Homework 5

EE 290n - Advanced Topics in Systems Theory

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1. This problem considers parallel and concurrent scheduling of the synchronous dataflow model in figure 1. Assume that actors a and c have execution times of 1 and actor b has an execution time of 4.

   (a) Write the balance equations for this model in matrix form.

   (b) Find the least positive integer solution to the balance equations.

   (c) An **acyclic precedence graph** (**APG**) is a graph that shows the dependencies of actor firings. Specifically, it contains one node for each firing of an actor, and a directed arc from one firing to the next if the first firing must occur before the second. Give the APG for the firings represented by your solution in (b). You should assume that each firing of an actor depends on previous firings of the actor (so that the actor can have state).

   (d) The **latency** between actor a and actor c is defined as the maximum time between the start of a firing of a and the start of a firing of actor c that depends directly on that firing of a. Find a single processor, non-preemptive schedule that minimizes this latency. What is the latency?

   (e) Suppose that actors a and c represent real-time interactions via sensors and actuators and that they must execute at regular time intervals. That is, actor a must fire every τ time units, like clockwork. What is the minimum value for τ given the schedule in the previous part? Given this value, what is the processor utilization?

   (f) Consider a single-processor schedule that allows preemption. Is a smaller value for τ achievable? What is the minimum? What is the processor utilization? Assume the overhead associated with a preempting an execution is zero.

   (g) The **makespan** of a set of firings is the time it takes to complete the entire set. Give a two-processor schedule that minimizes the makespan for the set of firings in your solution to part (b). If you assume no overlap between executions of successive iterations, what is the processor utilization?

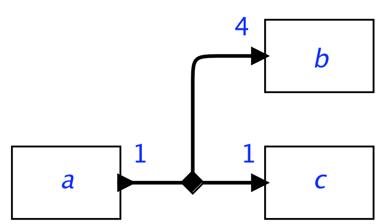


Figure 1: A synchronous dataflow model with production and consumption rates shown.

(h) Find the optimal N-processor schedule that allows overlapped execution of successive iterations (pipelining). What is N? What is the time τ between firings of a? What is the processor utilization? Assume the cost of interprocessor communication is zero.

Solution.

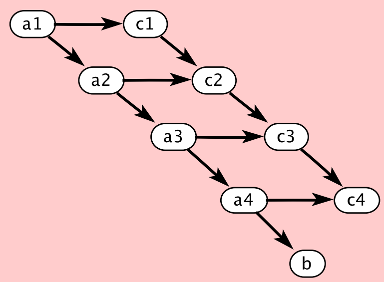
(a)

$$\begin{bmatrix} 1 & -4 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 4 \end{bmatrix}$$

(c) The APG is as follows:



(d) A minimum latency schedule is $(a_1, c_1, a_2, c_2, a_3, c_3, a_4, c_4, b)$. The latency is one time unit.

(e) The minimum value is $\tau = 6$ time units. With this value, it takes $6 \times 4 = 24$ time units to complete one iteration, and the total computation time is 12 time units, so the processor utilization is $1/2$.

(f) With preemption, a feasible schedule is $(a_1, c_1, b, a_2, c_2, b, a_3, c_3, b, a_4, c_4, b)$, where the four occurrences of b represent one execution that is preempted three times. We now see that a executes every $\tau = 3$ time units, and that the processor utilization is 100%. There is also no increase in latency from a to c.

(g) An optimal schedule executes (a_1, a_2, a_3, a_4, b) on the first processor, and (c_1, c_2, c_3, c_4) on the second. The makespan is 8 time units, which is the total computation in the critical path of the APG. The processor utilization is 75%.

(h) With $N = 3$ processors, we can devote one processor to each actor. Each processor simply firing the actor repeatedly. The processor executing c begins executing after 1 time unit. The processor executing b begins executing after 4 time units. The time between firings of a is $\tau = 1$. The processor utilization is 100%.

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2. This problem explores the use of the SDF model of computation together with modal models to improve expressiveness. In particular, you are to implement a simple run-length coder us-

ing no director other than SDF, leveraging modal models with state refinements. Specifically, given an input sequence, such as

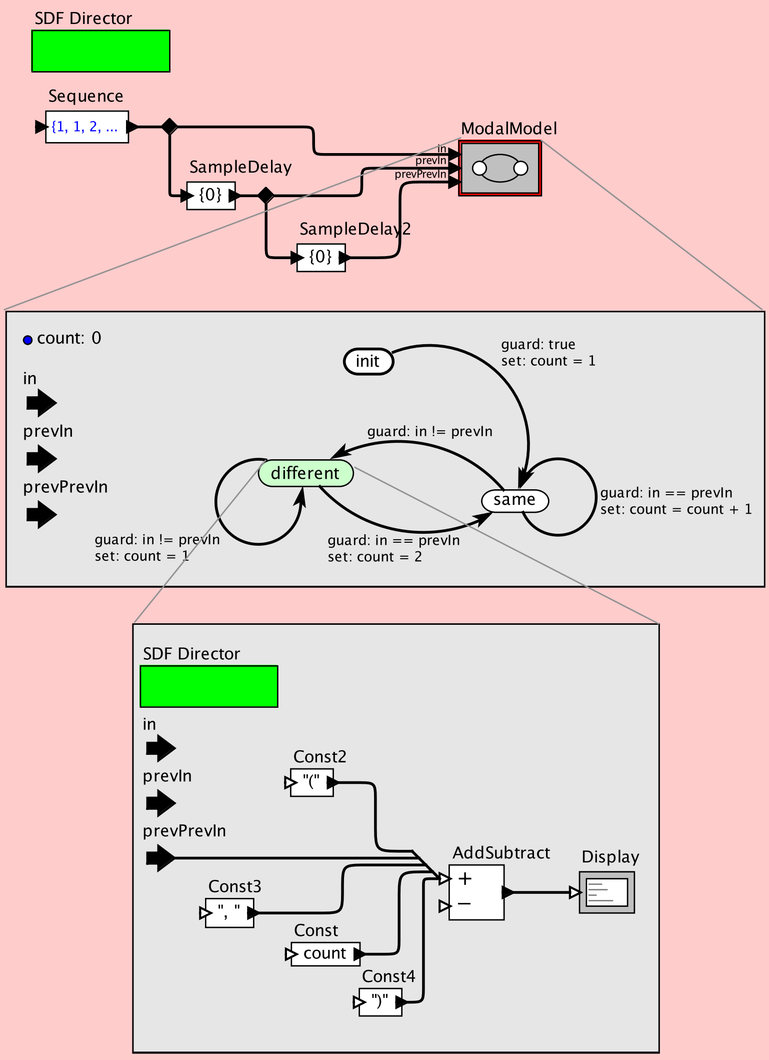
$$(1,1,2,3,3,3,3,4,4,4)$$

you are to display a sequence of pairs (i,n), where i is a number from the input sequence and n is the number of times that number is repeated consecutively. For the above sequence, your output should be

$$((1,2),(2,1),(3,4),(4,3)).$$

Note that this pattern arises commonly in many coding applications, including image and video coding.

Solution.



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