Decentralized Coordination of Controlled Loads and Transformers in a hierarchical structure ⋆

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Abstract: This paper considers the coordination of controlled loads in a framework that loads connect to the distribution network through transformers. Our objective is designing a decentralized control method that can motivate selfish loads to achieve global benefits. We formulate this problem as a hierarchical model. In the lower level, each transformer broadcasts a price signal to the loads connect to it, under which loads implement individual best strategies. While in the upper level, transformers communicate with the distribution network and obtain a price reflecting the system generation cost. Each transformer determines a price including this price and another part reflecting individual characteristics. By proposing a dynamic update algorithm, our results build that the system converges to the unique and efficient solution with fast convergence speed.

Keywords: Load control, transformers, efficient coordination, consensus, decentralized optimization, convergence.

1. INTRODUCTION

With the restructuring of power systems, pricing has become a new paradigm which results in the emergence of a variaty of transactive techniques, such as pricing, market clearing, bidding and settlements Samadi et al. [2012], Torriti [2012], Rahimi and Ipakchi [2012]. With the advent of smart grids technologies, another new paradigm is characterized by active demand side participantion in response to environmental policies and prices Parvania et al. [2013], Fanti et al. [2015], Walawalkar et al. [2010]. Demand response has been regarded as one of the most effective ways to improve the efficiency and reliability of smart grids, while it could have a significant impacts to the grid if not carefully integrated Aalami et al. [2010], Liu and Tomsovic [2014], Aghaei and Alizadeh [2013]. Hence, transactive smart grid systems require control techniques to manage demand side participation, dispatch optimization, and other services.

To deal with the control issues in transactive smart girds, we consider a system where loads are served with energy through transformers possessing limited capacities. Various centralized methods, such as traditional direct load control, for scheduling the load behaviours have been widely developed, see Li et al. [2014], Han et al. [2010], Hao et al. [2015], Zhang et al. [2013] and references therein.

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Due to the desire of loads for privacy and autonomy, as well as the fast communication requirments and high computational burden of centralized methods, decentralized methods are potentially more practical. Hence our work is dedicated to formulating the models of loads and transformers, and designing a decentralized method that can motivate the system to converge to the efficient (global optimal) strategy.

In this paper, we explore a decentralized approach motivated by a real-time price model, proposed in Ma et al. [2016], where participating loads simultaneously determine their optimal strategies with respect to a given price. While strategies that rely on time or a fixed-price schedule tend to result in suboptimal solutions, a real-time price model has been widely applied for demand response management Mohsenian-Rad and Leon-Garcia [2010], Samadi et al. [2010] and EV charging/discharging coordination Ma et al. [2015, 2016, 2013], Gan et al. [2013], Wu et al. [2012]. First, each individual load calculates its best strategy with respect to a price profile broadcasted by the transformer that it connects, and then each transformer communicates with the distribution network and obtains a price reflecting the system generation cost. By adding the other part of price reflecting their own characteristics, each transformer determines a revised price profile which is reapplied to loads for computing optimal strategies, and the process repeats. It is shown that under mild conditions, the system convergence is ensured by applying the proposed iterative process. The converged price is coincident with the optimal system price. As a consequence, the resulting collection of load strategies is efficient. Moreover, we quantify the convergence rate of our algorithm.

The rest of the paper is organized as follows. In Section 2, we formalize a cost-based economic model for transformers and loads, followed by a centralized load coordination in Section 3. A decentralized coordination method is constructed in Section 4 to implement the efficient strategy and the convergence of the proposed algorithm is analyzed. In Section 5, simulation results are presented to demonstrate the results developed in this paper. Section 6 concludes the paper and discusses ongoing researches.

2. MODEL FORMULATION

In this paper, we study a load coordination problem under the framework that loads are connected to the distribution network through transformers, see Fig. 1 for an illustration. Our objective is to coordinate the loads to minimize the global system cost over a time horizon. Let $\mathcal{T} = \{1, \cdots, T\}$ denote the time horizon, and ΔT the time length of a time period.

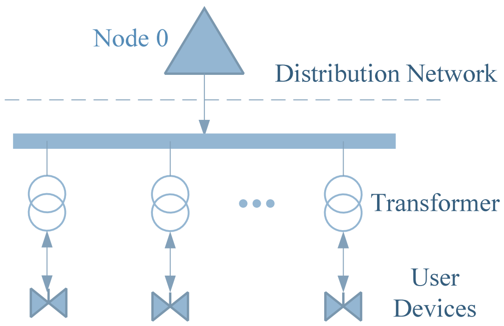

Fig. 1. Electricity transaction architecture via transformers.

2.1 Transformer model

Loads get electricity power from transformers, which may cause losses and the higher the power is, the more the losses are. Thus the loss can be reduced by motivating lower power flow level through transformers. Moreover, transformer overloading will bring harm and undesirable effects on distribution networks. We make efforts to avoid this overloading while in some urgent situations, transformers can be overloaded temporarily to ease the system pressure.

We consider the distribution network as node 0 that can communicate with transformers. Suppose that there are M transformers and $\mathcal{M} = \{1, \cdots, M\}$ is the transformer set.

In order to describe these phenomena, we introduce a local cost for each transformer $m \in \mathcal{M}$ at time t, denoted by $r_{mt}(\cdot)$, as a piecewise function as below.

$$r_{mt}(x) = \begin{cases} f_{mt}(x), & \text{if } x \leq \varphi_m \\ f_{mt}(x) + \beta_{mt}(x - \varphi_m)^2, & \text{otherwise,} \end{cases} \tag{1}$$

where x denotes the power through transformers m at t, φ_m is the capacity of transformer m, β_{mt} is a weighting factor reflecting a penalty exceeding the transformer capacity and f_{mt} represents the power loss on transformer m at t with respect to the power through it at this time.

We assume that the function $f_{mt}(x)$ satisfies:

(A1) $f_{mt}(x)$ is increasing, differentiable, and convex, for all $m \in \mathcal{M}$, $t \in \mathcal{T}$.

Due to $r_{mt}(\varphi_m) = f_{mt}(\varphi_m)$ and $\lim_{x \to \varphi_m} r_{mt}(x) = f_{mt}(\varphi_m)$, $r_{mt}(x)$ is continuous at the point $x = \varphi_m$; then we have $r_{mt}(x)$ is continuous. Furthermore, $r_{mt}(x)$ is of class C^1, and its derivative has the form:

$$r'_{mt}(x) = \begin{cases} f'_{mt}(x), & \text{if } x \leq \varphi_m \\ f'_{mt}(x) + 2\beta_{mt}(x - \varphi_m), & \text{otherwise.} \end{cases} \tag{2}$$

2.2 Load model and preference

In the system, there are critical loads that are unwilling to adjust their power consumption no matter what the electricity price is. We consider them as a background demand, and denote by $d_{mt}, m \in \mathcal{M}, t \in \mathcal{T}$ the background demand connected to transformer m at time t.

Suppose that each controllable load is equipped with a smart controller that can specify its own energy consumption and communicate with the transformer it connects to. Denote by $\mathcal{N} = \{1, \cdots, N\}$ the population set of all the controllable loads in the system, and $\mathcal{N}_m$ the set of loads connected to transformer m. Hence we have $\bigcup_{m \in \mathcal{M}} \mathcal{N}_m = \mathcal{N}$.

For each load $n \in \mathcal{N}_m$, the power consumption at time t is denoted by $u_{mn,t}$ (with units of kW, and the energy delivered over this period is $u_{m,nt}\Delta T$). A load consumption strategy $\boldsymbol{u}_{mn} \equiv (u_{mn,t}; t \in \mathcal{T})$ is *admissible* if,

$$u_{mn,t} \begin{cases} \in [0, \bar{u}_{mn,t}], & t \in \mathcal{T}_{mn} \\ = 0, & \text{otherwise,} \end{cases} \tag{3a}$$

and

$$\|\boldsymbol{u}_{mn}\|_1 \Delta T \equiv \sum_{t \in \mathcal{T}} u_{mn,t} \Delta T \leq \Xi_{mn}, \tag{3b}$$

where $\bar{u}_{mn,t}$ represents the maximum power consumed by load n, $\mathcal{T}_{mn} \subset \mathcal{T}$ is the time horizon that load n is "on" and $\Xi_{mn} = \Gamma_{mn}\Delta T$ is the maximum total demand of load n.

The objective of each load is to implement a demand schedule that maximizes its individual payoff, i.e. loads' benefit by consuming energy minus individual costs. Typically, the coordination problem across a group of loads has generally sought to minimize total generation cost over the time horizon, e.g. Denholm and Short [2006], Ma et al. [2013], Gan et al. [2013]. In practice, loads suffer from some local cost fee arising from high distribution-level demand and device degradation etc. To consider this point, we define a valuation function for each load that captures the load benefit and costs on its energy consumption.

Let the value assigned by load $n \in \mathcal{N}_m$ be denoted by $v_{mn}(\boldsymbol{u}_{mn})$, such that:

$$v_{mn}(\boldsymbol{u}_{mn}) = h_{mn}(\|\boldsymbol{u}_{mn}\|_1) - \sum_{t \in \mathcal{T}} g_{mn,t}(u_{mn,t}), \tag{4}$$

where $h_{mn}(\|\boldsymbol{u}_{mn}\|_1)$ describes the benefit derived from the total consumption over the time horizon, and $g_{mn,t}(\cdot)$ is the local cost at time t reflecting demand charge and degradation cost etc. Consistent with Kirschen [2003], we consider that:

(A2) $g_{mn,t}(u_{mn,t})$ is monotonically increasing, differentiable, and convex, for all $m \in \mathcal{M}$, $n \in \mathcal{N}_m$, $t \in \mathcal{T}$.

In Han et al. [2010], the benefit function $h_{mn}(\cdot)$ has the quadratic form,

$$h_{mn}(\|\boldsymbol{u}_{mn}\|_1) = -\delta_{mn}(\|\boldsymbol{u}_{mn}\|_1 - \Gamma_{mn})^2, \tag{5}$$

with the factor δ_{mn} reflecting the relative importance to consume maximum energy over the horizon. Moreover, the marginal valuation of load n is specified as:

$$\begin{aligned}\nu_{mn,t}(\boldsymbol{u}_{mn}) \equiv& \frac{\partial}{\partial u_{mn,t}} v_{mn}(\boldsymbol{u}_{mn})\\ =& h'_{mn}(\|\boldsymbol{u}_{mn}\|_1) - g'_{mn,t}(u_{mn,t}),\end{aligned} \tag{6}$$

where for notational simplicity, $h'_{mn}(\cdot)$ represents the partial derivative of function $h_{mn}(\cdot)$ with respect to $u_{mn,t}$.

3. CENTRALIZED LOAD COORDINATION PROBLEM

Consider a load coordination problem where the system operator allocates energy to loads aiming at minimizing the total system cost under the structure specified in Fig. 1. Given a collection of load consumption strategies $\boldsymbol{u} \equiv (u_{mn,t}, \forall m, n, t)$, the system cost function can be expressed as,

$$\begin{aligned}&J(\boldsymbol{u})\\ &\triangleq \sum_{t\in\mathcal{T}} \Big\{ c_t(D_t) + \sum_{m\in\mathcal{M}} r_{mt}(D_{mt}) \Big\} - \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}_m} v_{mn}(\boldsymbol{u}_{mn})\\ &= \sum_{t\in\mathcal{T}} \Big\{ c_t(D_t) + \sum_{m\in\mathcal{M}} \big\{ r_{mt}(D_{mt}) + \sum_{n\in\mathcal{N}_m} g_{mn,t}(u_{mn,t}) \big\} \Big\}\\ &\quad - \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}_m} \Big\{ h_{mn}(\|\boldsymbol{u}_{mn}\|_1) \Big\},\end{aligned} \tag{7}$$

where $c_t(\cdot)$ gives the generation cost with respect to the total demand $D_t = \sum_{m\in\mathcal{M}} D_{mt}$ with $D_{mt} = d_{mt} + \sum_{n\in\mathcal{N}_m} u_{mn,t}$ at time t.

In the literature, see Bompard et al. [2007], Gountis and Bakirtzis [2004] and references therein, the electricity generation cost is supposed to satisfy:

(A3) $c_t(\cdot)$ is monotonically increasing, differentiable and strictly convex for all $t \in \mathcal{T}$.

It is widely assumed that the generation cost can be approximated by the quadratic form,

$$c_t(D_t) = \frac{1}{2}\mathsf{a}_t D_t^2 + \mathsf{b}_t D_t + \mathsf{c}_t, \tag{8}$$

with parameters $\mathsf{a}_t, \mathsf{b}_t$ and c_t which reflect system conditions.

Hence, the centralized load coordination problem can be formulated as the following optimization problem:

Problem 1.

$$\boldsymbol{u}^{**} = \underset{\boldsymbol{u} \text{ s.t. } (3)}{\operatorname{argmin}}\ J(\boldsymbol{u}). \tag{9}$$

That is, the objective is to implement an efficient (socially optimal) collection of consumption strategies for all the loads by minimizing the system cost (7). ■

When the benefit function $h_{mn}(\cdot)$ takes the form (5), the solution obtained by minimizing $J(\boldsymbol{u})$ subject only to (3a) always satisfies (3b), and hence is also the solution to (9). To see this, we define a set of demand schedule for load n,

$$\mathcal{S}_{mn} \triangleq \big\{ \boldsymbol{u}_{mn};\ \text{s.t. constraint (3a)} \big\},$$

and let $\mathcal{S}$ denote the collection of such sets for all $m \in \mathcal{M}, n \in \mathcal{N}_m$. Consider an optimization problem below,

$$\boldsymbol{u}^{\diamond *} = \underset{\boldsymbol{u}\in\mathcal{S}}{\operatorname{argmin}}\ J(\boldsymbol{u}).$$

Based on Assumptions (A1,A2,A3), the optimal solution $\boldsymbol{u}^{\diamond *}$ is unique and can be characterized by its associated KKT conditions below Boyd and Vandenberghe [2004].

- $c'_t(D_t) + r'_{mt}(D_{mt}) - \nu_{mn,t}(\boldsymbol{u}_{mn}) + \lambda_{mn,t} \ge 0$, $u_{mn,t} \ge 0$, with complementary slackness,
- $u_{mn,t} \le \bar{u}_{mn,t}$, $\lambda_{mn,t} \ge 0$, with complementary slackness,

for all $m \in \mathcal{M}$, $n \in \mathcal{N}_m$ and $t \in \mathcal{T}_{mn}$, where $\lambda_{mn,t}$ is the Lagrange multiplier associated with $u_{mn,t} \le \bar{u}_{mn,t}$. Thus it follows from KKT that the optimal solution $\boldsymbol{u}^{\diamond}$ is uniquely specified by:

$$c'_t(D^{\diamond}_t) + r'_{mt}(D^{\diamond}_{mt}) \begin{cases} = \nu_{mn,t}(\boldsymbol{u}^{\diamond}_{mn}), & \text{if } u^{\diamond}_{mn,t} \in (0, \bar{u}_{mn,t}) \\ \ge \nu_{mn,t}(\boldsymbol{u}^{\diamond}_{mn}), & \text{if } u^{\diamond}_{mn,t} = 0 \\ \le \nu_{mn,t}(\boldsymbol{u}^{\diamond}_{mn}), & \text{if } u^{\diamond}_{mn,t} = \bar{u}_{mn,t} \end{cases} \tag{10}$$

where $D^{\diamond}_t = \sum_{m\in\mathcal{M}} D^{\diamond}_{mt}$ and $D^{\diamond}_{mt} = d_{mt} + \sum_{n\in\mathcal{N}_m} u^{\diamond}_{mn,t}$.

Note that if $\|\boldsymbol{u}^{\diamond}_{mn}\|_1 > \Gamma_{mn}$, (5) together with Assumption (A2) ensures that $\nu_{mn,t}(\boldsymbol{u}^{\diamond}_{mn}) < 0$. But $c'_t(D^{\diamond}_t), r'_{mt}(D^{\diamond}_{mt}) \ge 0$, so (10) implies that $\boldsymbol{u}^{\diamond}_{mn} = \boldsymbol{0}$. Hence we obtain a contradiction. Accordingly, $\|\boldsymbol{u}^{\diamond}_{mn}\|_1 \le \Gamma_n$, i.e. (3b) is always satisfied. It implies that $\boldsymbol{u}^{\diamond} = \boldsymbol{u}^{**}$. Hence, the efficient consumption strategies $\boldsymbol{u}^{**}$ is uniquely given by:

$$\rho^{**}_{mt} \begin{cases} = \nu_{nt}(\boldsymbol{u}^{**}_n), & \text{if } u^{**}_{mn,t} \in (0, \bar{u}_{mn,t}) \\ \ge \nu_{nt}(\boldsymbol{u}^{**}_n), & \text{if } u^{**}_{mn,t} = 0 \\ \le \nu_{nt}(\boldsymbol{u}^{**}_n), & \text{if } u^{**}_{mn,t} = \bar{u}_{mn,t} \end{cases}, \tag{11}$$

where $\rho^{**}_{mt} \equiv c'_t(D^{**}_t) + r'_{mt}(D^{**}_{mt})$, and $D^{**}_t = \sum_{m\in\mathcal{M}} D^{**}_{mt}$ with $D^{**}_{mt} = d_{mt} + \sum_{n\in\mathcal{N}_m} u^{**}_{mn,t}$.

Centralized coordination is only possible when the system operator has complete information while loads would be unwilling to share such private information. Also, for a large population, centralized control may be infeasible in communication and computation. Alternatively, we develop a hierarchical structure to coordinate loads in a decentralized process. In the lower level, each load adjusts their consumption in response to a common electricity price broadcasted by its transformer, while in the upper level, transformers determine their private price through a consensus-based method.

4. DECENTRALIZED LOAD COORDINATION METHOD

4.1 Best responses of individual loads with respect to a fixed price profile

Given a price profile $\boldsymbol{p}$, we define an individual cost function for load $n \in \mathcal{N}_m$ as:

$$J_{mn}(\boldsymbol{u}_{mn};\boldsymbol{p}) \triangleq \sum_{t\in\mathcal{T}} p_t u_{mn,t} - v_{mn}(\boldsymbol{u}_{mn})$$

$$= \sum_{t\in\mathcal{T}} \Big\{ p_t u_{mn,t} + g_{mn,t}(u_{mn,t}) \Big\} - h_{mn}(\|\boldsymbol{u}_{mn}\|_1). \quad (12)$$

The optimal strategy of load n, with respect to $\boldsymbol{p}$, is obtained by minimizing the cost function (12),

$$\boldsymbol{u}^*_{mn}(\boldsymbol{p}) = \underset{\boldsymbol{u}_{mn} \text{ s.t. } (3)}{\text{argmin}} \; J_{mn}(\boldsymbol{u}_{mn};\boldsymbol{p}). \quad (13)$$

Suppose that the total delivered power takes a fixed value $\|\boldsymbol{u}_{mn}\|_1 = \omega$, and in such cases, the benefit $h_{mn}(\|\boldsymbol{u}_{mn}\|_1)$ can be neglected since it is equal across all such scenarios. To begin with, consider the cost for load n, but excluding h_{mn} below.

$$F_{mn}(\boldsymbol{u}_{mn};\boldsymbol{p}) \triangleq \sum_{t\in\mathcal{T}} \Big\{ p_t u_{mn,t} + g_{mn,t}(u_{mn,t}) \Big\}. \quad (14)$$

Accordingly, consider an optimization problem to minimize the cost function (14) subject to the constraint that the total power delivered to load n is ω,

$$\min_{\boldsymbol{u}_{mn}\in\mathcal{U}_{mn}(\omega)} F_{mn}(\boldsymbol{u}_{mn};\boldsymbol{p}), \quad (15)$$

where $\mathcal{U}_{mn}(\omega) \triangleq \{\boldsymbol{u}_{mn};\ \text{s.t. (3) and } \|\boldsymbol{u}_{mn}\|_1 = \omega\}$ with $0 \le \omega \le \min\{\Gamma_{mn}, \|\bar{\boldsymbol{u}}_{mn}\|_1\}$. Then by the KKT conditions, it gives the optimal solution of (15) has the form:

$$u_{mn,t}(\boldsymbol{p}, A_{mn}) = \begin{cases} \min\left\{\bar{u}_{mn,t}, \max\{0, y\}\right\}, & t \in \mathcal{T}_{mn} \\ 0, & t \notin \mathcal{T}_{mn}, \end{cases} \quad (16)$$

where A_{mn} is the Lagrangian multiplier associated with the constraint $\|\boldsymbol{u}_{mn}\|_1 = \omega$, $y = [g'_{mn,t}]^{-1}(A_{mn} - p_t)\}$ and $[g'_{mn,t}]^{-1}$ is the inverse function of the derivative of $g_{mn,t}$. By applying the results in [Ma et al. [2016], Section 4.1], we can calculate an optimal $A^*_{mn}(\boldsymbol{p})$ with respect to a given $\boldsymbol{p}$ as below.

Define $f_{mn}(\boldsymbol{p},\omega) \triangleq \mathcal{A}_{mn}(\boldsymbol{p},\omega) - h'_{mn}(\omega)$, with $\mathcal{A}_{mn}(\boldsymbol{p},\cdot)$ given by

$$\frac{d}{d\omega} F^*_{mn}(\boldsymbol{p},\omega) = \mathcal{A}_{mn}(\boldsymbol{p},\omega), \qquad \text{with } \omega \in [0, \bar{\omega}], \quad (17)$$

where $F^*_{mn}(\boldsymbol{p},\omega) \triangleq \min_{\boldsymbol{u}_{mn}\in\mathcal{U}_{mn}(\omega)} F_{mn}(\boldsymbol{u}_{mn};\boldsymbol{p})$, and $\bar{\omega} = \min\{\Gamma_{mn}, \|\bar{\boldsymbol{u}}_{mn}\|_1\}$. Then

$$A^*_{mn}(\boldsymbol{p}) = \begin{cases} \mathcal{A}_{mn}(\boldsymbol{p},\bar{\omega}), & \text{if } f_{mn}(\boldsymbol{p},\bar{\omega}) \le 0 \\ \mathcal{A}_{mn}(\boldsymbol{p},0), & \text{if } f_{mn}(\boldsymbol{p},0) \ge 0 \\ \mathcal{A}_{mn}(\boldsymbol{p},\omega^*), & \text{if } f_{mn}(\boldsymbol{p},\omega^*) = 0 \end{cases} \quad (18)$$

where $0 < \omega^* < \bar{\omega}$.

Hence the strategy $\boldsymbol{u}_{mn}(\boldsymbol{p}, A^*_{mn}(\boldsymbol{p}))$ defined in (16) uniquely minimizes the cost function (12) with respect to a given $\boldsymbol{p}$, i.e. $\boldsymbol{u}^*_{mn}(\boldsymbol{p}) = \boldsymbol{u}_{mn}(\boldsymbol{p}, A^*_{mn}(\boldsymbol{p}))$.

4.2 Price profile update mechanism

For load $n \in \mathcal{N}_m$, consider its given price profile as $\rho^{**}_{mt}, \forall t$, i.e. the summation of optimal (efficient) marginal generation cost and transformer cost. By (11) and (16), we have the collection of load strategies $\boldsymbol{u}^*(\boldsymbol{\rho}^{**})$ given by (16) and (18) is efficient.

However, this efficient price ρ^{**}_{mt} cannot be determined in advance. Hence we would like to explore an update mechanism that guarantees convergence of the price profile to the efficient one.

Denote by $\boldsymbol{\rho}_m$ the price profile provided for load $n \in \mathcal{N}_m$. Here, we consider the scheme,

$$\rho^+_{mt}(\boldsymbol{\rho}_m) = c'_t(D_t(\boldsymbol{\rho}_m)) + r'_{mt}(D_{mt}(\boldsymbol{\rho}_m)), \quad (19)$$

where $D_t(\boldsymbol{\rho}_m) = \sum_{m\in\mathcal{M}} D_{mt}(\boldsymbol{\rho}_m)$, $D_{mt}(\boldsymbol{\rho}_m) = d_{mt} + \sum_{n\in\mathcal{N}_m} u^*_{mn,t}(\boldsymbol{\rho}_m)$ with $u^*_{mn,t}(\boldsymbol{\rho}_m)$ defined in (13). Under this scheme, the price can reflect the marginal generation and transformer cost with respect to the latest load strategies.

4.3 Decentralized coordination of PEV charging

It is now possible to formalize a decentralized coordination algorithm for determining the optimal strategy for loads in Algorithm 1.

Algorithm 1 Decentralized coordination process.

Require:
A set of loads $\mathcal{N}$, a set of transformers $\mathcal{M}$, and time horizon $\mathcal{T}$;
Aggregate background demand $(d_{mt}, \forall m, t)$;
An initial price profile $(\boldsymbol{\rho}^{(0)}_m, \forall m)$;
Set $\varepsilon > \varepsilon_{\text{stop}}$;
Set $k = 0$;

Ensure:
Strategy for loads $\boldsymbol{u}$;

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1: while ε > ε_stop do
2:   All the loads update individual strategy simultaneously w.r.t. ρ_m^(k) for all n ∈ N_m and m ∈ M by minimizing their individual costs (12).
3:   Update ρ_m^(k+1) from u^(k+1)(ρ^(k)) using (19),
         ρ_mt^(k+1) = c'_t(D_t(ρ^(k))) + r'_mt(D_mt(ρ_m^(k))), ∀t;
4:   Update ε := Σ_{m∈M} ‖ρ_m^(k+1) − ρ_m^(k)‖_1;
5:   Update k := k + 1;
6: end while
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In the following, we establish the convergence of Algorithm 1 in Theorem 1. Before this, we define κ_t as the Lipschitz constant for $c'_t(\cdot)$ and η_{mt} as the Lipschitz constant for $r'_{mt}(\cdot)$ over the typical range in the total demand, and $\theta_{mn,t}$ as the Lipschitz constant for $[g'_{mn,t}]^{-1}(\cdot)$ over the interval $[g'_{mn,t}(0), g'_{mn,t}(\Gamma_{mn})]$. Then we have the following parameters.

$$\kappa = \max_{t\in\mathcal{T}} \kappa_t,$$
$$\eta = \max_{m\in\mathcal{M}, t\in\mathcal{T}} \eta_{mt},$$
$$\theta = \max_{m\in\mathcal{M}, n\in\mathcal{N}_m, t\in\mathcal{T}} \theta_{mn,t}.$$

Theorem 1. (Convergence of Algorithm 1 to the efficient solution)

Suppose $\alpha \equiv 2\bar{N}\theta(\kappa M + \eta) < 1$ with $\bar{N} = \max_{i\in\mathcal{M}} N_i$ and consider any initial charging price $\boldsymbol{\rho}^{(0)}$. Then Algorithm 1 converges to the efficient solution $\boldsymbol{u}^{**}$ specified in (11).

Proof. For each transformer m, consider a pair of price profiles $\boldsymbol{\rho}_m$ and $\boldsymbol{\varrho}_m$, and the respective updated price profiles $\boldsymbol{\rho}^+_m$ and $\boldsymbol{\varrho}^+_m$ given by (19). In the beginning, we could verify (20) by the result given in [Ma et al. [2016], Lemma 4.4].

$$\|\boldsymbol{u}^*_{mn}(\boldsymbol{\rho}_m) - \boldsymbol{u}^*_{mn}(\boldsymbol{\varrho}_m)\|_1 \leq 2\theta\|\boldsymbol{\rho}_m - \boldsymbol{\varrho}_m\|_1. \tag{20}$$

Then, we have the following analysis:

$$\begin{aligned}\|\boldsymbol{\rho}^+_m - \boldsymbol{\varrho}^+_m\|_1 &= \sum_{t\in\mathcal{T}}\Big|c'_t(D_t(\boldsymbol{\rho})) - c'_t(D_t(\boldsymbol{\varrho})) \\ &\quad + r'_{mt}(D_{mt}(\boldsymbol{\rho}_m)) - r'_{mt}(D_{mt}(\boldsymbol{\varrho}_m))\Big| \\ &\leq \sum_{t\in\mathcal{T}}\Big|c'_t(D_t(\boldsymbol{\rho})) - c'_t(D_t(\boldsymbol{\varrho}))\Big| \\ &\quad + \sum_{t\in\mathcal{T}}\Big|r'_{mt}(D_{mt}(\boldsymbol{\rho}_m)) - r'_{mt}(D_{mt}(\boldsymbol{\varrho}_m))\Big|.\end{aligned} \tag{21}$$

By the definition of κ, it follows that,

$$\begin{aligned}&\sum_{t\in\mathcal{T}}\Big|c'_t(D_t(\boldsymbol{\rho})) - c'_t(D_t(\boldsymbol{\varrho}))\Big| \\ &\leq \kappa\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{M}}\sum_{n\in\mathcal{N}_i}\Big|u^*_{in,t}(\boldsymbol{\rho}_i) - u^*_{in,t}(\boldsymbol{\varrho}_i)\Big| \\ &= \kappa\sum_{i\in\mathcal{M}}\sum_{n\in\mathcal{N}_i}\|\boldsymbol{u}^*_{in}(\boldsymbol{\rho}_i) - \boldsymbol{u}^*_{in}(\boldsymbol{\varrho}_i)\|_1 \\ &\leq 2\kappa\theta\sum_{i\in\mathcal{M}}\sum_{n\in\mathcal{N}_i}\|\boldsymbol{\rho}_i - \boldsymbol{\varrho}_i\|_1, \text{ by (20)} \\ &= 2\kappa\theta\sum_{i\in\mathcal{M}}N_i\|\boldsymbol{\rho}_i - \boldsymbol{\varrho}_i\|_1.\end{aligned}$$

Similarly, by the definition of η, we have

$$\begin{aligned}&\sum_{t\in\mathcal{T}}\Big|r'_{mt}(D_{mt}(\boldsymbol{\rho}_m)) - r'_{mt}(D_{mt}(\boldsymbol{\varrho}_m))\Big| \\ &\leq \eta\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}_m}\Big|u^*_{mn,t}(\boldsymbol{\rho}_m) - u^*_{mn,t}(\boldsymbol{\varrho}_m)\Big| \\ &= \eta\sum_{n\in\mathcal{N}_m}\|\boldsymbol{u}^*_{mn}(\boldsymbol{\rho}_m) - \boldsymbol{u}^*_{mn}(\boldsymbol{\varrho}_m)\|_1 \\ &\leq 2\eta\theta\sum_{n\in\mathcal{N}_m}\|\boldsymbol{\rho}_m - \boldsymbol{\varrho}_m\|_1, \text{ by (20)} \\ &= 2N_m\eta\theta\|\boldsymbol{\rho}_m - \boldsymbol{\varrho}_m\|_1.\end{aligned}$$

Hence with (21), it implies that,

$$\|\boldsymbol{\rho}^+_m - \boldsymbol{\varrho}^+_m\|_1 \leq 2\kappa\theta\bar{N}\sum_{i\in\mathcal{M}}\|\boldsymbol{\rho}_i - \boldsymbol{\varrho}_i\|_1 + 2\bar{N}\eta\theta\|\boldsymbol{\rho}_m - \boldsymbol{\varrho}_m\|_1.$$

Further,

$$\begin{aligned}&\sum_{m\in\mathcal{M}}\|\boldsymbol{\rho}^+_m - \boldsymbol{\varrho}^+_m\|_1 \\ &\leq 2\bar{N}M\kappa\theta\sum_{m\in\mathcal{M}}\|\boldsymbol{\rho}_m - \boldsymbol{\varrho}_m\|_1 + 2\bar{N}\eta\theta\sum_{m\in\mathcal{M}}\|\boldsymbol{\rho}_m - \boldsymbol{\varrho}_m\|_1 \\ &= 2\bar{N}\theta(\kappa M + \eta)\sum_{m\in\mathcal{M}}\|\boldsymbol{\rho}_m - \boldsymbol{\varrho}_m\|_1.\end{aligned}$$

Denote $\alpha \equiv 2\bar{N}\theta(\kappa M + \eta)$, then we have

$$\sum_{m\in\mathcal{M}}\|\boldsymbol{\rho}^+_m - \boldsymbol{\varrho}^+_m\|_1 \leq \alpha\sum_{m\in\mathcal{M}}\|\boldsymbol{\rho}_m - \boldsymbol{\varrho}_m\|_1. \tag{22}$$

Hence by the contraction mapping theorem Smart [1974], if $\alpha < 1$, the price profile $\boldsymbol{\rho}^{(k)}$ under the update scheme (19) converges to a unique price profile $\boldsymbol{\rho}^*$ from any initial price $\boldsymbol{\rho}^{(0)}$.

Furthermore, it will now be shown that $\boldsymbol{u}^*(\boldsymbol{\rho}^*)$ is the efficient strategy that minimizes the central optimization problem (9). At the converged price, $\boldsymbol{\rho}^+_m(\boldsymbol{\rho}^*) = \boldsymbol{\rho}^*_m$, and so (19) implies,

$$c'_t(D_t(\boldsymbol{\rho}^*)) + r'_{mt}(D_{mt}(\boldsymbol{\rho}^*_m)) = \rho^*_{mt}, \quad \forall t\in\mathcal{T}. \tag{23}$$

Let $\omega^* = \|\boldsymbol{u}^*_{mn}(\boldsymbol{\rho}^*_m)\|_1$. If $0 < \omega^* < \min\{\Gamma_{mn}, \|\bar{\boldsymbol{u}}_{mn}\|_1\}$ then by (18), $A^*_{mn}(\boldsymbol{\rho}^*_m) = h'_{mn}(\omega^*)$. According to (16), (11) is satisfied. If $\omega^* = 0$ then by (18), $A^*_{mn}(\boldsymbol{\rho}^*_m) \geq h'_{mn}(\omega^*)$. Also, in this case, $u^*_{mn,t}(\boldsymbol{\rho}^*_m) = 0$ for all $t\in\mathcal{T}$. Therefore, from (16),

$$\rho^*_{mt} + g'_{mn,t}(u^*_{mn,t}(\boldsymbol{\rho}^*_m)) \geq A^*_{mn}(\boldsymbol{\rho}^*_m) \geq h'_{mn}(\omega^*),$$

which again satisfies (11). $\boldsymbol{u}^*(\boldsymbol{\rho}^*)$ also satisfies (11) in case $\omega^* = \min\{\Gamma_{mn}, \|\bar{\boldsymbol{u}}_{mn}\|_1\}$, by applying the same technique.

Therefore $\boldsymbol{u}^* = \boldsymbol{u}^{**}$ with respect to the converged price $\boldsymbol{\rho}^* = \boldsymbol{\rho}^{**}$. ∎

We can further quantify an upper bound of the iteration steps that Algorithm 1 converges.

Corollary 2. (Convergence rate of Algorithm 1)

In case $\alpha \equiv 2\bar{N}\theta(\kappa M + \eta) < 1$, then for any $\varepsilon > 0$, the system converges to a price profile $\boldsymbol{\rho}$, such that $\|\boldsymbol{\rho} - \boldsymbol{\rho}^{**}\|_1 \leq \varepsilon$, in $K(\varepsilon)$ iterations, with

$$K(\varepsilon) = \left\lceil \frac{1}{\ln(\alpha)}\Big(\ln(\varepsilon) - \ln(MT\varrho_{max})\Big)\right\rceil, \tag{24}$$

where ϱ_{max} denotes the maximum possible price, and $\lceil x\rceil$ represents the minimal integer value larger than or equal to x.

Proof. Given $\alpha \equiv 2\bar{N}\theta(\kappa M + \eta) < 1$, (22) implies,

$$\sum_{m\in\mathcal{M}}\|\boldsymbol{\rho}^{(k)}_m - \boldsymbol{\rho}^{**}_m\|_1 \leq \alpha^k\sum_{m\in\mathcal{M}}\|\boldsymbol{\rho}^{(0)}_m - \boldsymbol{\rho}^{**}_m\|_1.$$

With all the entries of $\boldsymbol{\rho}^{(0)}, \boldsymbol{\rho}^{**}$ in $[0, \varrho_{max}]$, this gives,

$$\sum_{m\in\mathcal{M}}\|\boldsymbol{\rho}^{(k)}_m - \boldsymbol{\rho}^{**}_m\|_1 \leq \alpha^k MT\varrho_{max}.$$

To insure $\sum_{m\in\mathcal{M}}\|\boldsymbol{\rho}^{(k)}_m - \boldsymbol{\rho}^{**}_m\|_1 \leq \varepsilon$, we can derive that k satisfying (24). ∎

5. CASE STUDIES

In this section, we evaluate the performance of the proposed decentralized method, and show the effects of transformers on the price and load strategies through case studies. The system used for the case study, as shown in Fig. 2, is a typical 9 bus distribution network with 3 transformers. The hourly load demand data is obtained from Atwa and El-Saadany [2010], and we assume a period of $T = 24$ time horizon.

For transformers T1, T2 and T3, we don't consider their loss costs, and suppose that their capacity limit at each instant is $[680, 1000, 900]$ with the factor $\beta_{mt} = 3\times10^{-5}$ for all m, t. Thus the Lipschitz constant for $r'_{mt}(\cdot)$ is $\eta = 6\times10^{-5}$. Suppose that the generation cost has the quadratic form of $c_t(x) = 3\times10^{-5}x^2 + 0.05x$, and the corresponding Lipschitz constant is $\kappa = 6\times10^{-5}$.

In order to demonstrate the transformer effects on the price and load strategies, we suppose that transformers T1 and T2 connect 5 loads, and transformer T3 does

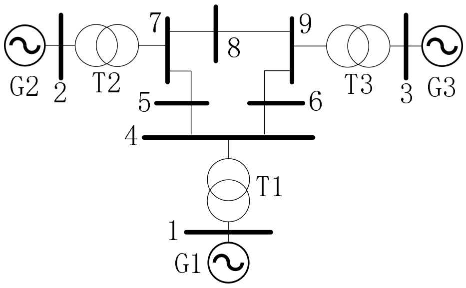

Fig. 2. A typical 9 bus distribution network.

not connect loads. Suppose all the loads have the same local cost function $g_{mn,t}(x) = 0.001x^2$; then the Lipschitz constant for $[g'_{mn,t}]^{-1}(\cdot)$ is $\theta = 500$.

Given these values, we have $\alpha = 0.9 < 1$ and $K(\varepsilon = 0.0001) = 112$. Thus by Theorem 1 and Corollary 2, the system is guaranteed to converge to the efficient solution within 112 iterations. Fig. 3 shows the evolution of $\|\boldsymbol{\rho}^{(k)} - \boldsymbol{\rho}^{**}\|_1$, and the updates of the total aggregate demand in the whole system achieved by Algorithm 1 are shown in Fig. 4. These are consistent with the results developed in Theorem 1 and Corollary 2.

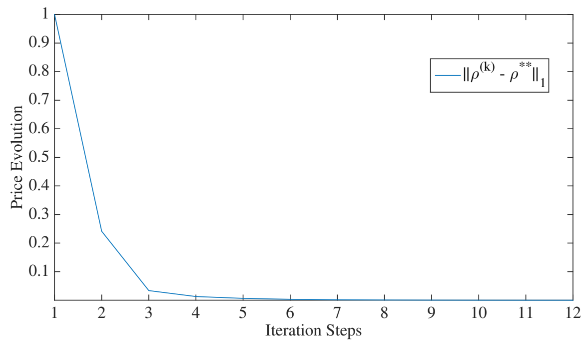

Fig. 3. Convergence of $\|\boldsymbol{\rho}^{(k)} - \boldsymbol{\rho}^{**}\|_1$.

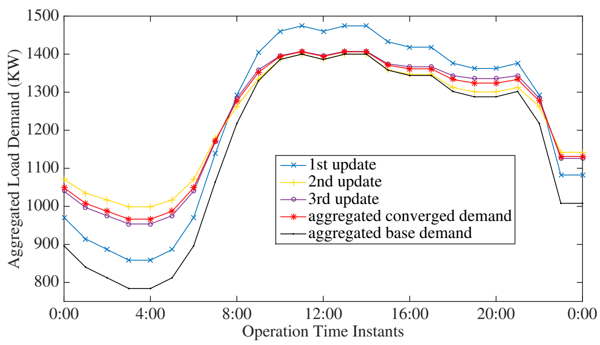

Fig. 4. Updates of the total aggregated demand of transformers T1 and T2 achieved by Algorithm 1.

Figure 5 and Fig. 6 show the converged prices and demands of transformers respectively. The prices and demands of transformers show difference due to their different maximum capacity limits.

If all the three transformers connect loads, the updates of the total aggregate demand in the whole system, the converged prices and demands of each transformer are illustrated in Fig. 7, Fig. 8 and Fig. 9 respectively. The prices of T2 and T3 are the same, since their total demand don't exceed their capacity limit, and then their prices equal the group consensus price.

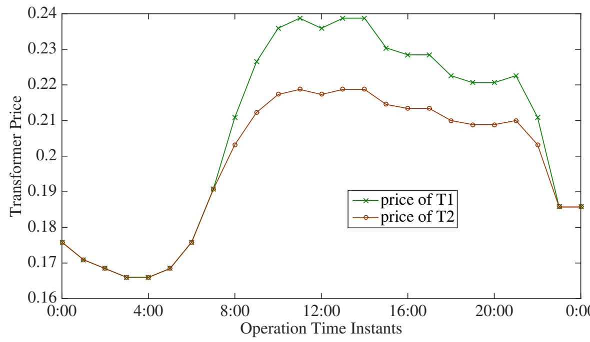

Fig. 5. Prices of transformers T1 and T2 achieved by Algorithm 1.

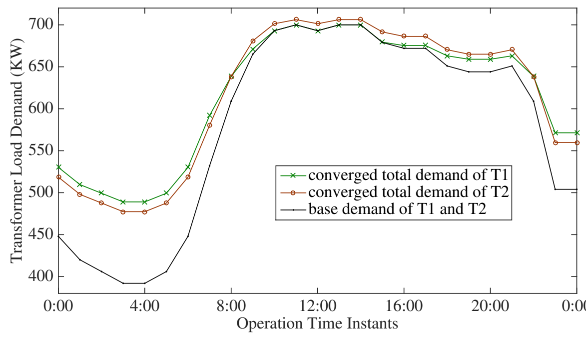

Fig. 6. Demands of transformers T1 and T2 achieved by Algorithm 1.

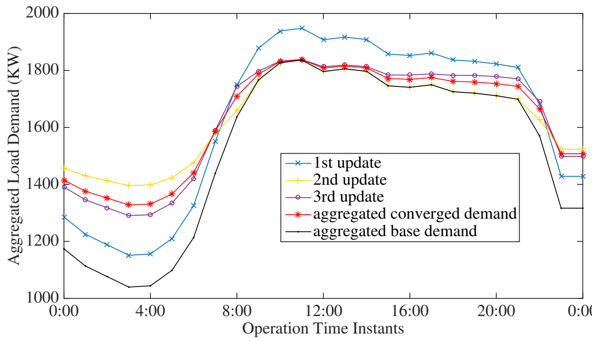

Fig. 7. Updates of the total aggregated demand when T3 connects loads.

Consider the loss costs of transformers by setting $f_{mt} = 0.0005x^2$ for all m, t. The converged prices and demands of each transformer are illustrated in Fig. 10 and Fig. 11 respectively. In this case, the price of T2 is higher than that of T3, as the loss cost of T2 is higher. Moreover, due to the existence of losses, the total load demands of each transformer are lower than those without considering the losses.

6. CONCLUSIONS

A price based method has been proposed to coordinate controlled loads in a framework that loads connect to

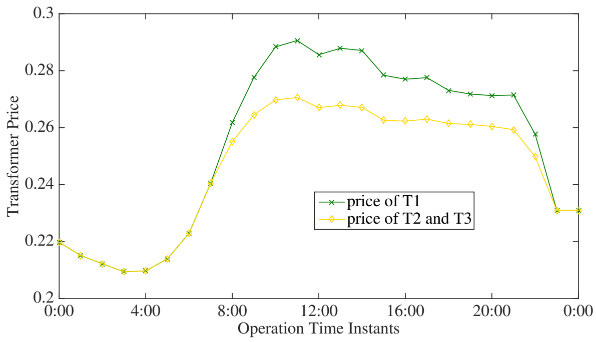

Fig. 8. Prices of transformers T1, T2 and T3.

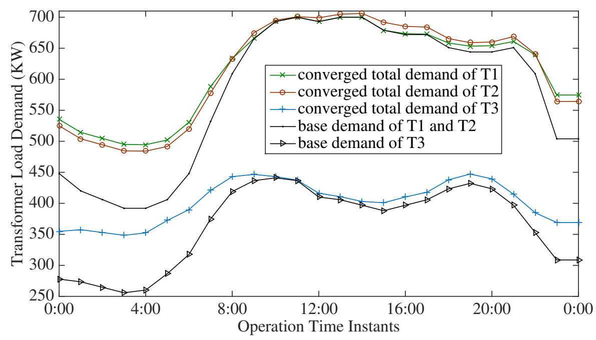

Fig. 9. Demands of transformers T1, T2 and T3.

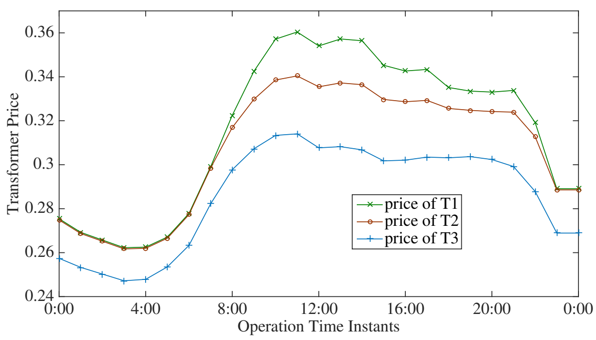

Fig. 10. Prices of transformers T1, T2 and T3 considering transformer loss costs.

transformers. Also, the system considered the tradeoffs between the electricity generation cost and local individual costs. By formulating this problem as a hierarchical model, individual loads implemented their best strategies under a price signal broadcasted by the transformer in the lower level. While in the upper level, transformers communicated with the distribution network and got a price that reflected the system generation cost. Thus each transformer determined a private price based upon not only the individual characteristics but also the price reflecting the system's characteristics. By proposing a dynamic update algorithm, our results built that the system converged to the unique and efficient solution with fast convergence speed.

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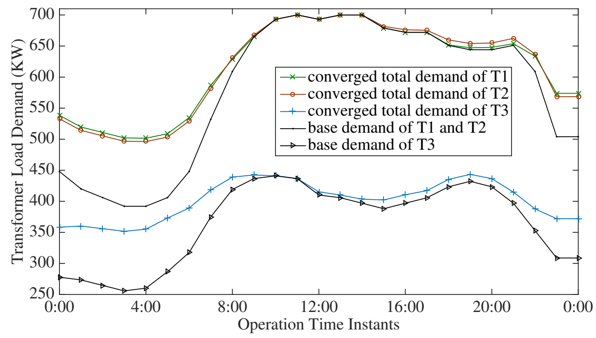

Fig. 11. Demands of transformers T1, T2 and T3 considering transformer loss costs.

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