A Hierarchical Approach to CPS Resilience
based on Game Theory, Stochastic Control, and Theory of Incentives

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Motivation: CPS resilience, security

Research plan: Three-layer hierarchical approach
- Upper layer: Game theory
- Middle layer: Stochastic control & Theory of incentives
- Lower layer: Control Theory

Will concentrate on the upper and middle layers
Failures in CPS

- Simultaneous attacks [security failures]
  - Targeted cyber-attacks
  - Non-targeted cyber-attacks
  - Coordinated physical attacks

- Simultaneous faults [reliability failures]
  - Common-mode failures
  - Random failures due to nature
  - Operator errors

- Cascading failures
  - Failure of nodes in one subnet $\Rightarrow$ progressive failures in other subnets

Observation
Due to cyber-physical interactions, it is extremely difficult to distinguish reliability & security failures using imperfect diagnostic information.
Salient features of CPSs

CPSs are multi-agent systems, where

- Agents (players) are strategic, utility-maximizing entities
- Incomplete and also asymmetric (private) information is present
- CPSs are subject to security failures and reliability failures
- Defense strategies include both control and IT security tools
- Players face regulatory impositions for ensuring efficiency & safety

A hierarchical approach

The above features, along with the social objectives of resilient CPS operation, motivate a hierarchical approach.
Research plan: Three-layer hierarchical approach

Upper layer

- How the collection of CPS's agents deal with external strategic adversary(-ies)
- Network games that model both security failures and reliability failures

Middle layer

- How strategic agents contribute to CPS efficiency and safety, while protecting their conflicting individual objectives
- Joint stochastic control and incentive-theoretic design, coupled with the outcome of the upper layer game

Lower layer

- Control at each individual agent's site.
Upper hierarchical layer

Game with security-reliability failures

Game played on a graph $G = (V, E, w)$ representing the topological structure of CPS

- Attacker(s)
  - Strategic adversary
  - Nature
- Defender: CPS network designer
Game with security-reliability failures

Graph $G$ representing CPS topology
- $V$: Set of nodes
- $E$: Set of edges
- $w$: Set of weights on edges

Attacker’s strategy space
- $E$: set of graph’s edges
- Attacker chooses an edge $e \in E$
  - Failure comes from nature with probability $\pi$
  - Failure comes from a strategic adversary with probability $(1 - \pi)$

Defender’s strategy space
- $T$: Set of graph’s spanning trees
- Defender chooses $\tau \in T$
Game with security-reliability failures

Payoffs for a choice of $\tau \in \mathcal{T}$ and $e \in \mathcal{E}$

$$\Pi_D(\tau, e) = v(\tau) - (1 - \pi) \left[ w(e) \mathbf{1}_{\{e \in \tau\}} \right] - \pi \left[ \sum_{e' \in \mathcal{E}} \gamma_{e'} w(e') \mathbf{1}_{\{e' \in \tau\}} \right]$$

$$\Pi_A(\tau, e) = w(e) \mathbf{1}_{\{e \in \tau\}}$$

- $v(\tau)$: value of an operational spanning tree $\tau \in \mathcal{T}$
- $w(e)$: Weight/importance of edge $e \in \mathcal{E}$
- $\mathbf{1}_{\{e' \in \tau\}}$: Indicator function of the even $\{e \in \tau\}$
- $\gamma_{e'}$: Probability of reliability failure of $e' \in \mathcal{E}$
Upper hierarchical layer - Game Theory

Assumptions

- Imperfect information: defender faces aggregate failure probabilities:
  \[ P(f_e) = \pi \gamma_e + (1 - \pi) \beta_e, \quad \forall e \in \mathcal{E}, \]

  where \( \pi \) reliability and \( 1 - \pi \) security

- Given failure probabilities due to nature: \( \gamma = (\gamma_{e_1}, \ldots, \gamma_{e_m}) \)
- Equilibrium failure probabilities due to attacker: \( \beta = (\beta_{e_1}, \ldots, \beta_{e_m}) \)

- Common knowledge: Payoff functions \( \Pi_A \) and \( \Pi_D \)

Objectives

- Determine Nash equilibria (NE) of the one-stage game within the class of mixed strategies
- Determine equilibria for the finitely or infinitely repeated game
References

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G. A. Schwartz, S. Amin, A. Gueye, J. Walrand (2011)
Network design game with both reliability and security failures.

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Upper layer $\rightarrow$ Middle layer

How to embed the outcomes of upper layer into the middle layer failure models for the design of resilient CPS strategies using stochastic control and incentive-theoretic formulations?
Upper layer → Middle layer

Outcome of upper layer game
- Equilibrium strategies for attacker and defender \((\alpha, \beta)\)
- Edge failure probabilities:
  \[ P(f_e) = \pi \gamma_e + (1 - \pi) \beta_e, \quad \forall e \in \mathcal{E} \]

Embedding \(P(f_e)\) into middle layer model
- Physical: structural failures
- Cyber: sensor-actuator failures

Middle hierarchical layer
Resulting failure models are used to design of resilient strategies.
Middle hierarchical layer

Stochastic control and incentives

- Stochastic control: Performance benchmark against CPS failures
- Theory of Incentives: implement in appropriate equilibria the optimal control strategies of the stochastic control problem

![Diagram showing the flow from CPS dynamics & failures models to Decisions (Control + Security) and then to Theory of Incentives Game Form, with connections to Upper layer (Game Theory), Middle layer (Stochastic Control Theory) and Lower layer (Local Control).]
Middle hierarchical layer - CPS model

Agent $i$’s dynamics modeled by

- A controlled stochastic vector difference equation
- A controlled multi-dimensional Markov chain

\[
X_{t+1}^i = f_t^i \left( X_t^i, U_1^i, \ldots, U_N^i, W_t^i, P_{s,t}^i, P_{u,t}^i \right)
\]

- $N$: # of agents in CPS
- $\mathcal{N} = \{1, \ldots, N\}$: set of agents
- $X_t^i \in \mathcal{X}_i$: state of agent $i$ at time $t$ and $\mathcal{X}_i$ finite
- $U_t^i$: control action of agent $i$
- $W_t^i$: noise in component $i$ at $t$
- $P_{s,t}^i$ and $P_{u,t}^i$: probabilities of structural failure & actuation failure at $t$
Middle hierarchical layer - CPS model

State of CPS with $N$ agents at time $t$

\[ X_t = (X_1^t, X_2^t, \ldots, X_N^t) \]

Sensing model

\[ Y_t^i = h_t^i \left( X_t^i, W_t^{o,i}, P_{i,t}^o \right), \quad i \in \mathcal{N} \]

- $Y_t^i$: observation of agent $i$ at $t$
- $W_t^{o,i}$: observation noise of $i$ at $t$
- $P_{i,t}^o$: probability of sensing failure at $t$
Middle hierarchical layer - CPS model

Decision strategies

\[ U_t^i = g_t^i \left( Y_{1:t}^1, Y_{1:t}^2, \ldots, Y_{1:t}^N, U_{1:t-1}^1, U_{1:t-1}^2, \ldots, U_{1:t-1}^N \right), \quad i \in \mathcal{N}, \quad t = 1, \ldots, T \]

- \( T \): time horizon (finite or infinite)
- \( Y_{1:t}^i = (Y_1^i, Y_2^i, \ldots, Y_t^i) \)
- \( U_{1:t-1}^i = (U_1^i, U_2^i, \ldots, U_{t-1}^i) \)
- \( g^i = (g_1^i, g_2^i, \ldots, g_T^i) \): control/decision strategy of agent \( i \)
- \( g = (g_1^1, g_2^2, \ldots, g_N^N) \): control strategy for the CPS
Middle hierarchical layer - CPS model

Reward Functions

- Reward function for agent $i$

$$R^i = \sum_{t=1}^{T} R_t^i \left( X_t^i, U_{t}^1, U_{t}^2, \ldots, U_{t}^N \right)$$

- Total reward

$$R = \sum_{i=1}^{N} \sum_{t=1}^{T} R_t^i \left( X_t^i, U_{t}^1, U_{t}^2, \ldots, U_{t}^N \right)$$
CPS model: An example

- CPS system - system consisting of $N$ energy suppliers
  - Each supplier is strategic (selfish, self-utility optimizer)
  - Each supplier has private information (e.g. production technology)
  - Efficient operation so as to achieve a social objective

- $X^i_t$: energy producing capability of power supplier $i$ at $t$
- $U^i_t$: energy produced by power suppliers $i$ at $t$
- $X^i_{t+1} = f^i_t (X^i_t, U^i_t, W^i_t, P^i_{s,t}, P^i_{u,t})$, i.e., $X^i_{t+1}$ depends on $X^i_t$, $U^i_t$, failures due to nature, failures due to strategic adversary, repairs.

- Profit of power supplier $i$ at time $t$

$$R^i_t (X^i_t, U^1_t, U^2_t, \ldots, U^N_t) = \lambda_t \left( U^1_t, U^2_t, \ldots, U^N_t \right) \cdot U^i_t - \hat{c}^i_t (X^i_t) \cdot U^i_t$$

- $\lambda_t (U^1_t, U^2_t, \ldots, U^N_t)$: price charged per unit of produced energy
- $\hat{c}^i_t (X^i_t)$: cost per unit of energy produced when state is $X^i_t$. 
Middle hierarchical layer - Objectives

Determine \( g = (g^1, g^2, \ldots, g^N) \) to maximize \( E^g[R] \), subject to
- Informational constraints (agent \( i \)'s information at \( t \) is \( (Y_{1:t}^i, U_{1:t}^i) \))
- Taking strategy behavior into account

To achieve the objective
- Derive performance benchmark using stochastic control
- Achieve performance benchmark by a mechanism/game form which satisfies the problem’s constraints using the theory of incentives
Middle hierarchical Layer - Stochastic control

Consider a central authority that has all the information, including

- Agents’ utilities/reward functions
- Observations & control actions, i.e. \( \mathcal{I}_t = (Y^1_{1:t}, \ldots, Y^N_{1:t}, U^1_{1:t}, \ldots, U^N_{1:t}) \)
- CPS dynamics

Stochastic control problem

- Central authority chooses \( g = (g^1, g^2, \ldots, g^N) \) to maximize \( E[g][R] \)
  subject to
    - Sensor-actuator failures
    - Structural failures

- Solution provides a performance benchmark
- Achievable if all agents were willing to cooperate & share information
- However, CPS agents are strategic, selfish!
References

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Security of interdependent and identical networked control systems.
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Theory of incentives / Mechanism design

- **E** environment space (space of agents’ utilities, network topologies)
- **U** Action / alloc. / control space
- \((M, h)\): game form/mechanism
  - **M**: message / strategy space
  - **h**: outcome function
- \(\mu\): message correspondence
- \(\forall e \in E, (M, h, e)\) is the game induced by \((M, h)\)
Let $M^*(e) = \{m^* \in M : m^* \text{ is an equilibrium message/strategy of } (M, h, e)\}$

Objective: Design $(M, h)$ so that

$$\forall e \in E, \forall m^* \in M^*(e), h(m^*)(e) = g(e)$$

That is, design a game form/mechanism (if exists) that accounts for the

- Information structure of the CPS,
- Agents’ strategic behavior
- Achieves the same performance as the performance benchmark (i.e., the solution of the stochastic control problem)
Incentives: Achieving the upper bound

Approach

- Restrict attention to direct revelation mechanisms invoking the revelation principle.
- **Revelation principle**: If a game form \((\mathbf{M}, h)\) implements \(g : \mathbf{E} \rightarrow \mathbf{U}\) in a certain equilibrium concept \(\hat{\Lambda}\) (e.g. BNE), then there is a direct revelation mechanism \((\mathbf{E}, h^*)\) which has the following property:

  Reporting one’s true environment \(e\) is an equilibrium message/strategy of \((\mathbf{E}, h^*, e)\) in the same equilibrium concept \(\hat{\Lambda}\), and \(h^*(e) \in g(e)\) for all \(e \in \mathbf{E}\).

- We are looking for **truthful implementation** of \(g\) (optimal control strategy for the stochastic control problem).
- We consider agents \(i \in \mathcal{N} = \{1, 2, \ldots, N\}\) with quasi-linear utilities

\[
V^i_t(X^i_t, U^1_t, \ldots, U^N_t, (tx)^i_t) = R^i_t(X^i_t, U^1_t, \ldots, U^N_t) - (tx)^i_t
\]
Dynamic Incentives: Achieving the upper bound

Determine a dynamic direct revelation mechanism \((E, h_1, h_2, \ldots, h_T)\) [if it exists] that has the following properties:

(i) It is incentive compatible (i.e., truth telling is a BNE of the game induced by the mechanism)
(ii) It is budget-balanced

\[
\sum_{i=1}^{N} (tx)^i_t = 0 \quad \forall t \quad \text{OR} \quad \sum_{t=1}^{T} \sum_{i=1}^{N} (tx)^i_t = 0 \quad \text{at truthful equilibrium}
\]

(iii) Decisions/control actions at truthful equilibrium are the same as the decisions made by \(g\) (the optimal control law).