price $\beta_{nt}^* = \frac{\partial}{\partial x_{nt}} w_n(\boldsymbol{x}_n^{**})$ can be written,

$$\beta_{nt}^* = 2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} x_{nt}^{**}\Big) - f_n'(x_{nt}^{**}).$$

Because $\beta_{nt}^* \geq 0$ and $f_n'(x_{nt}^{**}) > 0$ by Assumption A2, then $\sum_{t\in\mathcal{T}} x_{nt}^{**} - \Gamma_n < 0$. Together with $\left(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n\right)\lambda_n = 0$ in (5b), this gives $\lambda_n^{**} = 0$.

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With $\beta^*_{nt} = \frac{\partial}{\partial x_{nt}} w_n(\boldsymbol{x}^{**}_n)$ in (5c), the complementarity condition (5a) gives,

$$\beta^*_{nt} \begin{cases} = c'_t(D_t + \sum_{n\in\mathcal{N}} x^{**}_{nt}), & \text{when } x^{**}_{nt} > 0 \\ \leq c'_t(D_t + \sum_{n\in\mathcal{N}} x^{**}_{nt}), & \text{when } x^{**}_{nt} = 0, \end{cases} \tag{40}$$

for all $n \in \mathcal{N}, t \in \mathcal{T}$. Substituting $d^*_{nt} = x^{**}_{nt}$ into (8b) gives,

$$x^*_{nt} - x^{**}_{nt} \leq 0, \quad \sigma^*_{nt} \geq 0, \quad (x^*_{nt} - x^{**}_{nt})\sigma^*_{nt} = 0. \tag{41}$$

If $x^{**}_{nt} = 0$ then $x^*_{nt} = 0 = x^{**}_{nt}$. If $x^{**}_{nt} > 0$ then (40) indicates that $\beta^*_{nt} = c'_t(D_t + \sum_{n\in\mathcal{N}} x^{**}_{nt})$. The following argument shows that $x^*_{nt} < x^{**}_{nt}$ cannot occur.

By the convexity property of $c_t(\cdot)$ and if $x^*_{nt} < x^{**}_{nt}$, then,

$$c'_t(D_t + \sum_{n\in\mathcal{N}} x^*_{nt}) - c'_t(D_t + \sum_{n\in\mathcal{N}} x^{**}_{nt}) < 0. \tag{42}$$

Also, if $x^*_{nt} < x^{**}_{nt}$ then (41) indicates that $\sigma^*_{nt} = 0$. Together with $\frac{\partial L^a}{\partial x_{nt}} \geq 0$ in (8), this gives,

$$c'_t(D_t + \sum_{n\in\mathcal{N}} x^*_{nt}) - c'_t(D_t + \sum_{n\in\mathcal{N}} x^{**}_{nt}) \geq 0,$$

which contradicts (42). Hence, $x^*_{nt} < x^{**}_{nt}$ cannot occur, implying $x^*_{nt} = x^{**}_{nt}$. In summary, under the bid profile $\boldsymbol{b}^*$, the associated optimal allocation $\boldsymbol{x}^*(\boldsymbol{b}^*)$ for Problem 2 is identical to the socially optimal solution to Problem 1.

Also, (10) follows from (40) with $\boldsymbol{x}^* = \boldsymbol{x}^{**}$. *End of Proof.*

B. Proof of Lemma 4.3

If $d^*_{nt} = 0$ then $\widehat{d}_{nt} = 0$, according to (22). In that case, $\widehat{x}_{nt} = x^*_{nt} = 0$ trivially. The remainder of the proof assumes $d^*_{nt} > 0$.

The first step is to establish $u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$. From (18),

$$\widehat{\beta}_{nt} \triangleq \beta_{nt}(\widehat{d}_{nt}; A) > \beta_{nt}(\widehat{d}_{nt}; \sum_{t\in\mathcal{T}} d^*_{nt}),$$

because $A < \sum_{t\in\mathcal{T}} d^*_{nt}$. Also, the constraint $\widehat{d}_{nt} < d^*_{nt}$ ensures that,

$$\beta_{nt}(\widehat{d}_{nt}; \sum_{t\in\mathcal{T}} d^*_{nt}) > \beta_{nt}(d^*_{nt}; \sum_{t\in\mathcal{T}} d^*_{nt}) = \beta^*_{nt}.$$

Therefore $\widehat{\beta}_{nt} > \beta^*_{nt}$, as illustrated in Fig. 6. Also, from Lemma 3.2, $\beta^*_{nt} \geq \beta^*_{mt}$ for all $m \in \mathcal{N} \setminus \{n\}$, so $\widehat{\beta}_{nt} > \beta^*_{mt}$.

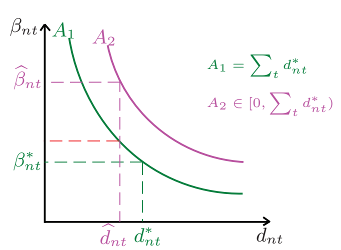

Fig. 6. Relationship between $\widehat{\beta}_{nt}$ and β^*_{nt} when $0 \leq A < \sum_{t\in\mathcal{T}} d^*_{nt}$.

Because $\widehat{x}_{nt} \leq \widehat{d}_{nt} < d^*_{nt} = x^*_{nt}$ and $\widehat{x}_{mt} \leq d^*_{mt} = x^*_{mt}$ for all $m \in \mathcal{N} \setminus \{n\}$, then,

$$c'_t(D_t + \sum_{k\in\mathcal{N}} \widehat{x}_{kt}) < c'_t(D_t + \sum_{k\in\mathcal{N}} x^*_{kt}) = \beta^*_{jt}, \tag{43}$$

for all $j \in \mathcal{N}$ where $d^*_{jt} > 0$. From (8a),(8c), $c'_t(D_t + \sum_{k\in\mathcal{N}} \widehat{x}_{kt}) - \beta^*_{mt} + \widehat{\sigma}_{mt} \geq 0$. Therefore (43) implies $\widehat{\sigma}_{mt} > 0$, and hence from (8b) that $\widehat{x}_{mt} = d^*_{mt}$. Likewise, because $\widehat{\beta}_{nt} > \beta^*_{nt}$, $c'_t(D_t + \sum_{k\in\mathcal{N}} \widehat{x}_{kt}) - \widehat{\beta}_{nt} + \widehat{\sigma}_{nt} \geq 0$ implies $\widehat{\sigma}_{nt} > 0$, and therefore that $\widehat{x}_{nt} = \widehat{d}_{nt}$. Consequently, all players are fully allocated, and (24) is established.

The payment of the n-th player is given by (13), with the above argument indicating that $x^{*,-n}_{mt} = x^*_{mt}$ for all $m \in \mathcal{N} \setminus \{n\}$. Therefore,

$$\tau_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) = \sum_{t\in\mathcal{T}} \Big\{ c_t(D_t + \sum_{m\neq n} d^*_{mt} + \widehat{d}_{nt}) - c_t(D_t + \sum_{m\neq n} d^*_{mt}) \Big\}.$$

The payoff of player n then follows from (14) as,

$$u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) = w_n(\widehat{\boldsymbol{d}}_n) - \tau_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}).$$

The auctioneer needs to find an optimal bid $\widehat{\boldsymbol{b}}^*_n$ that maximizes the payoff $u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$. This is achieved by the following restatement of the optimization problem (23):

Problem 3:

$$\max_{\widehat{\boldsymbol{b}}_n} \; u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}),$$

such that (22) is satisfied together with,

$$\sum_{t\in\mathcal{T}} \widehat{d}_{nt} = A. \tag{44}$$

The Lagrangian for Problem 3 is given by,

$$L^s(\widehat{\boldsymbol{d}}_n, \boldsymbol{\lambda}, \mu) = u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) + \sum_{t\in\mathcal{T}} \lambda_t \left(\widehat{d}_{nt} - d^*_{nt}\right) + \mu\Big(\sum_{t\in\mathcal{T}} \widehat{d}_{nt} - A\Big),$$

where λ_t and μ are the Lagrangian multipliers associated with the constraints (22) and (44), respectively.

The KKT conditions for Problem 3 are given by,

$$\frac{\partial}{\partial \widehat{d}_{nt}} L^s(\widehat{\boldsymbol{d}}_n, \boldsymbol{\lambda}, \mu) \leq 0, \;\; \widehat{d}_{nt} \geq 0, \;\; \frac{\partial}{\partial \widehat{d}_{nt}} L^s(\widehat{\boldsymbol{d}}_n, \boldsymbol{\lambda}, \mu)\widehat{d}_{nt} = 0,$$
$$\widehat{d}_{nt} - d^*_{nt} \leq 0, \quad \lambda_t \geq 0, \quad (\widehat{d}_{nt} - d^*_{nt})\lambda_t = 0,$$
$$\sum_{t\in\mathcal{T}} \widehat{d}_{nt} - A = 0,$$

for all $t \in \mathcal{T}$, and,

$$\frac{\partial}{\partial \widehat{d}_{nt}} L^s(\widehat{\boldsymbol{d}}_n, \boldsymbol{\lambda}, \mu) = -f'_n(\widehat{d}_{nt}) - c'_t(D_t + \sum_{m\neq n} d^*_{mt} + \widehat{d}_{nt}) + \lambda_t + \mu. \tag{45}$$

Since $\widehat{d}_{nt} - d^*_{nt} < 0$, then $\lambda^*_t = 0$, and so,

$$g'_{nt}(\widehat{d}^*_{nt}) = f'_n(\widehat{d}^*_{nt}) + c'_t(D_t + \sum_{m\neq n} d^*_{mt} + \widehat{d}^*_{nt}) \geq \mu,$$

where $g_{nt}(d_{nt})$ is defined in (25). If $\widehat{d}^*_{nt} > 0$ then (45) equals zero, and so $g'_{nt}(\widehat{d}^*_{nt}) = \mu$. Therefore (26) holds.

Under Assumptions (A1, A2), the necessary KKT conditions are also sufficient for optimality. Consequently, (26) is sufficient for the optimal bid $\widehat{\boldsymbol{b}}^*_n$ of player n. *End of proof.*

C. Verification of inequality (28) in Theorem 4.2

For notational simplicity, $\widetilde{\boldsymbol{x}}$ and $\widehat{\boldsymbol{x}}$ denote the optimal allocations with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$, respectively.

Consider the allocations of all EVs at time t_1 with respect to the bid profile $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}$. Because $\widehat{d}_{nt_1} < \widehat{d}^*_{nt_1} < d^*_{nt_1} < \widehat{d}^2_{nt_1}$, (18a) gives,

$$\widehat{\beta}_{nt_1} \triangleq \beta_{nt_1}(\widehat{d}_{nt_1}; A) > \beta_{nt_1}(\widehat{d}^2_{nt_1}; A) = \beta^*_{nt_1}.$$

Also, by Lemma 3.2, $\beta^*_{nt} \geq \beta^*_{mt}$ for all $m \in \mathcal{N} \backslash \{n\}$ when $d^*_{nt} > 0$. Therefore, $\widehat{\beta}_{nt_1} > \beta^*_{mt_1}$. Using an argument similar to that following (43), it is straightforward to show,

$$\widehat{x}_{nt_1} = \widehat{d}_{nt_1}, \qquad \widehat{x}_{mt_1} = d^*_{mt_1} \text{ for all } m \in \mathcal{N} \setminus \{n\}. \tag{46}$$

Hence, at time t_1, all EVs are fully allocated with respect to $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}$. Similarly, with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}$,

$$\widetilde{x}_{nt_1} = \widetilde{d}_{nt_1}, \qquad \widetilde{x}_{mt_1} = d^*_{mt_1} \text{ for all } m \in \mathcal{N} \setminus \{n\}. \tag{47}$$

By (46) and (47), the difference in the payments of the n-th EV at time t_1 with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}$ is given by,

$$\begin{aligned}\Delta\tau_{nt_1} &\triangleq \tau_{nt_1}\Big((\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}\Big) - \tau_{nt_1}\Big((\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}\Big)\\ &= c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widetilde{d}_{nt_1})\\ &\qquad - c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widehat{d}_{nt_1}).\end{aligned} \tag{48}$$

For the n-th EV at time t_2, the difference in payments with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$ is given by,

$$\begin{aligned}\Delta\tau_{nt_2} &\triangleq \tau_{nt_2}\Big((\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}\Big) - \tau_{nt_2}\Big((\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}\Big)\\ &= c_t(D_{t_2} + \sum_{m\neq n} \widetilde{x}_{mt_2} + \widetilde{x}_{nt_2})\\ &\qquad - c_t(D_{t_2} + \sum_{m\neq n} \widehat{x}_{mt_2} + \widehat{x}_{nt_2})\\ &\qquad + \sum_{m\neq n} \beta^*_{mt_2}(\widehat{x}_{mt_2} - \widetilde{x}_{mt_2}).\end{aligned} \tag{49}$$

The last term of (49) can be simplified by recalling from Lemma 3.2 that all EVs, $k \in \mathcal{N}$, with $d^*_{kt_2} > 0$ share the same value for $\beta^*_{kt_2}$. Denoting that common value by $\beta^*_{\diamond t_2}$ allows the last term to be expressed as $\beta^*_{\diamond t_2} \sum_{m\neq n}(\widehat{x}_{mt_2} - \widetilde{x}_{mt_2})$.

It follows from (27d) that for the n-th EV, the difference in payments at times $t \neq t_1, t_2$, with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_t$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_t$ is,

$$\Delta\tau_{nt} \triangleq \tau_{nt}\big((\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_t\big) - \tau_{nt}\big((\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_t\big) = 0, \ \forall t \neq t_1, t_2. \tag{50}$$

Thus, by (48)-(50), the difference in the payments of the n-th EV with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$ satisfies,

$$\Delta\tau_n \triangleq \tau_n(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) - \tau_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) = \Delta\tau_{nt_1} + \Delta\tau_{nt_2}. \tag{51}$$

The difference in utility of the n-th EV, with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$, is given by,

$$\begin{aligned}\Delta w_n &\triangleq w_n(\widetilde{\boldsymbol{x}}_n) - w_n(\widehat{\boldsymbol{x}}_n)\\ &= -\delta_n\Big(\sum_{t\in\mathcal{T}} \widetilde{x}_{nt} - \Gamma_n\Big)^2 + \delta_n\Big(\sum_{t\in\mathcal{T}} \widehat{x}_{nt} - \Gamma_n\Big)^2\\ &\qquad + f_n(\widehat{d}_{nt_1}) - f_n(\widetilde{d}_{nt_1}) + f_n(\widehat{x}_{nt_2}) - f_n(\widetilde{x}_{nt_2}).\end{aligned} \tag{52}$$

By (51) and (52), the difference in the payoff of the n-th EV, subject to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$, becomes,

$$\Delta u_n \triangleq u_n(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) - u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) = \Delta w_n - \Delta\tau_n. \tag{53}$$

To establish (28), firstly the case with $\widehat{d}_{nt_2} = d^*_{nt_2}$ will be addressed, then the three cases $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in \mathcal{R}_i, i = 1, 2, 3$ will be considered separately.

Case I, $\widehat{d}^*_{nt_2} < \widetilde{d}_{nt_2} < \widehat{d}_{nt_2} = d^*_{nt_2}$

Because $\widehat{d}_{nt_2} = d^*_{nt_2}$,

$$\widehat{\beta}_{nt_2} = \beta_{nt_2}(d^*_{nt_2}, A) > \beta_{nt_2}(d^*_{nt_2}, \sum_{t\in\mathcal{T}} d^*_{nt}) = \beta^*_{nt_2}.$$

Likewise, with $\widehat{d}^*_{nt_2} < \widetilde{d}_{nt_2} < \widehat{d}_{nt_2} = d^*_{nt_2}$,

$$\widetilde{\beta}_{nt_2} = \beta_{nt_2}(\widetilde{d}_{nt_2}, A) > \beta_{nt_2}(\widehat{d}^2_{nt_2}, A) = \beta^*_{nt_2}.$$

A similar argument to that used to establish (46),(47) for t_1 shows that all EVs are fully allocated at t_2 with respect to both $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$ and $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$:

$$\begin{aligned}&\widehat{x}_{nt_2} = \widehat{d}_{nt_2} = d^*_{nt_2}, \qquad \widehat{x}_{mt_2} = d^*_{mt_2} \text{ for all } m \in \mathcal{N} \setminus \{n\},\\ &\widetilde{x}_{nt_2} = \widetilde{d}_{nt_2}, \qquad\qquad\quad \widetilde{x}_{mt_2} = d^*_{mt_2} \text{ for all } m \in \mathcal{N} \setminus \{n\}.\end{aligned}$$

Substituting these allocations into (53) gives,

$$\begin{aligned}\Delta u_n &= f_n(\widehat{d}_{nt_1}) - f_n(\widetilde{d}_{nt_1}) + f_n(\widehat{d}_{nt_2}) - f_n(\widetilde{d}_{nt_2})\\ &\quad - \Big(c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widetilde{d}_{nt_1})\\ &\qquad - c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widehat{d}_{nt_1})\\ &\qquad + c_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widetilde{d}_{nt_2})\\ &\qquad - c_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{d}_{nt_2})\Big)\\ &= g_{nt_1}(\widehat{d}_{nt_1}) - g_{nt_1}(\widetilde{d}_{nt_1}) + g_{nt_2}(\widehat{d}_{nt_2}) - g_{nt_2}(\widetilde{d}_{nt_2})\\ &> g'_{nt_1}(\widehat{d}^*_{nt_1})\big(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}\big) + g'_{nt_2}(\widehat{d}^*_{nt_2})\big(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}\big)\\ &= \mu(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1} + \widehat{d}_{nt_2} - \widetilde{d}_{nt_2})\\ &= 0,\end{aligned}$$

where the inequality holds due to the convexity of $g_{nt}(\cdot)$ and the subsequent equality follows from (26). Therefore, (28) is satisfied in this case.

Case II, $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in \mathcal{R}_1$

The initial step in showing (28) is to determine the allocations of all EVs at time t_2 with respect to the bid profile $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$. Firstly, consider $\widehat{d}_{nt_2} \in \text{Int}(\mathcal{R}_1)$. Then,

$$\begin{aligned}\widehat{\beta}_{nt_2} > \widehat{\beta}^1_{nt_2} &= c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{d}^1_{nt_2})\\ &> c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{d}_{nt_2}),\end{aligned}$$

so it follows from the KKT conditions (8) that $\widehat{x}_{nt_2} = \widehat{d}_{nt_2}$.

Now consider the case with $\widehat{d}_{nt_2} = \widehat{d}^1_{nt_2}$, the upper boundary of $\mathcal{R}_1$. In this case, $\widehat{\beta}_{nt_2} = c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{d}_{nt_2})$. Assume $\widehat{x}_{nt_2} < \widehat{d}_{nt_2}$. Then due to the convexity of $c_t(\cdot)$,

$$c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{x}_{nt_2}) < c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{d}_{nt_2}) = \widehat{\beta}_{nt_2}.$$

But (8) then implies $\widehat{\sigma}_{nt_2} > 0$ and therefore that $\widehat{x}_{nt_2} = \widehat{d}_{nt_2}$. Hence a contradiction, so $\widehat{x}_{nt_2} = \widehat{d}_{nt_2}$.

If $\widehat{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}$ then it can be shown by contradiction that $\sum_{m\neq n} \widehat{x}_{mt_2} = 0$. Assuming $\sum_{m\neq n} \widehat{x}_{mt_2} > 0$ gives,

$$c'_t(D_{t_2} + \sum_{k\neq n} \widehat{x}_{kt_2} + \widehat{x}_{nt_2}) > c'_t(D_{t_2} + \sum_{k\in\mathcal{N}} d^*_{kt_2}) \geq \beta^*_{mt_2},$$

for all $m \in \mathcal{N} \setminus \{n\}$. But (8) then implies $\widehat{x}_{mt_2} = 0$ for all $m \in \mathcal{N} \setminus \{n\}$, hence a contradiction. Alternatively, if $\widehat{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}$

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then it can be shown, once again by contradiction, that $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} = \sum_{k\in\mathcal{N}} d^*_{kt_2}$. Consider $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} > \sum_{k\in\mathcal{N}} d^*_{kt_2}$. Then $c'_t(D_{t_2} + \sum_{k\in\mathcal{N}} \widehat{x}_{kt_2}) > \beta^*_{mt_2}$ for $m \in \mathcal{N}\backslash\{n\}$, with (8) implying $\widehat{x}_{mt_2} = 0$, hence a contradiction. If $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}$, then $c'_t(D_{t_2} + \sum_{k\in\mathcal{N}} \widehat{x}_{kt_2}) < \beta^*_{mt_2}$, with (8) implying $\widehat{x}_{mt_2} = d^*_{mt_2}$. This leads to another contradiction, as $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} = \sum_{m\neq n} d^*_{mt_2} + \widehat{d}_{nt_2} > \sum_{k\in\mathcal{N}} d^*_{kt_2}$. Summarizing,

$$\widehat{x}_{nt_2} = \widehat{d}_{nt_2}, \quad \sum_{m\neq n} \widehat{x}_{mt_2} > 0, \quad \sum_{m\neq n} \widehat{x}_{mt_2} + \widehat{d}_{nt_2} = \sum_{k\in\mathcal{N}} d^*_{kt_2}, \quad \text{if } \widehat{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}, \tag{54a}$$

$$\widehat{x}_{nt_2} = \widehat{d}_{nt_2}, \quad \sum_{m\neq n} \widehat{x}_{mt_2} = 0, \quad \sum_{m\neq n} \widehat{x}_{mt_2} + \widehat{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}, \quad \text{if } \widehat{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}. \tag{54b}$$

Similarly, the above analysis also holds for the bid profile $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$ so,

$$\widetilde{x}_{nt_2} = \widetilde{d}_{nt_2}, \quad \sum_{m\neq n} \widetilde{x}_{mt_2} > 0, \quad \sum_{m\neq n} \widetilde{x}_{mt_2} + \widetilde{d}_{nt_2} = \sum_{k\in\mathcal{N}} d^*_{kt_2}, \quad \text{if } \widetilde{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}, \tag{55a}$$

$$\widetilde{x}_{nt_2} = \widetilde{d}_{nt_2}, \quad \sum_{m\neq n} \widetilde{x}_{mt_2} = 0, \quad \sum_{m\neq n} \widetilde{x}_{mt_2} + \widetilde{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}, \quad \text{if } \widetilde{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}. \tag{55b}$$

Substituting into (51) gives,

$$\begin{aligned}\Delta\tau_n = {} & c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widetilde{d}_{nt_1}) - c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widehat{d}_{nt_1}) \\ & + c_t(D_{t_2} + \sum_{m\neq n} \widetilde{x}_{mt_2} + \widetilde{d}_{nt_2}) + \beta^*_{\diamond t_2} \sum_{m\neq n} (\widehat{x}_{mt_2} - \widetilde{x}_{mt_2}) \\ & - c_t(D_{t_2} + \sum_{m\neq n} \widehat{x}_{mt_2} + \widehat{d}_{nt_2}).\end{aligned}$$

Because $\widehat{x}_{nt_1} + \widehat{x}_{nt_2} = \widetilde{x}_{nt_1} + \widetilde{x}_{nt_2}$ and $\widehat{x}_{nt} = \widetilde{x}_{nt}$ for all $t \neq t_1, t_2$, it follows that $\sum_t \widehat{x}_{nt} = \sum_t \widetilde{x}_{nt}$, and so (52) becomes,

$$\Delta w_n = f_n(\widehat{d}_{nt_1}) + f_n(\widehat{d}_{nt_2}) - f_n(\widetilde{d}_{nt_1}) - f_n(\widetilde{d}_{nt_2}).$$

Three subcases must be considered, depending on the relative values of $\widetilde{d}_{nt_2}$, $\widehat{d}_{nt_2}$ and $\sum_{k\in\mathcal{N}} d^*_{kt_2}$.

Case II.1, $\widetilde{d}_{nt_2} < \widehat{d}_{nt_2} < \sum_{k=1}^{N} d^*_{kt_2}$: In this case, Δu_n defined in (53) is established using (54a) and (55a),

$$\begin{aligned}\Delta u_n &= g_{nt_1}(\widehat{d}_{nt_1}) - g_{nt_1}(\widetilde{d}_{nt_1}) + f_n(\widehat{d}_{nt_2}) - f_n(\widetilde{d}_{nt_2}) \\ &\quad - \beta^*_{\diamond t_2}(\widetilde{d}_{nt_2} - \widehat{d}_{nt_2}) \qquad (56a)\\ &> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) \\ &\quad + \left(f'_n(d^*_{nt_2}) + \beta^*_{\diamond t_2}\right)(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \qquad (56b)\\ &= g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + g'_{nt_2}(d^*_{nt_2})(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}), \qquad (56c)\end{aligned}$$

where (56a) holds by the specification of $g_{nt}(\cdot)$ given in (25) and substitution from (54a) and (55a); (56b) holds by the convexity of $g_{nt}(\cdot)$ together with (27a), and the convexity of $f_n(\cdot)$ together with (29); and (56c) holds by (10) in Lemma 3.2 and (25).

From (22), $\widehat{d}^*_{nt_2} < d^*_{nt_2}$, so $g'_{nt_2}(d^*_{nt_2}) > g'_{nt_2}(\widehat{d}^*_{nt_2})$ due to the convexity of $g_{nt_2}(\cdot)$. By construction, $\widehat{d}^*_{nt_1} > 0$, so (26) gives $g'_{nt_2}(\widehat{d}^*_{nt_2}) \geq g'_{nt_1}(\widehat{d}^*_{nt_1}) = \mu$. Therefore, because (29) ensures $\widehat{d}_{nt_2} > \widetilde{d}_{nt_2}$, (56c) gives,

$$\Delta u_n > \mu(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1} + \widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) = 0, \tag{57}$$

where the final equality holds by (27c).

Case II.2, $\widetilde{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2} \leq \widehat{d}_{nt_2}$: In this case, Δu_n is governed by (54b) and (55a), giving,

$$\begin{aligned}\Delta u_n &= g_{nt_1}(\widehat{d}_{nt_1}) - g_{nt_1}(\widetilde{d}_{nt_1}) + f_n(\widehat{d}_{nt_2}) - f_n(\widetilde{d}_{nt_2}) \\ &\quad - c_t(D_{t_2} + \sum_{m\neq n} \widetilde{x}_{mt_2} + \widetilde{d}_{nt_2}) + c_t(D_{t_2} + \widehat{d}_{nt_2}) \\ &\quad + \beta^*_{\diamond t_2} \sum_{m\neq n} \widetilde{x}_{mt_2} \qquad (58a)\\ &> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(d^*_{nt_2})(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \\ &\quad + c'_t(D_{t_2} + \sum_{k\in\mathcal{N}} d^*_{kt_2})(\widehat{d}_{nt_2} - \sum_{m\neq n} \widetilde{x}_{mt_2} - \widetilde{d}_{nt_2}) \\ &\quad + \beta^*_{\diamond t_2} \sum_{m\neq n} \widetilde{x}_{mt_2} \qquad (58b)\\ &= g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(d^*_{nt_2})(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \\ &\quad + \beta^*_{\diamond t_2}(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}), \qquad (58c)\end{aligned}$$

where (58a) holds by (53) and (25); (58b) holds by the convexity of $g_{nt}(\cdot)$ together with (27a), the convexity of $f_n(\cdot)$ together with (29), and the convexity of $c_t(\cdot)$ using (55a); and (58c) holds by (10). Proceeding as in (56c),(57) yields $\Delta u_n > 0$.

Case II.3, $\sum_{k\in\mathcal{N}} d^*_{kt_2} \leq \widetilde{d}_{nt_2} < \widehat{d}_{nt_2}$: In this case, Δu_n uses (54b) and (55b) to give,

$$\begin{aligned}\Delta u_n &= g_{nt_1}(\widehat{d}_{nt_1}) - g_{nt_1}(\widetilde{d}_{nt_1}) + f_n(\widehat{d}_{nt_2}) - f_n(\widetilde{d}_{nt_2}) \\ &\quad - c_t(D_{t_2} + \widetilde{d}_{nt_2}) + c_t(D_{t_2} + \widehat{d}_{nt_2}) \qquad (59a)\\ &> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(d^*_{nt_2})(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \\ &\quad + c'_t(D_{t_2} + \widetilde{d}_{nt_2})(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \qquad (59b)\\ &\geq g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) \\ &\quad + \Big(f'_n(d^*_{nt_2}) + c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + d^*_{nt_2})\Big) \\ &\quad \times (\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \qquad (59c)\\ &= g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + g'_{nt_2}(d^*_{nt_2})(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}), \qquad (59d)\end{aligned}$$

where (59a) holds by (53) and (25); (59b) holds by the convexity of $g_{nt}(\cdot)$ together with (27a), and the convexity of $f_n(\cdot)$ and $c_t(\cdot)$ together with (29); (59c) holds by the convexity of $c_t(\cdot)$ with $\widetilde{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}$; and (59d) holds by (25). Proceeding as in (57) yields $\Delta u_n > 0$.

Hence, $\Delta u_n > 0$ whenever $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in \mathcal{R}_1$.

Case III, $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in Int(\mathcal{R}_2)$

The situation where $\widehat{d}_{nt_2} \in \mathcal{R}_2$ will be considered as two separate cases. Case III, presented here, discusses $\widehat{d}_{nt_2} \in \text{Int}(\mathcal{R}_2)$, while Case IV addresses $\widehat{d}_{nt_2} = \widehat{d}^2_{nt_2}$, the upper boundary of $\mathcal{R}_2$.

Consider the allocations of all EVs at time t_2 with respect to the bid profile $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$. If $\widehat{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}$, then the argument presented in Case II can again be used to show that $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} = \sum_{k\in\mathcal{N}} d^*_{kt_2}$. Also, because $\widehat{\beta}_{nt_2} > \beta^*_{nt_2} = c'_t(D_{t_2} + \sum_{k\in\mathcal{N}} d^*_{kt_2})$, (8) implies $\widehat{x}_{nt_2} = \widehat{d}_{nt_2}$. Similar outcomes hold for the bid profile $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$ as $\widetilde{d}_{nt_2} < \widehat{d}_{nt_2}$. Therefore, (54a) and (55a) are again applicable.

However, if $\widehat{d}_{nt_2} > \sum_{k\in\mathcal{N}} d^*_{kt_2}$, then because $\widehat{\beta}_{nt_2} < \widehat{\beta}^1_{nt_2} = c'_t(D_t + \sum_{m\neq n} d^*_{mt_2} + \widehat{d}^1_{nt_2})$, there is no guarantee that $\widehat{x}_{nt_2} = \widehat{d}_{nt_2}$. Whether or not (54b) holds depends on the comparison between $\widehat{\beta}_{nt_2}$ and $c'_t(D_{t_2} + \sum_{m\neq n} \widehat{x}_{mt_2} + \widehat{d}_{nt_2})$. Similarly, for the bid profile $(\widetilde{b}_{nt_2}, \boldsymbol{b}^*_{-n,t_2})$, there is no guarantee that (55b) holds.

Three subcases must be considered for $\widehat{\boldsymbol{x}}_{t_2}$ and $\widetilde{\boldsymbol{x}}_{t_2}$, depending on the relative values of $\widetilde{d}_{nt_2}$, $\widehat{d}_{nt_2}$ and $\sum_{k\in\mathcal{N}} d^*_{kt_2}$.

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Case III.1, $\widetilde{d}_{nt_2} < \widehat{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}$: Analysis of Δu_n in this case is identical to that of Case II.1, so $\Delta u_n > 0$.

Case III.2, $\widetilde{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2} \le \widehat{d}_{nt_2}$: Because $\widehat{\beta}_{nt_2} > \beta^*_{nt_2}$, satisfying (8) for the n-th EV results in $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} \ge \sum_{k\in\mathcal{N}} d^*_{kt_2}$, with equality holding only if $\widehat{x}_{nt_2} = \widehat{d}_{nt_2} = \sum_{k\in\mathcal{N}} d^*_{kt_2}$ and $\sum_{m\neq n} \widehat{x}_{mt_2} = 0$. If the inequality is strict, then $c'_t(D_{t_2} + \sum_{k\in\mathcal{N}} \widehat{x}_{kt_2}) > \beta^*_{mt_2}$ for all $m \in \mathcal{N}\setminus\{n\}$, with (8) implying $\widehat{x}_{mt_2} = 0$. Hence,

$$\sum_{k\in\mathcal{N}} d^*_{kt_2} \le \widehat{x}_{nt_2} \le \widehat{d}_{nt_2}, \qquad \sum_{m\neq n} \widehat{x}_{mt_2} = 0. \tag{60}$$

The applicability of (54b) reverts to a comparison between $\widehat{\beta}_{nt_2}$ and $c'_t(D_{t_2} + \widehat{d}_{nt_2})$:

- If $\widehat{\beta}_{nt_2} \ge c'_t(D_{t_2} + \widehat{d}_{nt_2})$ then it can be verified that (54b) holds. Thus, $\Delta u_n > 0$, since the analysis in this case is identical to that developed in Case II.2.
- If $\widehat{\beta}_{nt_2} < c'_t(D_{t_2} + \widehat{d}_{nt_2})$, (54b) does not hold. Rather, Δu_n can be established using (53), (25), (55a) and (60),

$$\begin{aligned}
\Delta u_n &= g_{nt_1}(\widehat{d}_{nt_1}) - g_{nt_1}(\widetilde{d}_{nt_1}) - \delta_n\Big(\sum_{t\in\mathcal{T}} \widetilde{x}_{nt} - \Gamma_n\Big)^2 \\
&\quad + \delta_n\Big(\sum_{t\in\mathcal{T}} \widehat{x}_{nt} - \Gamma_n\Big)^2 + f_n(\widehat{x}_{nt_2}) - f_n(\widetilde{d}_{nt_2}) \\
&\quad - c_t(D_{t_2} + \sum_{m\neq n} \widetilde{x}_{mt_2} + \widetilde{d}_{nt_2}) + c_t(D_{t_2} + \widehat{x}_{nt_2}) \\
&\quad + \beta^*_{\diamond t_2} \sum_{m\neq n} \widetilde{x}_{mt_2} \\
&> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(d^*_{nt_2})(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) \\
&\quad + g'_{nt_2}(d^*_{nt_2})(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1} + \widetilde{d}_{nt_2} - \widehat{x}_{nt_2}) \\
&\quad + \beta^*_{\diamond t_2} \sum_{m\neq n} \widetilde{x}_{mt_2} + c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + d^*_{nt_2}) \\
&\qquad \times \big(\widehat{x}_{nt_2} - \sum_{m\neq n} \widetilde{x}_{mt_2} - \widetilde{d}_{nt_2}\big) \qquad (61a) \\
&= g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + g'_{nt_2}(d^*_{nt_2})(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}) \qquad (61b) \\
&> \mu(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + \mu(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}) \qquad (61c) \\
&= 0,
\end{aligned}$$

where (61a) holds by the convexity of $g_{nt}(\cdot)$ together with (27a), the convexity of $f_n(\cdot)$ together with (29) and (60), the convexity of $c_t(\cdot)$ together with (55a) and (60), and the concavity of $-\delta_n(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n)^2$ together with Lemma A.1 specified below, recalling that $\widehat{\boldsymbol{b}}_n, \widetilde{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ with $\sum_{t\in\mathcal{T}} d_{nt} = A < \sum_{t\in\mathcal{T}} d^*_{nt}$, and that $\sum_{t\in\mathcal{T}} \widetilde{x}_{nt} - \sum_{t\in\mathcal{T}} \widehat{x}_{nt} = \widetilde{d}_{nt_1} + \widetilde{d}_{nt_2} - (\widehat{d}_{nt_1} + \widehat{x}_{nt_2}) \ge 0$; (61b) holds by (10) in Lemma 3.2 together with (25); and (61c) follows the same justification as (57) though using (27a).

Lemma A.1: Consider an allocation $\boldsymbol{x}_n(\boldsymbol{b}) \equiv (x_{nt}, t\in\mathcal{T})$ with respect to a bid profile $\boldsymbol{b}$, such that $\sum_{t\in\mathcal{T}} d_{nt} < \sum_{t\in\mathcal{T}} d^*_{nt}$. Then:

$$\frac{\partial}{\partial x_{nt}}\Big(-\delta_n\Big(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n\Big)^2\Big) > g'_{nt}(d^*_{nt}) > \mu, \tag{62}$$

for all $t\in\mathcal{T}$, where g_{nt} is defined in Lemma 4.3.

Proof of Lemma A.1.

$$\begin{aligned}
\frac{\partial}{\partial x_{nt}}\Big(-\delta_n\Big(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n\Big)^2\Big) &= 2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} x_{nt}\Big) \\
&> 2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} d^*_{nt}\Big) \qquad (63a) \\
&= \beta^*_{nt} + f'_n(d^*_{nt}) \qquad (63b) \\
&= c'_t(D_t + \sum_{m\neq n} d^*_{mt} + d^*_{nt}) + f'_n(d^*_{nt}) \qquad (63c) \\
&= g'_{nt}(d^*_{nt}) \qquad (63d) \\
&> \mu, \qquad (63e)
\end{aligned}$$

where (63a) holds because $\sum_{t\in\mathcal{T}} x_{nt} \le \sum_{t\in\mathcal{T}} d_{nt} < \sum_{t\in\mathcal{T}} d^*_{nt}$; (63b) follows from $\beta^*_{nt} = \frac{\partial}{\partial d_{nt}} w_n(\boldsymbol{d}^*_n) = -f'_n(d^*_{nt}) + 2\delta_n(\Gamma_n - \sum_{t\in\mathcal{T}} d^*_{nt})$; (63c) holds by (10) in Lemma 3.2; (63d) holds by the specification of g_{nt} in (25); and (63e) holds by Lemma 4.3 and the convexity of g_{nt}.

End of proof of Lemma A.1.

Case III.3, $\sum_{k\in\mathcal{N}} d^*_{kt_2} \le \widetilde{d}_{nt_2} < \widehat{d}_{nt_2}$: Using the same argument as in Case III.2 gives $\sum_{m\neq n} \widehat{x}_{mt_2} = \sum_{m\neq n} \widetilde{x}_{mt_2} = 0$. Analysis of Δu_n depends on the relative values of $\widetilde{d}_{nt_2}$, $\widehat{x}_{nt_2}$ and $\widehat{d}_{nt_2}$, keeping in mind from Lemma 4.1 that $\beta_{nt_2}(\widetilde{d}_{nt_2}, A) > \beta_{nt_2}(\widehat{d}_{nt_2}, A)$.

- If $\widehat{x}_{nt_2} = \widehat{d}_{nt_2}$ then $\widetilde{x}_{nt_2} = \widetilde{d}_{nt_2}$ must also hold. Analysis of Δu_n in this case is identical to that developed in Case II.3.
- If $\widetilde{d}_{nt_2} \le \widehat{x}_{nt_2} < \widehat{d}_{nt_2}$ then $\widetilde{x}_{nt_2} = \widetilde{d}_{nt_2} \le \widehat{x}_{nt_2}$. Analysis of Δu_n follows that of Case III.2.
- If $\widehat{x}_{nt_2} < \widetilde{d}_{nt_2} < \widehat{d}_{nt_2}$ then $\widehat{x}_{nt_2} < \widetilde{x}_{nt_2}$, and Δu_n satisfies,

$$\begin{aligned}
\Delta u_n &> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(\widetilde{x}_{nt_2})(\widehat{x}_{nt_2} - \widetilde{x}_{nt_2}) \\
&\quad + c'_t(D_{t_2} + \widetilde{x}_{nt_2})(\widetilde{x}_{nt_2} - \widehat{x}_{nt_2}) \\
&\quad + g'_{nt_2}(d^*_{nt_2})(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}) \\
&\quad + 2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{x}_{nt}\Big)(\widetilde{x}_{nt_2} - \widehat{x}_{nt_2}) \qquad (64) \\
&> f'_n(\widetilde{x}_{nt_2})(\widehat{x}_{nt_2} - \widetilde{x}_{nt_2}) \\
&\quad + c'_t(D_{t_2} + \widetilde{x}_{nt_2})(\widetilde{x}_{nt_2} - \widehat{x}_{nt_2}) \\
&\quad + 2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{x}_{nt}\Big)(\widetilde{x}_{nt_2} - \widehat{x}_{nt_2}), \qquad (65)
\end{aligned}$$

where (64) holds by the convexity of $g_{nt_1}(\cdot)$ together with (27a), the convexity of $f_n(\cdot)$ and $c_t(\cdot)$, the concavity of $-\delta_n(\Gamma_n - \sum_{t\in\mathcal{T}} x_{nt})^2$ and Lemma A.1; and (65) makes use of (61b). Further analysis uses $\widetilde{x}_{nt} \le \widetilde{d}_{nt}$ for all $t\in\mathcal{T}$ to give,

$$\begin{aligned}
2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{x}_{nt}\Big) &\ge 2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{d}_{nt}\Big) \\
&= f'_n(\widetilde{d}_{nt_2}) + \widetilde{\beta}_{nt_2}, \qquad (66)
\end{aligned}$$

where the equality follows from (11). Because $\widetilde{x}_{nt_2} > 0$ and $\sum_{m\neq n} \widetilde{x}_{mt_2} = 0$, (8) gives $\widetilde{\beta}_{nt_2} \ge c'_t(D_{t_2} + \widetilde{x}_{nt_2})$. Therefore,

$$\begin{aligned}
2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{d}_{nt}\Big) &\ge f'_n(\widetilde{d}_{nt_2}) + c'_t(D_{t_2} + \widetilde{x}_{nt_2}) \\
&\ge f'_n(\widetilde{x}_{nt_2}) + c'_t(D_{t_2} + \widetilde{x}_{nt_2}). \qquad (67)
\end{aligned}$$

Because $\widehat{x}_{nt_2} < \widetilde{x}_{nt_2}$, (65) and (67) ensure $\Delta u_n > 0$.

Hence, $\Delta u_n > 0$ whenever $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in \text{Int}(\mathcal{R}_2)$.

Case IV, $\widehat{d}_{nt_2} = \widehat{d}^2_{nt_2}, \widetilde{d}_{nt_2} \in \mathcal{R}_2$

In this case, $\widehat{\beta}_{nt_2} = \beta^*_{nt_2}$, so (8) ensures that $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} = \sum_{k\in\mathcal{N}} d^*_{kt_2}$ and $d^*_{nt_2} \le \widehat{x}_{nt_2} \le \widehat{d}^2_{nt_2}$.

Case IV.1, $\widetilde{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}$: Using the same argument as in Case III, (55a) is again applicable.

If $\widehat{x}_{nt_2} > \widetilde{x}_{nt_2} = \widetilde{d}_{nt_2}$, Δu_n can be established by,

$$\begin{aligned}
\Delta u_n &> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(d^*_{nt_2})(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) \\
&\quad + g'_{nt_2}(d^*_{nt_2})(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1} + \widetilde{d}_{nt_2} - \widehat{x}_{nt_2}) \\
&\quad + \beta^*_{\diamond t_2}(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) \qquad (68a)
\end{aligned}$$

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$$= g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + g'_{nt_2}(d^*_{nt_2})(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}) \tag{68b}$$

$$> 0, \tag{68c}$$

where (68a) holds by the convexity of $g_{nt}(\cdot)$ together with (27a), the convexity of $f_n(\cdot)$ together with $\widehat{x}_{nt_2} > \widetilde{x}_{nt_2}$ and (55a), and the concavity of $-\delta_n(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n)^2$ together with Lemma A.1, recalling that $\widehat{\boldsymbol{b}}_n, \widetilde{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ with $\sum_{t\in\mathcal{T}} d_{nt} = A < \sum_{t\in\mathcal{T}} d^*_{nt}$; (68b) holds by (10) in Lemma 3.2 together with (25); and (68c) follows from (61b).

If $\widehat{x}_{nt_2} < \widetilde{x}_{nt_2} = \widetilde{d}_{nt_2}$, Δu_n is given by,

$$\begin{aligned}\Delta u_n &> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(\widetilde{d}_{nt_2})(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) \\ &\quad + g'_{nt_2}(d^*_{nt_2})(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}) + \beta^*_{\diamond t_2}(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) \\ &\quad + 2\delta_n(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{x}_{nt})(\widetilde{d}_{nt_2} - \widehat{x}_{nt_2}) \qquad (69a) \\ &> f'_n(\widetilde{d}_{nt_2})(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) + \beta^*_{\diamond t_2}(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) \\ &\quad + 2\delta_n(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{x}_{nt})(\widetilde{d}_{nt_2} - \widehat{x}_{nt_2}) \qquad (69b) \\ &> 0, \qquad (69c)\end{aligned}$$

where (69a) holds by the convexity of $g_{nt}(\cdot)$ together with (27a), the convexity of $f_n(\cdot)$ together with (55a), and the concavity of $-\delta_n(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n)^2$ together with Lemma A.1, recalling that $\widehat{\boldsymbol{b}}_n, \widetilde{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ with $\sum_{t\in\mathcal{T}} d_{nt} = A < \sum_{t\in\mathcal{T}} d^*_{nt}$; (69b) uses (68b); and (69c) uses (66) together with $\widetilde{\beta}_{nt_2} > \beta^*_{\diamond t_2}$.

Case IV.2, $\widetilde{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}$: Using the same argument as in Case III, (55b) is applicable. Then similar to the analysis of Case IV.1, $\Delta u_n > 0$.

Case V, $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in \mathcal{R}_3$

In this case, $\widehat{\beta}_{nt_2} < \widetilde{\beta}_{nt_2} < \beta^*_{nt_2}$, so (8) ensures that $\widehat{x}_{mt_2} = \widetilde{x}_{mt_2} = d^*_{mt_2}$ for all $m \in \mathcal{N}\setminus\{n\}$, and $\widehat{x}_{nt_2} \leq \widetilde{x}_{nt_2} < d^*_{nt_2}$.[4] Hence, (51) becomes,

$$\begin{aligned}&\Delta\tau_n = \\ &c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widetilde{d}_{nt_1}) - c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widehat{d}_{nt_1}) \\ &+ c_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widetilde{x}_{nt_2}) - c_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{x}_{nt_2}).\end{aligned} \tag{70}$$

Using (52) and (70) in (53) gives,

$$\begin{aligned}\Delta u_n &= g_{nt_1}(\widehat{d}_{nt_1}) - g_{nt_1}(\widetilde{d}_{nt_1}) + g_{nt_2}(\widehat{x}_{nt_2}) - g_{nt_2}(\widetilde{x}_{nt_2}) \\ &\quad - \delta_n\Big(\sum_{t\in\mathcal{T}} \widetilde{x}_{nt} - \Gamma_n\Big)^2 + \delta_n\Big(\sum_{t\in\mathcal{T}} \widehat{x}_{nt} - \Gamma_n\Big)^2 \\ &> -\mu\left(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}\right) - g'_{nt_2}(d^*_{nt_2})\left(\widetilde{x}_{nt_2} - \widehat{x}_{nt_2}\right) \\ &\quad - \delta_n\Big(\sum_{t\in\mathcal{T}} \widetilde{x}_{nt} - \Gamma_n\Big)^2 + \delta_n\Big(\sum_{t\in\mathcal{T}} \widehat{x}_{nt} - \Gamma_n\Big)^2,\end{aligned} \tag{71}$$

where the inequality holds by the convexity of $g_{nt}(\cdot)$ together with (27a) for the first term, and with $\widehat{x}_{nt_2} \leq \widetilde{x}_{nt_2} < d^*_{nt_2}$ for the second term.

Using (62) from Lemma A.1 together with (27a), the concavity of $-\delta_n(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n)^2$, and $\widetilde{x}_{nt_2} > \widehat{x}_{nt_2}$ gives,

$$\begin{aligned}&-\delta_n\Big(\sum_{t\in\mathcal{T}} \widetilde{x}_{nt} - \Gamma_n\Big)^2 + \delta_n\Big(\sum_{t\in\mathcal{T}} \widehat{x}_{nt} - \Gamma_n\Big)^2 \\ &> g'_{nt_2}(d^*_{nt_2})\left(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1} + \widetilde{x}_{nt_2} - \widehat{x}_{nt_2}\right) \\ &> \mu\left(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}\right) + g'_{nt_2}(d^*_{nt_2})\left(\widetilde{x}_{nt_2} - \widehat{x}_{nt_2}\right).\end{aligned} \tag{72}$$

[4]The equality $\widehat{x}_{nt_2} = \widetilde{x}_{nt_2} = 0$ can occur if $c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2}) \geq \widetilde{\beta}_{nt_2} > \widehat{\beta}_{nt_2}$.

Thus, it follows from (71) and (72) that $\Delta u_n > 0$ whenever $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in \mathcal{R}_3$.

In summary, the analysis presented in Cases I-V shows that inequality (28) holds for all $\widehat{d}_{nt_2} \geq d^*_{nt_2}$. *End of proof.*

D. Proof of Lemma 4.4

The total aggregate payment made by all EVs is given by (13),

$$\sum_{n\in\mathcal{N}} \tau_n(\boldsymbol{b}^*) = \sum_{n\in\mathcal{N}} \sum_{t\in\mathcal{T}} \tau_{nt}(\boldsymbol{b}^*_t),$$

with

$$\begin{aligned}\tau_{nt}(\boldsymbol{b}^*_t) = &- c_t\big(D_t + \sum_{m\neq n} x^{*,-n}_{mt}\big) + c_t\big(D_t + \sum_{k\in\mathcal{N}} x^*_{kt}\big) \\ &+ \sum_{m\neq n} \beta^*_{mt}\{x^{*,-n}_{mt} - x^*_{mt})\},\end{aligned} \tag{73}$$

where x^*_{kt} denotes the optimal charging allocation of the k-th EV given by Problem 2 with respect to $\boldsymbol{b}^*_t$, and $x^{*,-n}_{mt}$ denotes the optimal charging allocation of EV $m \neq n$ with respect to $(\mathbf{0}_n, \boldsymbol{b}^*_{-n})_t$. It follows from the KKT conditions (8) that $x^{*,-n}_{mt} = x^*_{mt}$ for all $m \neq n$. Therefore, (73) simplifies to,

$$\tau_{nt}(\boldsymbol{b}^*_t) = c_t\big(D_t + \sum_{k\in\mathcal{N}} x^*_{kt}\big) - c_t\big(D_t + \sum_{m\neq n} x^*_{mt}\big).$$

Because $0 < \sum_{m\neq n} x^*_{mt} < \sum_{k\in\mathcal{N}} x^*_{kt}$, Assumption (A1) implies,

$$\frac{c_t\big(D_t + \sum_{k\in\mathcal{N}} x^*_{kt}\big) - c\big(D_t + \sum_{m\neq n} x^*_{mt}\big)}{x^*_{nt}} > \frac{c_t\big(D_t + \sum_{k\in\mathcal{N}} x^*_{kt}\big) - c\big(D_t\big)}{\sum_{k\in\mathcal{N}} x^*_{kt}}.$$

Therefore,

$$\begin{aligned}&\sum_{n\in\mathcal{N}} \tau_n(\boldsymbol{b}^*) \\ &= \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} \frac{c_t\big(D_t + \sum_{k\in\mathcal{N}} x^*_{kt}\big) - c_t\big(D_t + \sum_{m\neq n} x^*_{mt}\big)}{x^*_{nt}} \times x^*_{nt} \\ &> \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} \frac{c_t\big(D_t + \sum_{k\in\mathcal{N}} x^*_{kt}\big) - c_t\big(D_t\big)}{\sum_{k\in\mathcal{N}} x^*_{kt}} \times x^*_{nt} \\ &= \sum_{t\in\mathcal{T}} \frac{c_t\big(D_t + \sum_{k\in\mathcal{N}} x^*_{kt}\big) - c_t\big(D_t\big)}{\sum_{k\in\mathcal{N}} x^*_{kt}} \times \sum_{n\in\mathcal{N}} x^*_{nt} \\ &= \sum_{t\in\mathcal{T}} \Big\{c_t\big(D_t + \sum_{k\in\mathcal{N}} x^*_{kt}\big) - c_t\big(D_t\big)\Big\}.\end{aligned}$$

End of Proof.

E. Proof of Theorem 4.4

It will be shown initially that for each EV, $n \in \mathcal{N}$, there always exists a best response $b_n \equiv (\beta_n, d_n)$ under which it is fully allocated, $x_n = d_n$. Denote by $\widehat{b}^*_n(\boldsymbol{b}_{-n}) \equiv (\widehat{\beta}^*_n, \widehat{d}^*_n)$ the best response of the n-th EV with respect to $\boldsymbol{b}_{-n}$,

$$\widehat{b}^*_n(\boldsymbol{b}_{-n}) = \operatorname*{argmax}_{\widehat{b}_n\in\mathcal{B}_n} u_n(\widehat{b}_n, \boldsymbol{b}_{-n}),$$

and let $\widehat{\boldsymbol{x}}^*$ be the allocation with respect to $(\widehat{b}^*_n, \boldsymbol{b}_{-n})$. As stated in Section III-A, incentive compatibility holds, so a bid with price satisfying $\beta_n = w'_n(d_n)$ is the best choice among all possible bids. Thus, it will be assumed all bids are truthful.

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Consider another bid $\widehat{b}_n = (\widehat{\beta}_n, \widehat{d}_n)$ with $\widehat{\beta}_n = w_n'(\widehat{x}_n^*)$ and $\widehat{d}_n = \widehat{x}_n^*$, and denote by $\widehat{\boldsymbol{x}}$ the allocation with respect to $(\widehat{b}_n, \boldsymbol{b}_{-n})$. The concavity of $w_n(\cdot)$ and $\widehat{x}_n^* \leq \widehat{d}_n^*$ ensure that $w_n'(\widehat{x}_n^*) \geq w_n'(\widehat{d}_n^*)$, which implies that $\widehat{\beta}_n \geq \widehat{\beta}_n^*$.

Assume $\widehat{x}_n < \widehat{d}_n = \widehat{x}_n^*$. Then,

$$c'(D + \sum_{m\neq n} \widehat{x}_m^* + \widehat{x}_n) < c'(D + \sum_{m\neq n} \widehat{x}_m^* + \widehat{x}_n^*) \leq \widehat{\beta}_n^* \leq \widehat{\beta}_n.$$

But (8c) then implies $\widehat{\sigma}_n > 0$ and so $\widehat{x}_n = \widehat{d}_n$. Hence a contradiction so $\widehat{x}_n \geq \widehat{d}_n$. But $\widehat{x}_n \leq \widehat{d}_n$, so $\widehat{x}_n = \widehat{d}_n = \widehat{x}_n^*$. That is to say, the bid $\widehat{b}_n$ results in full allocation. Since $\widehat{b}_n$ has the same allocation as $\widehat{b}_n^*$, it follows that $\widehat{b}_n$ has the same payoff as $\widehat{b}_n^*$. Therefore $\widehat{b}_n$ is also a best response. It may be concluded that for each EV, there exists a best response with a full allocation. Hence, the remainder of the proof assumes that EVs consider their best response with full allocation.

Suppose there exists another NE, denoted by $\boldsymbol{b}^0$, which differs from $\boldsymbol{b}^*$. By Lemma 3.2, $\boldsymbol{b}^0$ is inefficient and does not satisfy (10). Also, according to Definition 1, b_n^0 is the best response of the n-th EV with respect to $\boldsymbol{b}_{-n}^0$. From the above argument regarding full allocation, it may be assumed that $x_n^0 = d_n^0$ for all $n \in \mathcal{N}$. Moreover, since $\boldsymbol{b}^0$ satisfies (8):

$$\beta_n^0 \begin{cases} = c'(D + \sum_{k\in\mathcal{N}} x_k^0) + \sigma_n, & \text{if } x_n^0 > 0 \\ \leq c'(D + \sum_{k\in\mathcal{N}} x_k^0) + \sigma_n, & \text{if } x_n^0 = 0, \end{cases} \tag{74}$$

where $\sigma_n \geq 0$. Since $x_n^0 = d_n^0$ and $\boldsymbol{b}^0$ is inefficient, at least one EV, $l \in \mathcal{N}$, must satisfiy $\sigma_l > 0$.

Consider another bid for the l-th EV, denoted by $\widehat{b}_l = (\widehat{\beta}_l, \widehat{d}_l)$, such that $c'(D+\sum_{k\in\mathcal{N}} x_k^0) < \widehat{\beta}_l < \beta_l^0$. Let $\widehat{\boldsymbol{x}}$ be the allocation with respect to $(\widehat{b}_l, \boldsymbol{b}_{-l}^0)$. From (11), the concavity of $w_n(\cdot)$ gives $d_l^0 < \widehat{d}_l$. Then, because $x_m^0 = d_m^0$ for all $m \neq l$, and $c'(D+\sum_{k\in\mathcal{N}} x_k^0) < \widehat{\beta}_l$, it follows that:

$$0 \leq x_l^0 < \widehat{x}_l. \tag{75a}$$

Because $\widehat{x}_l > 0$, the KKT conditions (8) require:

$$c'(D + \sum_{k\in\mathcal{N}} \widehat{x}_k) \leq \widehat{\beta}_l. \tag{75b}$$

Furthermore, (8) implies:

$$\widehat{x}_m \begin{cases} = x_m^0, & \text{if } \beta_m^0 > \widehat{\beta}_l, \\ \leq x_m^0, & \text{otherwise,} \end{cases} \quad \text{for all } m \neq l, \tag{75c}$$

and

$$\sum_{k\in\mathcal{N}} \widehat{x}_k \geq \sum_{k\in\mathcal{N}} x_k^0. \tag{75d}$$

Hence the difference of payoffs between $\boldsymbol{b}^0$ and $(\widehat{b}_l, \boldsymbol{b}_{-l}^0)$, given by $\Delta u_l = u_l(\boldsymbol{b}^0) - u_l(\widehat{b}_l, \boldsymbol{b}_{-l}^0)$, satisfies the following analysis:

$$\begin{aligned} \Delta u_l &= f_l(\widehat{x}_l) - f_l(x_l^0) - \delta_l\big((x_l^0 - \Gamma_l)^2 - (\widehat{x}_l - \Gamma_l)^2\big) \\ &\quad + c\big(D + \sum_{k\in\mathcal{N}} \widehat{x}_k\big) - c\big(D + \sum_{k\in\mathcal{N}} x_k^0\big) \\ &\quad + \sum_{m\neq l} \beta_m^0 (x_m^0 - \widehat{x}_m) \\ &< f_l'(\widehat{x}_l)(\widehat{x}_l - x_l^0) + 2\delta_l(\widehat{x}_l - \Gamma_l)(\widehat{x}_l - x_l^0) \\ &\quad + c'\big(D + \sum_{k\in\mathcal{N}} \widehat{x}_k\big)\Big(\sum_{k\in\mathcal{N}} \widehat{x}_k - \sum_{k\in\mathcal{N}} x_k^0\Big) \\ &\quad + \widehat{\beta}_l \sum_{m\neq l} (x_m^0 - \widehat{x}_m) && (76a) \\ &\leq \big(f_l'(\widehat{d}_l) + 2\delta_l(\widehat{d}_l - \Gamma_l)\big)(\widehat{x}_l - x_l^0) \\ &\quad + c'\big(D + \sum_{k\in\mathcal{N}} \widehat{x}_k\big)\Big(\sum_{k\in\mathcal{N}} \widehat{x}_k - \sum_{k\in\mathcal{N}} x_k^0\Big) \\ &\quad + \widehat{\beta}_l \sum_{m\neq l} (x_m^0 - \widehat{x}_m) && (76b) \\ &= \Big(c'\big(D + \sum_{k\in\mathcal{N}} \widehat{x}_k\big) - \widehat{\beta}_l\Big)\Big(\sum_{k\in\mathcal{N}} \widehat{x}_k - \sum_{k\in\mathcal{N}} x_k^0\Big) && (76c) \\ &\leq 0, && (76d) \end{aligned}$$

where (76a) holds because of Assumptions (A1, A2) and (75c); (76b) follows from Assumption (A2), $\widehat{x}_l \leq \widehat{d}_l$ and (75a); (76c) uses (11); and finally (76d) uses (75b) and (75d).

This analysis implies that $u_l(\boldsymbol{b}^0) < u_l(\widehat{b}_l, \boldsymbol{b}_{-l}^0)$ which contradicts the statement that b_l^0 is the best response of the l-th EV with respect to $\boldsymbol{b}_{-l}^0$. Hence, no NE exist apart from $\boldsymbol{b}^*$. The NE for single-interval auction games is unique and efficient. *End of proof.*

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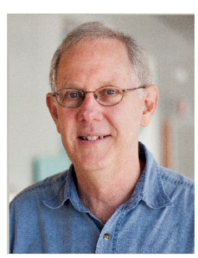

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