A Hybrid Approach to the Fast Positive Force Transient Tracking Problem in Automotive Engine Control[1]

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Abstract

The engine control problem can be decomposed into a set of sub-problems corresponding to *regions of operation* identified by the settings of the control devices available to the driver (for example accelerator pedal angle, selected gear). One of these regions is the so called Fast Positive Force Transient, where a quick acceleration is requested, maintaining certain comfort standards. In this paper, this control problem is formulated as a hybrid control problem and solved by approximation of an auxiliary continuous control problem. The quality of the results is backed by a set of simulations on a commercial car model.

1 Introduction

Automotive engine control is an important application domain for hybrid systems (see e.g. [5]) and for control in general. We argue that most of the engine control problems are hybrid control problems since the plant itself, having discrete as well as continuous components (e.g. engine cycle and power-train), is described by a hybrid model. In our approach, the system specifications are captured using a top-level Finite State Machine (FSM), whose states correspond to different regions of operation of the engine. The transitions are determined by driver's actions or by engine conditions. Each region of operation is characterized by a set of constraints, related to driving performance like comfort and safety or gas and noise emission, and a cost function that identifies the desired behavior of the controlled system. The goal of the controller is to act on the inputs to the plant so that it behaves according to the specifications summarized in the FSM.

In previous papers [1, 2], we introduced a hybrid model for the engine to solve the cut-off control problem, a sub-set of the control problem corresponding to the *Fast Negative Force Transient* region of operation. In this paper, in our quest for a general solution to the engine control problem, we consider the control problem corresponding to the *Fast Positive Force Tracking* region of operation. In this state, we have to react to a fast gas pedal motion that is interpreted as a request for a fast increase of the torque delivered by the engine while maintaining a reasonable level of comfort, specified in terms of vehicle acceleration and jerk. The goal is to control the evolution of the system from an initial condition characterized by the delivery of a torque u_0 to a final condition characterized by the delivery of a requested torque u_R in minimum time subject to constraints on acceleration and jerk. The available control actions are on fuel injection, spark ignition and, since we are considering cars equipped with drive-by-wire electronics, throttle angle (which regulates the air entering the cylinders). Our approach to the hybrid problem at hand is to first introduce an auxiliary relaxed problem: a continuous time problem with jerk as input. This problem is solved optimally. Then the solution is mapped back in the hybrid domain obtaining feedback laws for the throttle angle, the spark advance and the fuel injection inputs. The quality of the control law is demonstrated by a set of simulations on a model of a commercial car.

2 Problem formulation

2.1 Plant model

The hybrid model of a vehicle with a 4–stroke N–cylinder gasoline engine proposed in [2] is here reviewed. Such model is the composition of N sub–models, one per cylinder (see Figure 1). Each of them consists of: a FSM describing internal combustion engine's cycle (direct graph); a Discrete Event (DE) system modeling torque generation (dashed boxes); a Continuous Time (CT) dynamics modeling power-train and air dynamics (solid boxes).

Power-train model. The power-train behavior is described by the linear CT dynamics

$$\dot{\zeta} = A\,\zeta + b\,u \qquad (1)$$

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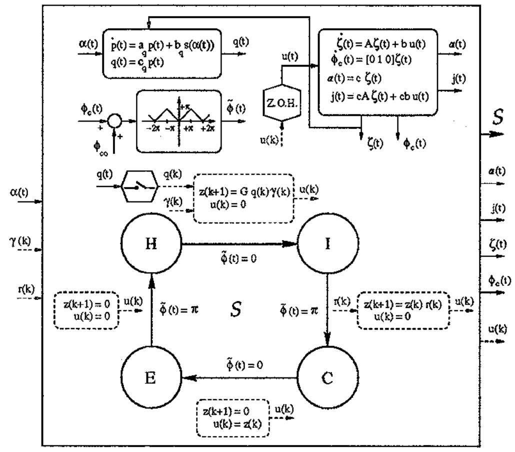

Figure 1: Hybrid model for a single cylinder engine.

$$\dot{\phi}_c \;=\; [0\;1\;0]\;\zeta \qquad (2)$$

where ζ components are: the axle torsion angle α_e, crankshaft revolution speed ω_c, and wheel revolution speed ω_p; and ϕ_c denotes the crankshaft angle. The input u is the torque acting on the crank. Being a passive system, dynamics (1) is asymptotically stable and has a real dominant pole λ_1, and a pair of complex poles $\lambda \pm j\mu$. Assuming vehicle speed equal to peripheral wheel speed, vehicle acceleration is $a = R_w\;\dot{\omega}_p$, where R_w is the wheel radius. Let $c \in \mathbb{R}^{1\times 3}$ be the product between R_w and the third row of A, and let j denote vehicle jerk. Since the third entry in b is 0, we have

$$a \;=\; c\,\zeta\;, \qquad (3)$$

$$j \;=\; \frac{da}{dt} = (cA)\;\zeta + (cb)\;u\;. \qquad (4)$$

Cylinder's behavior. The behavior of each cylinder in the internal combustion engine is represented by the FSM. State $\mathcal{S}$ assume values in the set $\{I, C, E, H\}$ related to the *intake*, *compression*, *expansion* and *exhaust* strokes. State transitions occur when the piston reaches the bottom or top dead center. Guard conditions are written in terms of the piston position expressed by $\tilde{\phi}$, which denotes the absolute value of the crank angle w.r.t. the upper dead center position. ϕ_{co} is the angle the crank is mechanically mounted on the shaft. The torque produced by the cylinder during its expansion phase is modeled as a constant signal.

Torque generation. The torque generation mechanism is characterized by a transport process represented by a DE system synchronized with FSM transitions. At each time t_k where a transition occurs the event counter k is incremented by one. The DE output $u(k)$ is converted by the zero-order hold block to the piece-wise constant signal $u(t) = u(k)$ for $t \in [t_k, t_{k+1})$, which feeds the power-train. DE inputs are: the mass of air $q(k) \in \mathbb{R}^+$ loaded in the intake stroke, which depends on air dynamics; the mix composition factor $\gamma(k) \in \{0\} \cup [\gamma_{\min}, \gamma_{\max}]$ which represents the ratio between the mass of fuel injected and the mass of air loaded, normalized w.r.t. the stoichiometric value; the spark modulation factor $r(k) \in [r_{\min}, 1]$ due to spark ignition timing. If $\gamma = 0$ no fuel is injected. At the $H \to I$ transition, the maximum amount of torque achievable during the next expansion phase, from the given air $q(k)$ and fuel $\gamma(k)$, is stored in the DE state $z \in \mathbb{R}$; then at the $I \to C$ transition the spark factor $r(k)$ is applied and at the $C \to E$ transition the torque $u(k)$ is output based on the value stored in z.

Air dynamics. The model of the quantity of air entering the cylinder during the intake stroke is obtained from the equation of the flow of a compressible fluid through a converging nozzle, whose section is controlled by the throttle valve [3]. Let $p(t)$,$\alpha(t)$ and $q(t)$ resp. denote the manifold pressure, the throttle angle, and the mass of air loaded by the cylinder at time t. We have

$$\dot{p} \;=\; a_q(\omega_c, p)\;p + b_q(p)s(\alpha) \qquad (5)$$

$$q \;=\; c_q(\omega_c, p)\;p \qquad (6)$$

where $s(\alpha)$ is the so called equivalent throttle area and

$$a_q(\omega_c, p) \;=\; -\frac{V_d}{4\pi(V_d + V_m)}\eta_v(\omega_c, p)\;\omega_c, \qquad (7)$$

$$b_q(p) \;=\; \frac{p_{atm}\sqrt{R_g T_{atm}}}{V_d + V_m}\beta\left(\frac{p}{p_{atm}}\right), \qquad (8)$$

$$c_q(\omega_c, p) \;=\; \frac{V_d}{4R_g T_{atm}}\eta_v(\omega_c, p), \qquad (9)$$

where V_d, V_m are resp. the displacement and the manifold volume, p_{atm}, T_{atm} are the ambient pressure and air temperature, R_g is the air gas constant, $\eta_v(\omega_c, p)$ is the volumetric efficiency in cylinder intakes and

$$\beta(y) = \begin{cases} \sqrt{\frac{2\gamma_c}{\gamma_c - 1}\left[y^{\frac{2}{\gamma_c}} - y^{\frac{\gamma_c+1}{\gamma_c}}\right]} & \text{if } y \geq \left(\frac{2}{\gamma_c+1}\right)^{\frac{\gamma_c}{\gamma_c-1}} \\ \sqrt{\gamma_c}\left(\frac{2}{\gamma_c+1}\right)^{\frac{\gamma_c+1}{2(\gamma_c-1)}} & \text{if } y < \left(\frac{2}{\gamma_c+1}\right)^{\frac{\gamma_c}{\gamma_c-1}} \end{cases}$$

with $\gamma_c = \frac{c_p}{c_v}$. While in the past the throttle valve was directly connected to the gas pedal, in modern cars equipped with drive–by–wire, the throttle valve is controlled by the engine control unit.

Engine hybrid model. The overall model of torque generation for a N–cylinder engine is the combination of N FSMs and of N DE systems representing the behavior of each cylinder. The hybrid model of the complete engine is obtained by adding to the torque generation model the power-train CT dynamics (1), (2) and air CT dynamics (5) which are shared among all cylinders. In this paper we focus on the most relevant case of a 4-cylinder engine, whose model is referred to as $\mathcal{M}_{4cyl}$. $\mathcal{M}_{4cyl}$ inputs are: the throttle angle α, a CT signal in the class $\mathcal{A}_{4cyl}$ of functions $\mathbb{R}^+ \to [0, \frac{\pi}{2}]$; the mix composition factors $\gamma =$

$[\gamma_1, \gamma_2, \gamma_3, \gamma_4]^T$, a DE signal in the class $\mathcal{G}_{4cyl}$ of functions $\mathbb{N} \to (\{0\} \cup [\gamma_{\min}, \gamma_{\max}])^4$, with γ_i synchronized with the i–th DE model; the spark modulation factors $\mathbf{r} = [r_1, r_2, r_3, r_4]^T$, a DE signal in the class $\mathcal{R}_{4cyl}$ of functions $\mathbb{N} \to [r_{\min}, 1]^4$, with r_i synchronized with the i–th DE model. $\mathcal{M}_{4cyl}$ states are: $\mathcal{S} = (\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4)$, $\mathbf{z} = [z_1, z_2, z_3, z_4]^T$ and (ζ, ϕ_c, p). The states of $\mathcal{S}$ are constrained by the mechanics of the four-stroke engine to the following set: (H,I,C,E), (I,C,E,H), (C,E,H,I), (E,H,I,C). Without loss of generality, let $\phi_{co_1} = \phi_{co_3} = \pi$, $\phi_{co_2} = \phi_{co_4} = 0$.

2.2 The optimization problem

The objective is to steer the system from a given point, characterized by the torque delivered to the crankshaft $u(0) = u_0$, to a new point with torque value $u_R > u_0$ in minimum time satisfying comfort requirements. The oscillating component of vehicle acceleration and the vehicle jerk have been shown experimentally to be the most important factors in passenger comfort in the FPFT region of operation. The control problem is formulated as follows: *steer the power-train elastic state, keeping the jerk bounded, i.e.,*

$$0 \le j(t) \le j_{\max} \quad (10)$$

to a point such that the application of the new requested value of transmitted torque produces an oscillating acceleration evolution $\tilde{a}(t)$ bounded above by a threshold of perception $\tilde{a}_{th} > 0$.

Introduce $v = u - u_R$ and the transformed state

$$\begin{bmatrix} x' \\ x \end{bmatrix} = N_{\zeta x} \, (\zeta + A^{-1} b \, u_R) \, , \quad (11)$$

with $x' \in \mathbb{R}$, $x \in \mathbb{R}^2$ and $N_{\zeta x} \in \mathbb{R}^{3\times3}$ such that dynamics (1),(3),(4) are rewritten as follows

$$\begin{bmatrix} \dot{x}' \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & A_x \end{bmatrix} \begin{bmatrix} x' \\ x \end{bmatrix} + \begin{bmatrix} b_{x'} \\ b_x \end{bmatrix} v \quad (12)$$

$$\begin{bmatrix} a \\ j \end{bmatrix} = \begin{bmatrix} c_{x'} & c_x \\ c_{x'}\lambda_1 & c_x A_x \end{bmatrix} \begin{bmatrix} x' \\ x \end{bmatrix} + \begin{bmatrix} -cA^{-1} b \, u_R \\ (cb) v \end{bmatrix} \quad (13)$$

with $A_x = \left[\begin{smallmatrix} \lambda & -\mu \\ \mu & \lambda \end{smallmatrix}\right]$, $b_x = N_{\zeta x} b$ and $[c_{x'} \; c_x] = c \; N_{\zeta x}^{-1}$. Under constant control v in (12), the oscillating component of the acceleration can then be expressed as

$$\tilde{a} = c_x \, x \, . \quad (14)$$

Without loss of generality, let $N_{\zeta x}$ in (11) be chosen such that $\|c_x\| = \tilde{a}_{th}$. Consider the manifold

$$\mathcal{C}_x = \left\{ \begin{bmatrix} x' \\ x \end{bmatrix} \in \mathbb{R}^3 | x' \in \mathbb{R}, x = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}, \theta \in [0, 2\pi) \right\} \quad (15)$$

Since the norm of $x(t)$ in (12) decreases over time when $v(t) = 0$, if at some time $\bar{t}$, the state has been driven to $x(\bar{t}) \in \mathcal{C}_x$, signal $v(t) = 0$, i.e. $u(t) = u_R$, keeps the trajectory inside $\mathcal{C}_x$ with acceleration $\tilde{a}(t)$ bounded above by the threshold value $\tilde{a}_{th}$, for all $t \ge \bar{t}$. Let

$$\mathcal{C}_\zeta = \left\{ \zeta \in \mathbb{R}^3 | \zeta = N_{x\zeta} \begin{bmatrix} x' \\ x \end{bmatrix} - A^{-1} b u_R, \begin{bmatrix} x' \\ x \end{bmatrix} \in \mathcal{C}_x \right\} \quad (16)$$

with $N_{x\zeta} = N_{\zeta x}^{-1}$ and $\mathcal{C}_x$ as in (15). The optimal control problem is:

Problem 1 *Given the engine hybrid model $\mathcal{M}_{4cyl}$, for any ζ_0 not inside the region delimited by $\mathcal{C}_\zeta$ as in (16),*

$$\min_{\substack{\alpha \in \mathcal{A}_{4cyl} \\ \gamma \in \mathcal{G}_{4cyl} \\ \mathbf{r} \in \mathcal{R}_{4cyl}}} \int_0^T dt$$

$$\textit{subject to}: \begin{cases} \text{Dynamics of Hybrid Model } \mathcal{M}_{4cyl} \text{ with} \\ \mathcal{S}^o = (H, I, C, E), \\ \mathbf{z}(0) = [0, Gq_0, Gq_0, Gq_0]^T, \\ \zeta(0) = \zeta_0, \;\; \phi_c(0) = 0, \;\; p(0) = p_0, \\ \zeta(T) \in \mathcal{C}_\zeta, \\ 0 \le j(t) \le j_{\max} \text{ for all } t \in [0, T], \end{cases}$$

with $(\zeta_0^T, 0)^T$ the power-train dynamics state value at the initial time and p_0, $q_0 = c(\omega_c(0), p_0)p_0$ the corresponding manifold pressure and mass of air.

3 Relaxed continuous-time problem

In this section a relaxation of Problem 1 to the continuous-time domain is defined and solved. The relaxed problem is concerned with comfort requirements, as specified in (10), for minimum time optimal trajectories of system (1) to manifold (16), assuming no constraint on torque signal. The solution is easily obtained by rewriting dynamics (1) with input j in place of u. From (1) and (4),

$$\dot{\zeta} = (I - b(cb)^{-1}c)A \, \zeta + (cb)^{-1} b \, j \, , \quad (17)$$

$$u = -(cb)^{-1}(cA)\zeta + (cb)^{-1} j \, . \quad (18)$$

System (17)-(18)corresponds to the inverse of system (1)-(4). For $a = c\zeta = 0$, hence for $j = 0$ (17) represents the zero-dynamics of system (1)-(4) which has a pole in the origin and two poles equal to the zeros of $c(sI - A)^{-1}b$. Since, c is proportional to the third row of A and the third entry in b is 0, pole $s = 0$ of (17) has multiplicity two. Let η denote the third real pole. The relaxed problem is easily solved in the transformed space $\xi = N_{\zeta\xi}(\zeta + A^{-1} b \, u_R)$ of the natural modes, where dynamics (17) is written as

$$\dot{\xi} = A_\xi \xi + b_\xi j \text{ with } A_\xi = \left[\begin{smallmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \eta \end{smallmatrix}\right]. \quad (19)$$

Let $\mathcal{J}$ be the class of functions $\mathbb{R}^+ \to [0, j_{\max}]$ and

$$\mathcal{C}_\xi = \left\{ \xi \in \mathbb{R}^3 | \xi = N_{x\xi} \begin{bmatrix} x' \\ x \end{bmatrix} \text{ with } \begin{bmatrix} x' \\ x \end{bmatrix} \in \mathcal{C}_x \right\} \quad (20)$$

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with $N_{x\xi} = N_{\zeta\xi} N_{x\zeta}$ and $\mathcal{C}_x$ as in (15). Define the relaxed optimal control problem

Problem 2 *For any ξ_0 not inside $\mathcal{C}_\xi$ as in (20)*

$$\min_{j \in \mathcal{J}} \int_0^T dt$$

$$subject\ to: \begin{cases} \dot{\xi} = A_\xi\ \xi + b_\xi\ j \\ \xi(0) = \xi_0 \\ \xi(T) \in \mathcal{C}_\xi \end{cases}$$

Proposition 1 *Let $\hat{j}(t)$, with $t \in [0,T]$, be an optimal solution to Problem 2 and let $\hat{\xi}(t)$ be the corresponding minimum time trajectory to manifold $\mathcal{C}_\xi$. Then, $\hat{j}(t) \in \{0, j_{\max}\}$ for all $t \in [0,T]$ and the final control value is*

$$\hat{j}(T) = \begin{cases} 0 & \text{if } b_\xi^T Q \xi(T) > 0 \\ j_{\max} & \text{if } b_\xi^T Q \xi(T) < 0 \end{cases}, \qquad (21)$$

where $Q = N_2^T N_2 + N_3^T N_3$, with N_2, N_3 the second and third row of $N_{x\xi}^{-1}$, respectively. Moreover there exist at most two times $t_1, t_2 \in [0,T]$ where a switching of $\hat{j}(t)$ takes place. Times t_1, t_2 are the solution to

$$[b_\xi^1 + b_\xi^2 (T - t_i),\ b_\xi^2,\ b_\xi^3 e^{\eta(T-t_i)}]\ Q\xi(T)\ . \qquad (22)$$

The proof of Proposition 1, omitted here for lack of space, is based on Pontryagin's Principle [6].
Integrating backwards (19), from a given $\xi(T) = \bar{\xi} \in \mathcal{C}_\xi$, with final control chosen according to (21) and switching instants obtained from (22), minimum time trajectories to $\mathcal{C}_\xi$ in the ξ state space are obtained. If (22) has no solutions in $(0,T)$ then $\hat{j}$ as in (21) for $t \in [0,T]$, is optimal. Otherwise, in the backward integration of (19) $\hat{j}$ switches at time t_1, and, if two solutions to (22) in $(0,T)$ exist, at time t_2. Points $\xi(t_1)$, $\xi(t_2)$ belong to 2–dimensional surfaces which define a partition $S_\xi^0 \cup S_\xi^{j_{\max}}$ of the set of points controllable to $\mathcal{C}_\xi$ in time lower than or equal to T, so that the solution to Problem 2 is expressed as:

$$\hat{j}(\xi) = \begin{cases} 0 & \text{if } \xi \in S_\xi^0 \\ j_{\max} & \text{if } \xi \in S_\xi^{j_{\max}} \end{cases}. \qquad (23)$$

From (18) and (23), mapping $S_\xi^0 \cup S_\xi^{j_{\max}}$ to the physical state space the minimum time torque is written as:

$$\hat{u}(\zeta) = \begin{cases} -\frac{cA}{cb}\zeta & \text{if } \zeta \in S_\zeta^0 \\ -\frac{cA}{cb}\zeta + \frac{j_{\max}}{cb} & \text{if } \zeta \in S_\zeta^{j_{\max}} \end{cases}. \qquad (24)$$

4 Hybrid system control scheme

In this Section we propose feasible feedback laws for throttle angle $\alpha(t)$, mix composition $\gamma(k)$ and spark modulation $\mathbf{r}(k)$ such that the torque u generated by

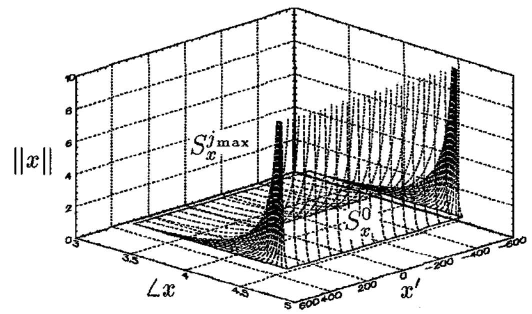

Figure 2: Partition $S_x^0, S_x^{j_{\max}}$, defining the minimum time jerk, in the (x', x) space represented in cylindric coordinate. Manifold $\mathcal{C}_x$ is the plane $\|x\| = 1$.

the hybrid model $\mathcal{M}_{4cyl}$ tracks (24). The main difficulties are:

(a) the generated torque is limited to $\{0\} \cup [r_{\min}\gamma_{\min} q, \gamma_{\max} q]$, where the air mass q is subject to manifold pressure dynamics;

(b) torque generation is synchronized with the power-train dynamics;

(c) there is a delay between the time at which γ and $\mathbf{r}$ are set and the time at which the torque is generated. As discussed in Section 2, the spark modulation $r_m(k)$, applied at m–th cylinder which enters the C state at time t_k, affects torque $u(k+1)$ generated at time t_{k+1}, while the mass $q(k)$ of air loaded and the mix composition $\gamma_\ell(k)$ of the ℓ–th cylinder, which enters the H state, affect torque $u(k+2)$. Hence, feedbacks for r_m and γ_ℓ are expressed in terms of a prediction of the point reached by ζ respectively at times t_{k+1} and t_{k+2}, obtained by integration of (1). The torque generation process can be viewed as a MIMO system composed of two interconnected sub–systems, as depicted in Figure 3. The air mass evolution $q(t)$ is subject to manifold pressure dynamics (5), which is controlled by throttle angle α and depends on the crankshaft revolution speed ω_c. The torque to the crankshaft is provided by a DE system, whose evolution depends on $q(k)$, with two dimensional state and inputs $\gamma_\ell(k)$, $r_m(k)$. The coupling between the DE system and pressure dynamics is given by the power-train dynamics (1), with input u and output ω_c. While the impact on the DE system of signal q is not negligible, the power-train impact on air dynamics is weak enough to allow a decentralized control (see [7]).

4.1 Pressure Dynamics Decentralized Control

The reference evolution $\hat{q}$ for the air mass is obtained considering a rigid model for the power-train and solving the minimum time problem assuming $r_m = \gamma_\ell = 1$. Let τ_{drv},M_v resp. denote the driveline transmission ratio and the vehicle equivalent mass. In the power-train rigid model, the force acting on the vehicle is $\frac{u}{R_w \tau_{drv}}$ and $j = (M_v R_w \tau_{drv})^{-1} \frac{du}{dt}$. Let $q_R = u_R / G$. The mini-

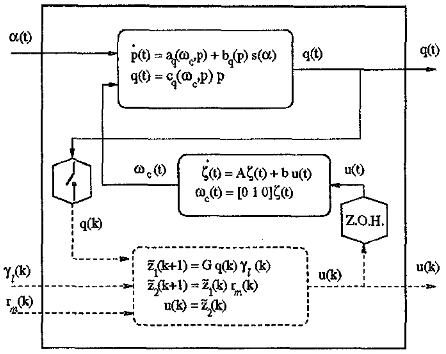

Figure 3: Torque generation model.

mum time air mass evolution is

$$\hat{q}(t)=\begin{cases} q(0)+\frac{j_{\max}M_v R_w \tau_{drv}}{G}t & \text{if } t<\frac{G(q_R-q(0))}{j_{\max}M_v R_w \tau_{drv}} \\ q_R & \text{if } t\geq \frac{G(q_R-q(0))}{j_{\max}M_v R_w \tau_{drv}} \end{cases} \quad (25)$$

A decentralized scheme for perfect tracking of reference trajectory (25) is presented (e.g. [4]). Rewrite (5) as

$$\dot{p}=a_q(\omega_c,p)\ p+w\ . \quad (26)$$

Consider $\hat{q}(t)$ as in (25) and introduce $\sigma(t)=q(t)-\hat{q}(t)$. From (25), $\sigma(0)=0$. The *equivalent control* (see [8])

$$w_{eq}=-a_q p+\left(c_q+\frac{\partial c_q}{\partial p}p\right)^{-1}\left(\dot{\hat{q}}-\frac{\partial c_q}{\partial \omega_c}p\ \dot{\omega}_c\right).$$

ensures $\sigma(t)=0$ if $\sigma(0)=0$. While $\dot{\hat{q}}$ is known, $\dot{\omega}_c$ is not available. However, power-train weak coupling allows us to employ a robust decentralized scheme where nonlinearities are compensated. Given a value $\bar{\eta}_v$, $\bar{\omega}_c$ of volumetric efficiency and crankshaft speed, introduce $a_{q1}=-\frac{V_d}{4\pi V_c}\bar{\eta}_v\bar{\omega}_c$, $c_{q1}=\frac{V_d}{4R_g T_{atm}}\bar{\eta}_v$, and $\tilde{a}_q(\omega_c,p)=a_q(\omega_c,p)-a_{q1}$, $\tilde{c}_q(\omega_c,p)=c_q(\omega_c,p)-c_{q1}$. If $\tilde{a}_q=\tilde{c}_q=0$, for any $W>0$, the variable structure control

$$w_{vsc}\ =\ -a_{q1}\ p+\frac{\dot{\hat{q}}}{c_{q1}}-\frac{W}{c_{q1}}\ \text{sign}\ (\sigma) \quad (27)$$

guarantees a sliding regime along the manifold $\sigma=0$, during which perfect tracking of $\hat{q}(t)$ is achieved.

Proposition 2 *Assume that $p,\omega_c,\dot{\omega}_c,\dot{\hat{q}}$ satisfy $p\in[p_1,p_{atm}]$, $\omega_c\in[\omega_1,\omega_2]$, $|\dot{\omega}_c|\leq\dot{\omega}_{\max}$, $|\dot{\hat{q}}|\leq\dot{\hat{q}}_{\max}$. Choose a $\bar{\eta}_v$, $\bar{\omega}$ and let $\tilde{C}_1,\tilde{C}_p,\tilde{C}_{\omega_c},\tilde{A}_q$ be upper bounds on $\left|\frac{\tilde{c}_q}{c_{q1}}\right|$, $\left|\frac{\partial\tilde{c}_q}{\partial p}\right|$, $\left|\frac{\partial\tilde{c}_q}{\partial\omega_c}\right|$, $|\tilde{a}_q|$, respectively. Given $\epsilon>0$, if $\tilde{C}_1+\tilde{C}_p\frac{p_{atm}}{c_{q1}}<1$ the VSC (27) with*

$$\begin{aligned} W\ &=\ \frac{(\tilde{C}_1+\tilde{C}_p\ \frac{p_{atm}}{c_{q1}})\dot{\hat{q}}_{\max}+(\tilde{C}_{\omega_c}\ p_{atm})\dot{\omega}_{\max}}{1-\tilde{C}-\tilde{C}_p\ \frac{p_{atm}}{c_{q1}}} \\ &\quad +c_{q1}\tilde{A}_q p_{atm}+(1-\tilde{C}-\tilde{C}_p\ \frac{p_{atm}}{c_{q1}})^{-1}\epsilon \end{aligned} \quad (28)$$

guarantees a sliding motion on $\sigma=0$, during which perfect tracking of $\hat{q}(t)$ as in (25) is achieved, robustly w.r.t. nonlinearities and power-train evolution.

To reduce the undesired chattering, typical in VSC, the sign function is replaced by $\frac{\sigma}{\delta+|\sigma|}$, where the smoothing parameter $\delta>0$ is properly tuned so to maintain satisfactory tracking. The throttle angle feedback is

$$\alpha=s^{-1}\left(\frac{1}{b_q(p)}\left(-a_{q1}\ p+\frac{\dot{\hat{q}}}{c_{q1}}-\frac{W}{c_{q1}}\ \frac{\sigma}{\delta+|\sigma|}\right)\right) \quad (29)$$

with $\sigma(t)=c_q(\omega_c,p)\ p-\hat{q}(t)$.

4.2 Torque Feedback Control

A feedback in terms of $\gamma(k)$ and $\mathbf{r}(k)$ according to law (24) is presented in the sequel. Assuming that crankshaft speed does not change significantly, i.e. $t_{k+1}-t_k\approx\tau_k=\pi/\omega_c(t_k)$, since $u(t)=u(k)$ for $t\in[t_k,t_{k+1})$, dynamics (1),(3) are discretized as follows

$$\begin{aligned} \zeta(k+1)\ &=\ \hat{A}\zeta(k)+\hat{b}u(k) &(30)\\ a(k)\ &=\ c\ \zeta(k) &(31)\end{aligned}$$

where $\hat{A}=e^{A\tau_k}$, $\hat{b}=(\hat{A}-I)A^{-1}b$. The jerk mean value in the time interval $[t_k,t_{k+1}]$ can be expressed as:

$$\frac{\int_{t_k}^{t_{k+1}} jdt}{t_{k+1}-t_k}\approx\frac{a(k+1)-a(k)}{\tau_k}=\frac{c(\hat{A}-I)}{\tau_k}\zeta(k)+\frac{c\hat{b}}{\tau_k}u(k).$$

Hence, the torque law $\hat{u}_d(\zeta(k))$ with

$$\hat{u}_d(\zeta)\ =\ (c\hat{b})^{-1}c(\hat{A}-I)\zeta+(c\hat{b})^{-1}\tau_k\ \hat{j}(\zeta)\ , \quad (32)$$

where $\hat{j}(\zeta)$ is chosen according to (23), that is

$$\hat{j}(\zeta)\ =\ \begin{cases} 0 & \text{if } \zeta\in S_\zeta^0 \\ j_{\max} & \text{if } \zeta\in S_\zeta^{j_{\max}} \end{cases}, \quad (33)$$

produces a jerk with mean value $\hat{j}(\zeta)$. However, the jerk exhibits a ripple on its average value due to the fact that, being $u(t)$ piecewise–constant, between two samples the natural modes of dynamics (1) evolve. Due to this ripple, the jerk exceeds interval $[0,j_{\max}]$, and a more conservative feedback than (32) has to be devised.

Proposition 3 *Define*

$$\begin{aligned} \hat{u}_c(\zeta)&=(cb)^{-1}(\hat{j}(\zeta)-cA\zeta)\\ \hat{u}_n(\zeta)&=(cA\hat{b}+cb)^{-1}(\hat{j}(\zeta)-cA\hat{A}\zeta)\\ \hat{u}_m(\zeta)&=(c(I+\tau_k A)b)^{-1}(j_{\max}-c(I+\tau_k A)A\zeta),\end{aligned}$$

with $\hat{j}(\zeta)$ as in (33). Let $j_d(t)$, $j_c(t)$, $j_n(t)$ and $j_m(t)$ denote the jerk profiles under feedbacks $\hat{u}_d(\zeta(k))$, $\hat{u}_c(\zeta(k))$, $\hat{u}_n(\zeta(k))$ and $\hat{u}_m(\zeta(k))$ respectively, and assume that $\frac{d^2j(t)}{dt^2}\leq 0$, $\forall t\in[t_k,\ t_{k+1}]$. The feedback

$$\hat{u}(k)=\begin{cases}\max(\hat{u}_c(\zeta),\hat{u}_n(\zeta)) & \textit{if } \zeta(k)\in S_\zeta^0\\ \begin{cases}\hat{u}_n(\zeta) \text{ if } j_d(t_{k+1})>j_{\max}\wedge\frac{dj_n}{dt}(t_{k+1})\geq 0\\ \hat{u}_c(\zeta) \text{ if } j_d(t_k)>j_{\max}\wedge\frac{dj_c}{dt}(t_k)\leq 0\\ \hat{u}_m(\zeta) \text{ otherwise}\end{cases} & \textit{if } \zeta(k)\in S_\zeta^{j_{\max}}\end{cases} \quad (34)$$

produces a jerk profile $j(t)\in[0,j_{\max}]$.

Feedback laws for the spark modulation $r_m(k)$ and mix composition factor $\gamma_\ell(k)$ which generate the torque $\hat{u}(k)$ as in (34) are reported below.

$$
\begin{array}{ll}
\zeta_k = \zeta(t_k), \quad q_k = q(t_k) & \text{(power-train state and air mass)} \\
\tau_k = \pi/[0\ 1\ 0]\,\zeta_k,\ \hat{A} = e^{A\,\tau_k},\ \hat{b} = (\hat{A} - I)A^{-1}b & \text{(discrete model)} \\
\zeta_{k+1} = \hat{A}\zeta(k) + \hat{b}\, z_2^c(k) & \text{(future power-train state)} \\
\text{if } \|N_{\zeta x}(\zeta_{k+1} + A^{-1}bu_R)\| > \hat{\rho} \text{ then} & \text{(exit condition: state } \zeta \text{ in } \mathcal{C}_\zeta) \\
\quad \hat{u}_{k+1} = \hat{u}(k+1) & \text{(as in (34) evaluated at } \zeta_{k+1}) \\
\text{else} & \\
\quad \hat{u}_{k+1} = u_R & \text{(target torque } u_R \text{ applied)} \\
\text{endif} & \\
r = \hat{u}_{k+1}/z_1^c(k) & \\
\text{if } r < r_{\min} \text{ then } r = r_{\min} \text{ elseif } r > 1 \text{ then } r = r_{\min} \text{ endif} & \\
r_m(k) = r & \text{(spark advance control)} \\
\zeta_{k+2} = \hat{A}\zeta_{k+1} + \hat{b}\, r_k\, z_1^c(k) & \text{(next power-train state)} \\
\text{if } \zeta_{k+2} \in S_\zeta^0 \text{ then} & \\
\quad \hat{j} = 0 & \\
\text{endif} & \\
\text{if } \|N_{\zeta x}(\zeta_{k+2} + A^{-1}bu_R)\| > \hat{\rho} \text{ then} & \text{(exit condition: state } \zeta \text{ in } \mathcal{C}_\zeta) \\
\quad \hat{u}_{k+2} = \hat{u}(k+2) & \text{(as in (34) evaluated at } \zeta_{k+2}) \\
\text{else} & \\
\quad \hat{u}_{k+2} = u_R & \text{(target torque } u_R \text{ applied)} \\
\text{endif} & \\
\gamma = \hat{u}_{k+2}/(Gq_k) & \\
\text{if } \gamma < \gamma_{\min} \text{ then } \gamma = \gamma_{\min} \text{ elseif } \gamma > \gamma_{\max} \text{ then } \gamma = \gamma_{\max} \text{ endif} & \\
\gamma_\ell(k) = \gamma & \text{(mix composition control)} \\
z_1^c(k+1) = G\, q_k\, \gamma & \text{(next potential torque)} \\
z_2^c(k+1) = z_1^c(k)\, r & \text{(next predicted torque)}
\end{array}
\qquad (35)
$$

5 Simulation Results

The performance of the our hybrid control approach has been evaluated in a number of simulations. $\mathcal{M}_{4cyl}$, with feedbacks α as in (29) and γ_ℓ, r_m as in (35), has been captured in the Xmath/SystemBuild environment (by Integrated Systems Inc.). Initial conditions $\zeta(0)$ in the set $Z_0 = \{\zeta_0 = -A^{-1}b\ u_0,\ u_0 = 20, 21, \ldots, 45\text{Nm}\}$, corresponding to power-train equilibrium points for $u = u_0$, were considered along with a target value $u_R = 100$Nm. The performances of the proposed control are evaluated by comparing, for all $\zeta_0 \in Z_0$, the optimal solution $T_{opt}^{CT}(\zeta_0)$ to the relaxed Problem 2 and the time $T^{\mathcal{M}_{4cyl}}(\zeta_0)$ needed to steer ζ_0 to $\mathcal{C}_\zeta$ in model $\mathcal{M}_{4cyl}$ under the proposed control. In fact, for the optimal solution $T^{\mathcal{M}_{4cyl}}(\zeta_0)$ to Problem 1, we have

$$T_{opt}^{CT}(\zeta_0) \leq T_{opt}^{\mathcal{M}_{4cyl}}(\zeta_0) \leq T^{\mathcal{M}_{4cyl}}(\zeta_0).$$

As expected for larger u_0, i.e. higher crankshaft speeds, $T^{\mathcal{M}_{4cyl}}(\zeta_0)$ is closer to $T_{opt}^{CT}(\zeta_0)$ and, hence, to $T_{opt}^{\mathcal{M}_{4cyl}}(\zeta_0)$ (see Figure 4).

6 Conclusions and future work

In this paper, we presented a novel approach to engine control for the Fast Positive Force Tracking problem, based on a hybrid model of the torque generation and of the power-train dynamics in a four-stroke engine. A control problem on this hybrid system is defined and solved using a sequence of approximations. The properties of the control law so obtained have been characterized, thus offering better confidence on the quality of the results with respect to commonly used heuristic approaches. In addition, since the control law is closed loop, expensive tuning processes can be avoided yielding a commercially appealing solution.

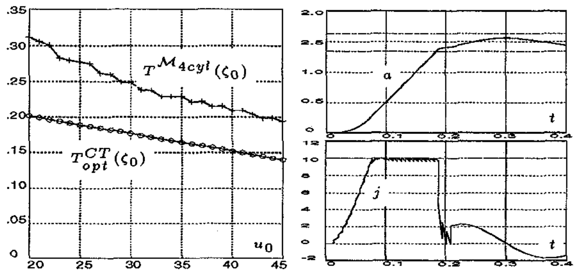

Figure 4: Relaxed optimal solution $T_{opt}^{CT}(\zeta_0)$ and hybrid solution $T^{\mathcal{M}_{4cyl}}(\zeta_0)$ for $\zeta_0 \in Z_0$, vs. u_0. (left); acceleration and jerk for $u_0 = 40$Nm (right).

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