Competitive Equilibrium in Electricity Markets with Heterogeneous Users and Ramping Constraints

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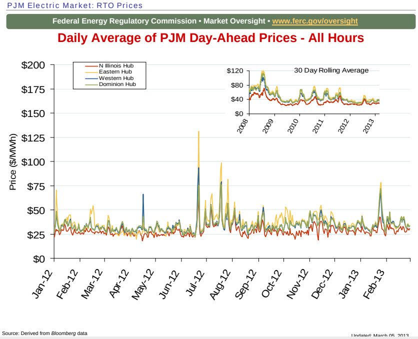
Motivation

- Much interest in engineering and economics in modeling and improving the functioning of energy networks.
  - Complicated by the fact that supply and demand are variable and often not flexible, leading to significant price fluctuations.
  - A market that involves significant heterogeneity across users and complex interactions between the supply at different dates because of the ramp-up/ramp-down constraints.



PJM day-ahead market prices Jan 2012 to Feb 2013 from FERC database.

This Talk

- We develop a flexible model of competitive equilibrium in the electricity market with heterogeneous users and supply incorporating ramp-up/ramp-down constraints.
- Model Features:
    - Heterogeneous (price-taking) users.
    - Limited flexibility on consumer side to shift demand over time as a function of the price sequence.
    - Supply side modeled with two cost components: cost of production; cost of ramp-up/ramp-down.

Main Results and Current Work

- Characterization of equilibrium prices and quantities.
- Primal-dual algorithm for decentralized and incentive compatible computation of prices.
- Though the equilibrium is efficient, it may feature high price volatility.
  - If cost of ramp-up/ramp-down is unimportant, there will typically be high load fluctuations and low price fluctuations.
  - If cost of ramp-up/ramp-down is important, low load fluctuations and much larger price fluctuations.
- Ongoing work:
  - Design of alternative allocation rules to reduce price fluctuations at limited welfare loss.

Related Literature

- [Cho, Meyn, 2010]: competitive equilibrium in the presence of exogenous stochastic demand (without flexibility of demand shifting) and ramp-up/ramp-down modeled as constraints rather than cost component.
- [Roozbehani, Dahleh, Mitter, 2012]: market dynamics under a specific pricing mechanism.
- Demand Management Mechanisms:
  - [Merugu, Prabhakar, Rama 2009], [Schwartz, Hamidou, Amin, Sastry 2012]: Raffle-based incentive schemes.
  - [Ma, Callaway, Hiskens, 2011]: Decentralized charging control of PEVs.
- Other recent related papers:
  - [Kizilkale, Mannor 2010, 2011], [Tsitsiklis, Xu 2012].

Demand Side

- Large number of (price taking) heterogenous consumers of type $1, \ldots, N$. Each type with total demand d_i to be allocated across time periods $t = 1, \ldots, T$.
- We use x_i^t to denote the allocated demand of type i consumer at time t.
  - Canonical Example: Utility of consumption of type i consumer is given by

    $$u_i^t(x_i^t) = v_i(|t - f_i|)u_i(x_i^t),$$

    where f_i is the favorite slot of type i consumer. Here v_i is a decreasing function and u_i is an increasing concave function.
  - Interpretation: Users have some flexibility in shifting their demand over time at some cost.
- Consumer demand comes from the following optimization problem:

$$\max_{\{x_t^i\}_{t=1,\ldots,T} \geq 0} \quad \sum_{t=1}^{T} u_i^t(x_i^t) - p^t x_i^t$$

$$\text{s.t.} \quad \sum_{t=1}^{T} x_i^t = d_i,$$

where p^t is the price of electricity at time t.
- This nests both elastic and inelastic demand.

Supply Side

- The supply side consists of a large number of (price-taking) suppliers which we represent as a single representative supplier.
- Costs are made up of two components:
  - Cost of production at t, $c^t(\bar{x}^t)$, where $\bar{x}^t$ is the total supply at t and c^t is an increasing convex function.
  - An intertemporal cost function $H(\bar{x}^t, \bar{x}^{t-1})$, which reflects the greater cost due to ramp-up/ramp-down (or start up-shutting down) of supply between times. H is assumed to be jointly convex in $\bar{x}^t$ and $\bar{x}^{t-1}$, and increasing in $\bar{x}^t$.
- Supply comes from the following optimization problem:

$$\max_{\{\bar{x}^t\}_{t=1,\ldots,T} \geq 0} \quad \sum_{t=1}^{T} p^t \bar{x}^t - c^t(\bar{x}^t) - H(\bar{x}^t, \bar{x}^{t-1}),$$

where $\bar{x}^0$ is a given initial supply.

Competitive Equilibrium

- A competitive equilibrium is a sequence of prices $\{p^t\}$ and supply $\{\bar{x}^t\}$ and demand $\{x_i^t\}$ sequences such that
  - x_i^t maximizes the utility of type i consumer.
  - $\bar{x}^t$ maximizes the profit of supplier.
  - p^t is such that the market clears, i.e., $\sum_{i=1}^N x_i^t = \bar{x}^t$ for all t.
- By the well-known first and second welfare theorems,
  - A competitive equilibrium is Pareto optimal.
  - Every Pareto optimal allocation can be decentralized as a competitive equilibrium.

Characterizing the Competitive Equilibrium

- By the second welfare theorem (and because utility is quasi-linear), a competitive equilibrium can be characterized by maximizing social welfare given as

$$\max_{\{x_i^t, \bar{x}^t\} \geq 0} \quad \sum_{t=1}^{T} \sum_{i=1}^{N} u_i^t(x_i^t) - c^t(\bar{x}^t) - H(\bar{x}^t, \bar{x}^{t-1}),$$

$$\text{s.t.} \quad \sum_{t=1}^{T} x_i^t = d_i \text{ for } i \in \{1, \ldots, N\}, \qquad \text{Demand satisfied over time T.}$$

$$\sum_{i=1}^{N} x_i^t = \bar{x}^t, \text{ for } t \in \{1, \ldots, T\}. \qquad \text{At each time, supply=demand.}$$

- The market clearing prices emerge as the dual variables to the last constraint.

Primal-Dual Approach

We use a subgradient based method to solve the social welfare maximization problem.

- Lagrangian function:

$$L(x_i^t, \bar{x}^t, p) = \sum_{i=1}^{N}\sum_{t=1}^{T} u_i^t(x_i^t) - \sum_{t=1}^{T} c^t(\bar{x}^t) - \sum_{t=1}^{T-1} H(\bar{x}^t, \bar{x}^{t+1}) - \sum_{t=1}^{T} p^t \left(\sum_{i=1}^{N} x_i^t - \bar{x}^t\right),$$

  where variables x_i^t satisfy $\sum_{t=1}^{T} x_i^t = d_i$ for $i \in \{1, \ldots, N\}$.

- The Lagrangian can be decomposed,

$$L(x_i^t, \bar{x}^t, p) = \sum_{i=1}^{N}\sum_{t=1}^{T} [u_i^t(x_i^t) - p^t x_i^t] + \sum_{t=1}^{T} p^t \bar{x}^t - c^t(\bar{x}^t) - \sum_{t=1}^{T-1} H(\bar{x}^t, \bar{x}^{t+1}).$$

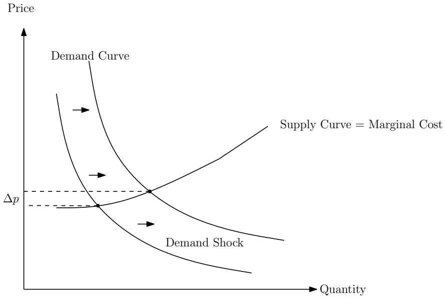
- Subgradient step: $p_k^t = p_{k+1}^t + \alpha \left(\sum_{i=1}^{N} {y_i}^t - \bar{y}^t\right)$, where ${y_i}^t$ and $\bar{y}^t$ are the optimal solutions to

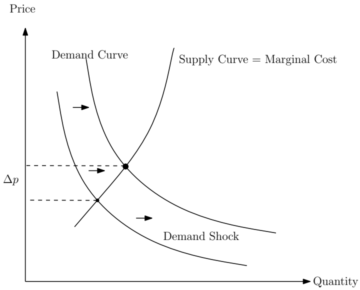
$$\max_{x_i^t, \bar{x}^t} L(x_i^t, \bar{x}^t, p^k).$$

- Converges when consumers and generators locally optimize utility function.

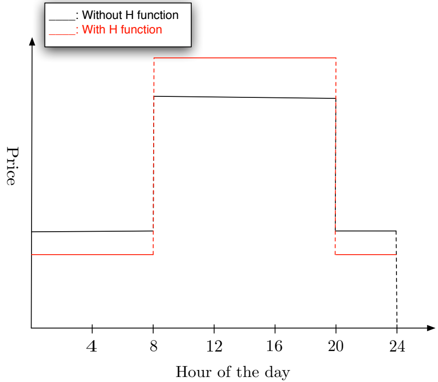
General Properties

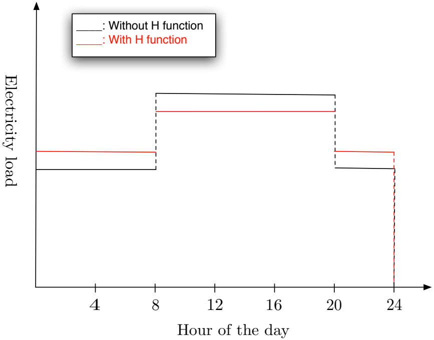
- The equilibrium and social welfare optimum involve different amount of fluctuations in quantities and prices depending on the form of the supplier cost functions.
  - If $H = 0$ and c^t is not very convex, there will be significant fluctuations in $\bar{x}^t$.
  - If the H function and the c^t function is highly convex, then $\bar{x}^t$ will fluctuate little and market prices will fluctuate a lot.





Illustration





Simulations - Demand Setup

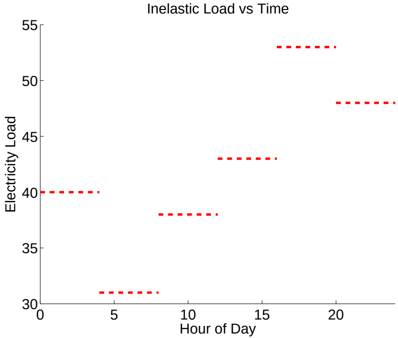


Figure: Inelastic component of the demand, discretized into 6 periods, 4 hours each. Values obtained from the demand from ERCOT, Aug 14, 2009.

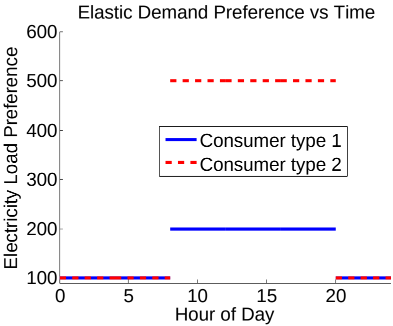
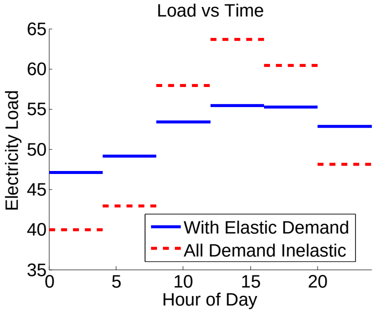


Figure: Elastic component of the demand x_i^t for $i = 1, 2$. Consumer type 1 slightly prefers peak hour while consumer type 2 much prefers peak hour. Hours 8AM to 8PM is peak hours.

Simulations - Price and load fluctuation

Used quadratic cost function $c(\bar{x}^t) = 0.25\bar{x}^t + 200\bar{x}^t$ and quadratic ancillary function $H(\bar{x}^t, \bar{x}^{t+1}) = 50(\bar{x}^t - \bar{x}^{t+1})^2$.
Each elastic demand has to sum to 30.



Load fluctuation with elastic demand vs when all demands are inelastic.

Simulations - Effect of Ramping Cost on Price and Load Fluctuation

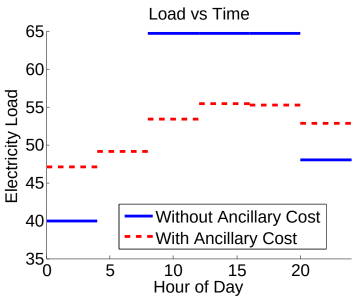


Figure: Load fluctuation with and without the ramping cost H.

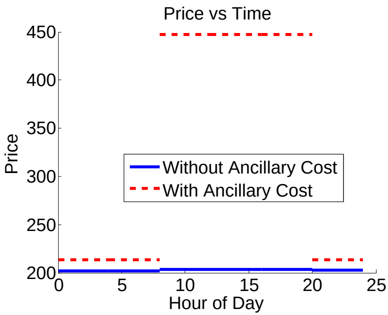


Figure: Price fluctuation with and without the ramping cost H.

Reducing Price Volatility

- Even though the equilibrium in this market is Pareto optimal and maximizes social welfare, it may deviate from other policy and social objectives, in particular it may involve significant price volatility as we have seen.
- This raises several new questions:
  - How can the price mechanism be designed such that price fluctuations are reduced?
- Idea:
  - Choose a rationing mechanism (supply will no longer be equal to demand).
  - Natural rationing mechanism: short side of the market is fully utilized, long side uniformly (randomly) rationed.
  - Introduce a price fluctuation penalty term in the dual formulation.
  - Compute whether price volatility can be reduced with small efficiency loss.

Extensions

- Stochastic demand: Instead of $u_i^t(x_i^t)$, we have $a_i^t u_i^t(x_i^t)$ where a_i^t is a random variable.
- Market power: Price taking both on supplier and demand side can be relaxed.
  - On the supply side, a few large suppliers.
  - On the demand side, both small and large buyers.
  - Equilibrium no longer efficient, raises the possibility that both welfare can be increased and price volatility can be reduced simultaneously.
- Online price dynamics with rationing.
- Network effects:
  - Introduce power flow constraints and treat each bus separately.
  - Price will be location dependent: LMP.