Consensus-Based Coordination of Electric Vehicle Charging ⋆

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Abstract: As the population of electric vehicles (EVs) grows, coordinating their charging over a finite time horizon will become increasingly important. Recent work established a framework for EV charging coordination where a central node broadcast a price signal that facilitated the tradeoff between the total generation cost and local costs associated with battery degradation and distribution network overloading. This paper considers a completely distributed protocol where the central node is eliminated. Instead, a consensus algorithm is used to fully distribute the price update mechanism. Each EV computes a local price through its estimate of the total EV charging demand, and exchanges this information with its neighbours. A consensus algorithm establishes the average over all the EV-based prices. It is shown that under a reasonable assumption, the price update mechanism is a Krasnoselskij iteration, and this iteration is guaranteed to converge to a fixed point. Furthermore, this iterative process converges to the unique and efficient solution.

Keywords: Electric vehicles; distributed protocol; efficient charging control; decentralized optimization; consensus price.

1. INTRODUCTION

The charging demand associated with a high penetration of electric vehicles (EVs) could have a significant impact on the grid if not carefully integrated Denholm and Short [2006], Rahman and Shrestha [1993], Koyanagi and Uriu [1997], Hadley and Tsvetkova [2008]. A wide range of control objectives have been considered at the distribution level where uncoordinated charging may induce localized overloading, excessive losses and voltage problems Fernández et al. [2011], Clement-Nyns et al. [2010], Galus and Andersson [2008], Hermans et al. [2012]. Various studies have considered tradeoffs between the electricity generation cost and local individual costs Gan et al. [2012], Ma et al. [2015a,b, 2016]. This paper extends a charging coordination scheme developed in Ma et al. [2015b, 2016] which considers the tradeoffs between system-wide economic efficiency, distribution-level limitations and battery degradation concerns.

Various centralized methods for scheduling the charging behaviour of EVs have been developed, see Clement-Nyns et al. [2010], Galus and Andersson [2008], Sundstrom and Binding [2010] and references therein. Due to the desire of EVs for privacy and autonomy, as well as the high communications and computational burden of centralized methods, decentralized methods are potentially more practical. Because strategies that rely on time or fixed-price schedules tend to result in suboptimal solutions, a real-time price model has been widely applied for demand response management Mohsenian-Rad and Leon-Garcia [2010], Samadi et al. [2010] and EV charging/discharging coordination Ma et al. [2015a,b, 2016, 2013], Gan et al. [2013b], Waraich et al. [2009], Wu et al. [2012], Fan [2012], Gan et al. [2013a]. In such formulations, the electricity price is given by the generation marginal cost which is a function of the total demand.

In this paper, we explore a decentralized approach motivated by a real-time price model, where participating EVs simultaneously determine their optimal charging strategies with respect to forecast price information and in turn these proposed charging strategies are used to estimate an updated price profile. In Ma et al. [2015a,b, 2016, 2013], Gan et al. [2013b], the authors suppose that a central entity exists to compute and broadcast this forecast price profile. In this paper, we establish a distributed protocol which avoids the need for a central entity. Coordination of multi-agent systems is often formulated as consensus or group agreement problems Andreasson et al. [2014], Hug et al. [2015], Olfati-Saber and Murray [2004], Olfati-Saber et al. [2007], Ren and Beard [2008], where the aim is to achieve agreement between communicating dynamic agents in a network. Hence, we adopt a typical consensus algorithm to reach an agreed price profile.

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First, each individual EV calculates its optimal charging strategy with respect to a given forecast price profile, and then estimates the total system demand based on its charging strategy. It uses that estimate to determine an updated price profile. All EVs then exchange their individual price profiles with their neighbours and proceed via a consensus algorithm to reach a group decision on the average price profile. This revised price profile is again used by the EVs to recompute their optimal charging strategy, and the process repeats.

The connectivity of the communications graph topology is key to achieving consensus. If the graph is connected, EVs will reach an average consensus asymptotically and the group decision will be the average of the individual price profiles. It will be shown that under mild conditions, the price update mechanism is a Krasnoselskij iteration, and this iteration is guaranteed to converge to a fixed point. Upon convergence, the price profile is coincident with the generation marginal cost over the charging horizon. As a consequence, the resulting collection of EV charging strategies is efficient.

The paper is organized as follows. Section 2 establishes the EV charging model, and a graph representation of the EV communications topology. Centralized optimal coordination of EV charging is considered in Section 3. Section 4 presents a novel decentralized charging coordination algorithm that uses a consensus process for determining the average price profile. Convergence and efficiency characteristics are also analyzed. Numerical illustrations are provided in Section 5, while Section 6 concludes the paper and discusses ongoing research.

2. MODEL FORMULATION FOR EV SYSTEMS

2.1 Individual EV charging model

This paper considers the optimal scheduling of EV charging. It is assumed that EVs negotiate (with each other) at the beginning of a time horizon to determine their charging profile over the horizon. Consider the scenario where a population $\mathcal{N} \equiv \{1, 2, \cdots, N\}$ of EVs needs to charge over the time horizon $\mathcal{T} \equiv \{0, 1, \cdots, T-1\}$. The aggregate inelastic background demand is denoted by d_t, for $t \in \mathcal{T}$.

The EVs are modelled as specified in Ma et al. [2015b, 2016], with the following summary provided for completeness. For each EV, $n \in \mathcal{N}$, the charging power over the time period $t \in \mathcal{T}$ is denoted by u_{nt} (with units of kW). A charging strategy $\boldsymbol{u}_n \equiv (u_{nt}; t \in \mathcal{T})$ is *admissible* if,

$$u_{nt} \begin{cases} \geq 0, & t \in \mathcal{T}_n \\ = 0, & t \in \mathcal{T} \setminus \mathcal{T}_n, \end{cases} \tag{1a}$$

and

$$\|\boldsymbol{u}_n\|_1 \equiv \sum_{t \in \mathcal{T}} u_{nt} \leq \Gamma_n, \tag{1b}$$

where $\mathcal{T}_n \subset \mathcal{T}$ is the charging horizon and Γ_n is the energy capacity of the n-th EV. The set of admissible charging controls for EV n is denoted by $\mathcal{U}_n$.

To reduce distribution-level impacts of EV charging, a demand charge is introduced:

$$Cost_{demand,nt} = g_{demand,nt}(u_{nt}), \tag{2}$$

whereby EVs incur a higher cost as their charging power increases, i.e. $g_{demand,nt}(\cdot)$ is a strictly increasing function. The battery degradation cost of EV n at time t can be expressed as,

$$Cost_{degrad,nt} = g_{cell,n}(u_{nt}) = a_n u_{nt}^2 + b_n u_{nt} + c_n. \tag{3}$$

This gives the monetary loss incurred by charging at a rate of u_{nt} for the period ΔT. Hence,

$$g_{nt}(u_{nt}) = g_{demand,nt}(u_{nt}) + g_{cell,n}(u_{nt}), \tag{4}$$

captures the demand charge (2) and battery degradation cost (3) of EV n. We regard $g_{nt}(u_{nt})$ as the individual local cost of EV n at time t.

Moreover, define a function $h_n(\|\boldsymbol{u}_n\|_1)$ with respect to the total energy delivered over the charging horizon, to indicate the benefit derived by EV n from the delivered energy. Also, we adopt the quadratic form specified in Ma et al. [2015b, 2016] and given by:

$$h_n(\|\boldsymbol{u}_n\|_1) = -\delta_n(\|\boldsymbol{u}_n\|_1 - \Gamma_n)^2, \tag{5}$$

with the factor δ_n reflecting the relative importance of delivering the full charge to the EV over the charging horizon.

The valuation function of an individual EV n, for a charging strategy $\boldsymbol{u}_n \in \mathcal{U}_n$, can be written,

$$v_n(\boldsymbol{u}_n) \triangleq h_n(\|\boldsymbol{u}_n\|_1) - \sum_{t \in \mathcal{T}} g_{nt}(u_{nt}). \tag{6}$$

2.2 Graph description of EV interaction topology

Suppose that the EVs interact with each other via local information transmissions. We would like to describe the distributed interaction protocol of EVs as a graph, as is widely the case for describing distributed networks, e.g. Olfati-Saber and Murray [2004], Olfati-Saber et al. [2007], Ren and Beard [2008]. Define the interaction topology of EVs as a graph denoted by $\mathcal{G} \triangleq \langle \mathcal{N}, \mathcal{E} \rangle$, where $\mathcal{N}$ is the set of EVs and $\mathcal{E}$ is the set of edges, i.e. the information flow among EVs. Each edge is denoted by $e = (n, m)$ with $n, m \in \mathcal{N}$, and implies that EV n can exchange information with EV m directly. In this case EV m is called a neighbour of EV n.

We shall assume that all the graphs considered in this paper are undirected, i.e. if EV n can exchange information with EV m, then EV m can also exchange information with EV n. Thus if we have $(n, m) \in \mathcal{E}$, the edge $e = (m, n)$ also belongs to $\mathcal{E}$. Hence, if EV m is a neighbour of EV n, then EV n is also a neighbour of EV m.

Let A denote the adjacency matrix of $\mathcal{G}$. Then $A \in \mathbb{R}^{N \times N}$ with

$$a_{nm} = \begin{cases} 1, & \text{if } (n, m) \in \mathcal{E} \\ 0, & \text{otherwise,} \end{cases}$$

where a_{nm} is the (n, m) entry of matrix A associated with $\mathcal{G}$. Since it is invalid that EVs exchange information with themselves, we suppose $a_{nn} = 0$. Also $a_{nm} = a_{mn}$ since $\mathcal{G}$ is an undirected graph.

Denote by $\mathcal{V}_n = \{m \in \mathcal{N}, (n, m) \in \mathcal{E}\}$ the set of neighbours of EV n, and let $D_n \triangleq \|\mathcal{V}_n\|$. Thus we have $D_n = \sum_{m \in \mathcal{N}} a_{nm}$.

The degree matrix is an $N \times N$ matrix defined as $\Delta = \Delta(\mathcal{G}) = \{\Delta_{nm}\}$, where:

$$\Delta_{nm} = \begin{cases} D_n, & \text{if } n = m \\ 0, & \text{otherwise.} \end{cases}$$

Thus the Laplacian of graph $\mathcal{G}$ is defined by

$$L = \Delta - A, \tag{7}$$

and the entries of L are given by:

$$l_{nm} = \begin{cases} -1, & \text{if } (n, m) \in \mathcal{E} \\ D_n, & \text{if } n = m \\ 0, & \text{otherwise.} \end{cases}$$

It is shown in Olfati-Saber and Murray [2004], Olfati-Saber et al. [2007], Ren and Beard [2008] that L is row stochastic, i.e. the summation of the entries in each row is zero.

Definition 1. A graph $\mathcal{G}$ is called connected if there exists a path between any two different nodes.

3. OPTIMAL COORDINATION OF EV CHARGING

The coordination problem of interest considers the tradeoff between the total cost of supplying energy to the EV population and the benefit derived from doing so. The total cost is composed of the generation cost and the local charging costs incurred by each EV. Given a collection of admissible charging strategies $\boldsymbol{u} \in \mathcal{U}$, the system cost function can be expressed as,

$$\begin{aligned} J(\boldsymbol{u}) &\triangleq \sum_{t\in\mathcal{T}} c\Big(d_t + \sum_{n\in\mathcal{N}} u_{nt}\Big) - \sum_{n\in\mathcal{N}} v_n(\boldsymbol{u}_n) \\ &= \sum_{t\in\mathcal{T}} \Big\{ c\Big(d_t + \sum_{n\in\mathcal{N}} u_{nt}\Big) + \sum_{n\in\mathcal{N}} g_{nt}(u_{nt}) \Big\} \\ &\quad - \sum_{n\in\mathcal{N}} \Big\{ h_n(\|\boldsymbol{u}_n\|_1) \Big\}, \end{aligned} \tag{8}$$

where $c(\cdot)$ gives the generation cost with respect to the total demand $d_t + \sum_{n\in\mathcal{N}} u_{nt}$.

Centralized EV charging coordination is discussed in detail in Ma et al. [2016]. A brief summary is provided here to establish the background necessary for later analysis. The centralized problem can be formulated as the optimization problem:

Problem 1.

$$\min_{\mathbf{u}\in\mathcal{U}} \; J(\boldsymbol{u}). \tag{9}$$

The objective is to implement a socially optimal collection of charging strategies for all EVs, denoted by $\boldsymbol{u}^{**}$, that minimizes the system cost (8). □

The following assumptions will apply throughout the paper:

(A1) $c(\cdot)$ is monotonically increasing, strictly convex and differentiable.

(A2) $g_{nt}(\cdot)$, for all $n \in \mathcal{N}$, $t \in \mathcal{T}$, is monotonically increasing, strictly convex and differentiable.

When the benefit function takes the form (5), the solution obtained by minimizing $J(\boldsymbol{u})$ subject only to (1a) always satisfies (1b), and hence is also the solution for Optimization Problem 1. Based on Assumptions (A1,A2), the KKT conditions imply that the efficient charging behavior $\boldsymbol{u}^{**}$ is uniquely specified by:

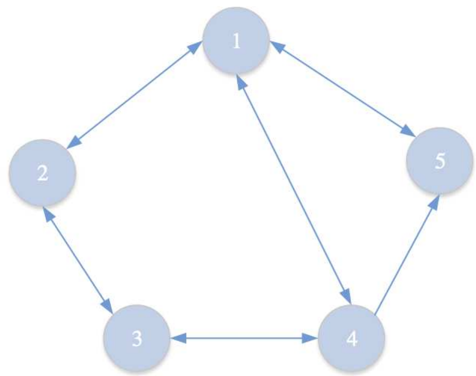

Fig. 1. Network topology of EVs.

$$p_t^{**} \begin{cases} = \dfrac{\partial}{\partial u_{nt}} v_n(\boldsymbol{u}_n^{**}), & \text{when } u_{nt}^{**} > 0 \\ \geq \dfrac{\partial}{\partial u_{nt}} v_n(\boldsymbol{u}_n^{**}), & \text{when } u_{nt}^{**} = 0, \end{cases} \tag{10}$$

where $p_t^{**} = c'(d_t + \sum_{n\in\mathcal{N}} u_{nt}^{**})$ is the generation marginal cost over the charging horizon with respect to the efficient allocation $\boldsymbol{u}^{**}$, and $\frac{\partial}{\partial u_{nt}} v_n(\cdot)$ is the marginal valuation based on (6).

It is commonly assumed, see Bompard et al. [2007], Gountis and Bakirtzis [2004], Wen and David [2001] and references therein, that the electricity generation cost can be approximated by the quadratic form,

$$c(y_t) = \frac{1}{2}\mathsf{a}y_t^2 + \mathsf{b}y_t + \mathsf{c}, \tag{11}$$

with parameters a, b and c which reflect system conditions. The marginal generation cost, which is the derivative of generation cost, therefore varies linearly with the total demand,

$$p(y_t) \triangleq c'(y_t) = \mathsf{a}y_t + \mathsf{b}. \tag{12}$$

We will assume this quadratic form for the generation cost throughout the paper.

Example 1. Consider the distributed topology of EVs shown in Fig. 1. Assume the charging period of all the EVs is from noon on one day to noon on the next.

In accordance with (11), we assume the generation cost has the quadratic form,

$$c(y_t) = 2.9 \times 10^{-4} y_t^2 + 0.06 y_t, \tag{13}$$

where $y_t = d_t + \sum_{n\in\mathcal{N}} u_{nt}$ is the total demand. The base demand $\boldsymbol{d}$, which is shown in Fig. 2, is representative of a typical hot summer day.

Assume all the EVs have a battery capacity of 40 kWh, and initially all EVs use the same weighting factor $\delta_n = 0.03$ in their valuation function $v_n(\boldsymbol{u}_n)$ given by (5) and (6). For simplicity, assume all the EVs have identical minimum and maximum state-of-charge (SoC), with $soc_\ell = 15\%$ and $soc_h = 90\%$. The upper limit (1b) on the energy that can be delivered to each EV is given by,

$$\Gamma_n = 40(soc_h - soc_{n0}),$$

which equals 30 kWh if $soc_{n0} = soc_\ell = 15\%$ for all $n \in \mathcal{N}$.

The local costs incurred by each EV at time t amount to,

$$g_{nt}(u_{nt}) = 0.003u_{nt}^2 + 0.11u_{nt} - 0.02. \tag{14}$$

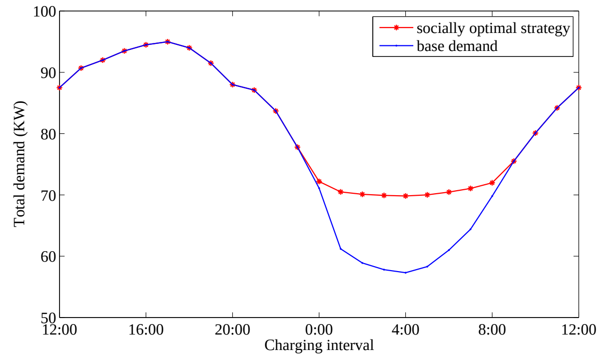

Fig. 2. Aggregate demand due to the efficient charging strategies.

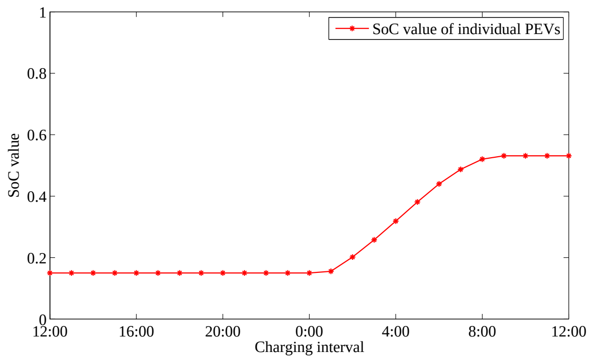

Fig. 3. Evolution of SoC due to the efficient charging strategies.

Thus the aggregate demand due to the efficient charging strategies of the EVs is shown in Fig. 2. The corresponding evolution of the SoC of each EV is illustrated in Fig. 3.

4. DECENTRALIZED CHARGING COORDINATION FOR EVS

Centralized coordination is only possible when the system operator has complete information, including the characteristics of EV batteries and the valuation functions of individual EVs. It is unlikely, however, that individuals would be willing to share such private information. Thus, the remainder of the paper is devoted to the development of a decentralized coordination process where each EV updates its charging strategy with respect to a price signal determined by the information it receives from its neighbours.

Decentralized coordination of EV charging can be achieved using an algorithm of the form:

(S1) Each EV $n \in \mathcal{N}$ autonomously determines its optimal charging strategy with respect to a given system price profile $\boldsymbol{p} \equiv (p_t, t \in \mathcal{T})$.

(S2) The electricity price profile $\boldsymbol{p}$ is updated to reflect the latest charging strategies determined by the EV population in (S1).

(S3) Steps (S1) and (S2) are repeated until the change in the price profile at (S2) is negligible.

4.1 EV optimal response for a given system price profile

The individual cost function of the n-th EV, under charging strategy $\boldsymbol{u}_n \in \mathcal{U}_n$ and with respect to the price profile $\boldsymbol{p}$, can be written:

$$\begin{aligned} J_n(\boldsymbol{u}_n; \boldsymbol{p}) &\triangleq \sum_{t\in\mathcal{T}} p_t u_{nt} - v_n(\boldsymbol{u}_n) \\ &= \sum_{t\in\mathcal{T}} \Big\{ p_t u_{nt} + g_{nt}(u_{nt}) \Big\} - h_n(\|\boldsymbol{u}_n\|_1). \quad (15) \end{aligned}$$

Note that this cost is composed of the total electricity cost $\sum_{t\in\mathcal{T}} p_t u_{nt}$, the total local cost $\sum_{t\in\mathcal{T}} g_{nt}(u_{nt})$, and the benefit derived from the total energy delivered over the charging horizon $h_n(\|\boldsymbol{u}_n\|_1)$.

The optimal charging strategy for EV n, with respect to $\boldsymbol{p}$, is obtained by minimizing the cost function (15),

$$\boldsymbol{u}_n^*(\boldsymbol{p}) = \underset{\boldsymbol{u}_n \in \mathcal{U}_n}{\text{argmin}}\ J_n(\boldsymbol{u}_n; \boldsymbol{p}). \quad (16)$$

Again, the KKT conditions imply that the EV optimal response with respect to a given $\boldsymbol{p}$ satisfies:

$$p_t \begin{cases} = \dfrac{\partial}{\partial u_{nt}} v_n(\boldsymbol{u}_n^*(\boldsymbol{p})), & \text{when } u_{nt}^* > 0 \\ \geq \dfrac{\partial}{\partial u_{nt}} v_n(\boldsymbol{u}_n^*(\boldsymbol{p})), & \text{when } u_{nt}^* = 0. \end{cases} \quad (17)$$

Hence, from (10) we have $\boldsymbol{u}_n^*(\boldsymbol{p}^{**}) = \boldsymbol{u}_n^{**}$, and

$$p_t^{**} = c'(d_t + \sum_{n\in\mathcal{N}} u_{nt}^*(\boldsymbol{p}^{**})). \quad (18)$$

A method of calculating the EV optimal response with respect to a given price profile $\boldsymbol{p}$ is derived in Ma et al. [2016].

Let g'_{nt} denote the derivative of g_{nt} given by (4) and $[g'_{nt}]^{-1}(\cdot)$ the corresponding inverse function. Define ν_{nt} as the Lipschitz constant for $[g'_{nt}]^{-1}(\cdot)$ over the interval $[g'_{nt}(0), g'_{nt}(\Gamma_n)]$, and

$$\nu = \max_{n\in\mathcal{N}, t\in\mathcal{T}} \nu_{nt}.$$

According to Ma et al. [2016], the optimal response $\boldsymbol{u}^*(\boldsymbol{p})$ has the following property:

Lemma 1. Assume the terminal valuation function h_n is increasing and strictly concave. Then,

$$\|\boldsymbol{u}_n^*(\boldsymbol{p}) - \boldsymbol{u}_n^*(\boldsymbol{\varrho})\|_1 \leq 2\nu\|\boldsymbol{p} - \boldsymbol{\varrho}\|_1 \quad (19)$$

where $\boldsymbol{p}$ and $\boldsymbol{\varrho}$ are two given price profiles, and $\|.\|_1$ denotes the l_1 norm of the associated vector.

4.2 Consensus price update process

In a truly distributed process where there is no centralized communications, EVs cannot obtain overall demand information. Thus, consider a scheme where each EV calculates its own price profile with respect to the given system price profile, according to:

$$\hat{p}_{nt}(\boldsymbol{p})[0] = p_t + \eta\Big(c'(d_t + N u_{nt}^*(\boldsymbol{p})) - p_t\Big),\ t \in \mathcal{T}, \quad (20)$$

where $\eta > 0$ is a fixed parameter, and $\boldsymbol{u}_n^*(\boldsymbol{p})$ is the optimal charging strategy for EV n with respect to $\boldsymbol{p}$.

This price profile update is based on each EV forming an estimate of the total system-wide charging demand that is equal to N times its own charging strategy. The resulting

Algorithm 1 Consensus price update process.

Require:

A set $\mathcal{N}$ of EVs, a time horizon $\mathcal{T}$ and a graph $\mathcal{G}$;

A base demand trajectory $\boldsymbol{d}$;

A system price profile $\boldsymbol{p}$ and the corresponding individual price profiles $\hat{\boldsymbol{p}}_n(\boldsymbol{p})[0]$ given by (20);

An initial $\epsilon > \epsilon_{\text{stop}}$ and $l := 0$;

Ensure:

Group decision price profile $\hat{\boldsymbol{p}}^*$;

while $\epsilon > \epsilon_{\text{stop}}$ **do**

Each EV exchanges their price profile estimate $\hat{\boldsymbol{p}}_n(\boldsymbol{p})[l]$ with their neighbours and updates their price using,

$$\hat{p}_{nt}(\boldsymbol{p})[l+1] = \sum_{m\in\mathcal{N}} q_{nm}\hat{p}_{mt}(\boldsymbol{p})[l],$$

where q_{nm} is the (n, m) entry of $Q = I - \mu L$, (with I the identity matrix) and $\mu > 0$ is the step size.

Update $\epsilon := \max_{n\in\mathcal{N}}\{\|\hat{\boldsymbol{p}}_n[l+1] - \hat{\boldsymbol{p}}_n[l]\|_1\}$;

Update $l := l + 1$;

end while

price profile $\hat{\boldsymbol{p}}_n(\boldsymbol{p})[0] \equiv (\hat{p}_{nt}(\boldsymbol{p})[0], t \in \mathcal{T})$ will be different for every EV.

To establish agreement between EV price profiles, we apply a consensus algorithm where EVs exchange information with their neighbours. The price profile $\hat{\boldsymbol{p}}_n(\boldsymbol{p})[0]$ calculated by each EV is set as the initial price information for the consensus process. The consensus price update process proceeds according to Algorithm 1.

In Olfati-Saber et al. [2007], matrix Q is referred to as the Perron matrix induced by a graph $\mathcal{G}$ with parameter μ. Denote by $\hat{\boldsymbol{p}}^* \equiv \hat{\boldsymbol{p}}^*(\boldsymbol{p})$ the group decision price profile achieved by Algorithm 1.

Lemma 2. (Ren and Beard [2008]) Consider a network of N agents with topology $\mathcal{G}$ applying the distributed consensus algorithm

$$x_n[l+1] = \sum_{m\in\mathcal{N}} q_{nm}x_m[l],$$

where q_{nm} is the (n, m) entry of $Q = I - \mu L$ and $0 < \mu < 1/\max_n\{D_n\}$. Then if $\mathcal{G}$ is connected, an average consensus is asymptotically reached and the group decision value is $x^* = \sum_n x_n[0]/N$.

Hence by Lemma 2, if graph $\mathcal{G}$ is connected, the group decision price profile achieved by Algorithm 1 will be:

$$\hat{p}_t^*(\boldsymbol{p}) = \frac{\sum_{n\in\mathcal{N}}\hat{p}_{nt}(\boldsymbol{p})[0]}{N}.$$

From (20), the group decision price profile is given by:

$$\hat{p}_t^*(\boldsymbol{p}) = \frac{1}{N}\sum_{n\in\mathcal{N}}\Big[p_t + \eta\Big(c'(d_t + Nu_{nt}^*(\boldsymbol{p})) - p_t\Big)\Big]. \quad (21)$$

Notice from (12) though that $c'(\cdot)$ is affine. Using this fact, straightforward manipulation allows (21) to be expressed as:

$$\hat{p}_t^*(\boldsymbol{p}) = (1-\eta)p_t + \eta c'\big(d_t + \sum_{n\in\mathcal{N}} u_{nt}^*(\boldsymbol{p})\big). \quad (22)$$

If $\eta \in (0, 1)$, this price update iteration is known as a Krasnoselskij iteration, see Berinde [2007].

Algorithm 2 Decentralized coordination of EV charging.

Require:

A set $\mathcal{N}$ of EVs and a time horizon $\mathcal{T}$;

A base demand trajectory $\boldsymbol{d}$;

An initial price profile $\boldsymbol{p}^{(0)} = (p_t^{(0)}, t \in \mathcal{T})$;

An initial $\varepsilon > \varepsilon_{\text{stop}}$ and $k := 0$;

Ensure:

Optimal price profile $\boldsymbol{p}^\star$;

while $\varepsilon > \varepsilon_{\text{stop}}$ **do**

Determine the optimal charging profile $\boldsymbol{u}_n^{(k+1)}$ w.r.t. $\boldsymbol{p}^{(k)}$ for all $n \in \mathcal{N}$ EVs simultaneously by minimizing their individual cost functions,

$$\boldsymbol{u}_n^{(k+1)}(\boldsymbol{p}^{(k)}) \triangleq \operatorname*{argmin}_{\boldsymbol{u}_n\in\mathcal{U}_n}\Big\{\sum_{t\in\mathcal{T}} p_t^{(k)}u_{nt} - v_n(\boldsymbol{u}_n)\Big\}; \quad (23)$$

Determine $\boldsymbol{p}^{(k+1)}$ by Algorithm 1 such that

$$\boldsymbol{p}^{(k+1)} = \hat{\boldsymbol{p}}^*(\boldsymbol{p}^{(k)}).$$

Update $\varepsilon := \|\boldsymbol{p}^{(k+1)} - \boldsymbol{p}^{(k)}\|_1$;

Update $k := k + 1$;

end while

4.3 Decentralized coordination of EV charging

It is now possible to formalize a decentralized coordination algorithm for determining the optimal charging strategy for a population of EVs.

Define a mapping,

$$\mathcal{P}(\boldsymbol{p}) : \boldsymbol{p} \to \{c'\big(d_t + \sum_{n\in\mathcal{N}} u_{nt}^*(\boldsymbol{p})\big), t \in \mathcal{T}\}.$$

By adopting Algorithm 2, the price iteration is guaranteed to converge to a fixed point of mapping $\mathcal{P}(\boldsymbol{p})$ if this mapping is non-expansive Berinde [2007], Grammatico et al. [2016].

Next we will specify the sufficient condition under which $\mathcal{P}(\boldsymbol{p})$ is non-expansive. Be Lemma 1, we have

$$\|\boldsymbol{u}_n^*(\boldsymbol{p}) - \boldsymbol{u}_n^*(\boldsymbol{\varrho})\|_1 \le 2\nu\|\boldsymbol{p} - \boldsymbol{\varrho}\|_1,$$

for any two price profiles $\boldsymbol{p}$ and $\boldsymbol{\varrho}$. By (12),

$$\mathcal{P}(\boldsymbol{p})|_t = \mathsf{a}\big(d_t + \sum_{n\in\mathcal{N}} u_{nt}^*(\boldsymbol{p})\big) + \mathsf{b}.$$

So we have the following:

$$\begin{aligned}\|\mathcal{P}(\boldsymbol{p}) - \mathcal{P}(\boldsymbol{\varrho})\|_1 &= \mathsf{a}\sum_{n\in\mathcal{N}}\|\boldsymbol{u}_n^*(\boldsymbol{p}) - \boldsymbol{u}_n^*(\boldsymbol{\varrho})\|_1 \\ &\le 2N\mathsf{a}\nu\|\boldsymbol{p} - \boldsymbol{\varrho}\|_1\end{aligned}$$

Then if $2N\mathsf{a}\nu \le 1$, $\mathcal{P}(\boldsymbol{p})$ is non-expansive.

Proposition 3. Consider any initial charging price profile $\boldsymbol{p}^{(0)}$. Suppose the graph $\mathcal{G}$ is connected, $2N\mathsf{a}\nu \le 1$ and $\eta \in (0, 1)$, then Algorithm 2 converges to the efficient solution $\boldsymbol{u}^{**}$ given by (10).

As stated in Berinde [2007], Grammatico et al. [2016], the price iteration converges to a fixed point of $\mathcal{P}(\boldsymbol{p})$, denoted by $\boldsymbol{p}^\star \equiv (p_t^\star, t \in \mathcal{T})$, which implies,

$$p_t^\star = \mathsf{a}\big(d_t + \sum_{n\in\mathcal{N}} u_{nt}^*(\boldsymbol{p}^\star)\big) + \mathsf{b}. \quad (24)$$

Then by (18), $\boldsymbol{p}^\star$ is equal to $\boldsymbol{p}^{**}$.

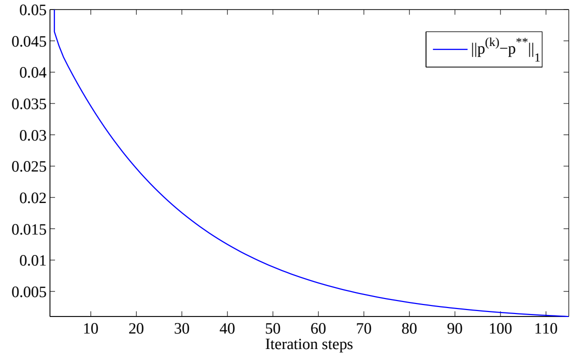

Fig. 4. Evolution of $\|\boldsymbol{p}^{(k)} - \boldsymbol{p}^{**}\|_1$.

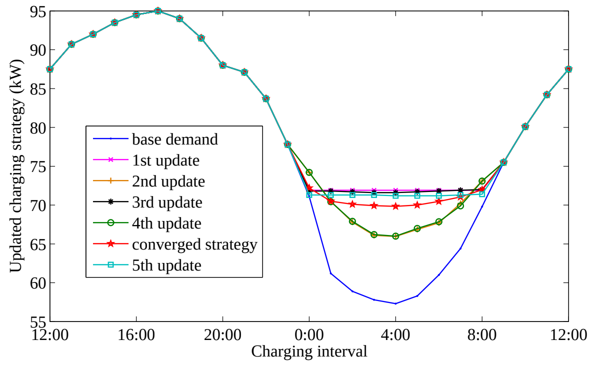

Fig. 5. Updates of the aggregate charging strategies.

5. NUMERICAL ILLUSTRATIONS

In order to illustrate the performance of the decentralized process for the coordination of EVs, we consider the network topology specified in Fig. 1, and adopt the parameters in Example 1.

For the generation cost function (13), the marginal cost is given by,

$$p(y_t) = c'(y_t) = 5.8 \times 10^{-4} y_t + 0.06.$$

From (14), the Lipschitz constant for $[g'_{nt}]^{-1}(\cdot)$ is $\nu = 1/0.006 = 166.7$. Given these values, $2N\mathsf{a}\nu < 1$. Thus by Proposition 3, the system is guaranteed to converge to the efficient solution.

Figure 4 shows the evolution of $\|\boldsymbol{p}^{(k)} - \boldsymbol{p}^{**}\|_1$, with $\eta = 1$, from an initial price profile $p_t^{(0)} = c'(d_t)$ for all $t \in \mathcal{T}$. It can be seen that convergence to the desired tolerance is achieved in about 110 iterations. The updates of aggregate charging strategies of EVs are shown in Fig. 5. The corresponding price profile update is illustrated in Fig. 6.

6. CONCLUSIONS

The paper explores the coordination of the charging behavior of electric vehicles (EVs) over a finite time horizon using a distributed protocol. In contrast to Ma et al. [2015b, 2016], there is no central entity which computes and broadcasts price. The paper first summarizes the centralized charging problem presented in Ma et al. [2015b, 2016] where there is a tradeoff between the total generation cost and the local costs associated with overloading and

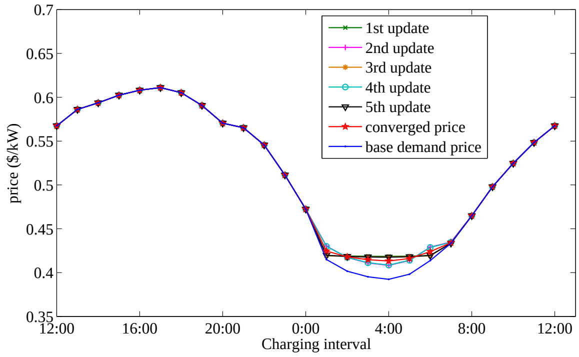

Fig. 6. Updates of the price profile.

battery degradation. A new consensus-based approach is used to update the price profile. Each EV computes a local price using an estimate of the total EV charging demand, and exchanges that price information with its neighbours. A group decision price is achieved through the consensus process. It is shown that if the graph topology describing EV interactions is connected, the EVs will agree with a price that is the average of their initial individual price profiles. This agreement price is consistent with the centrally optimal (efficient) price. Hence, the system converges to the unique, efficient solution.

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