Exercises

1. Suppose V is some set and $S = V^{**}$ is the set of finite and infinite sequences of elements of V. This exercise explores some of the properties of the CPO S^n with the pointwise prefix order, for some non-negative integer n. These properties are useful for understanding firing rules.

   (a) Show that any two elements $a, b \in S^n$ that have an upper bound have a least upper bound.

   (b) Let $U \subset S^n$ be such that no two distinct elements of U are joinable. Prove that for all $s \in S^n$ there is at most one $u \in U$ such that $u \sqsubseteq s$.

   (c) Given $s \in S^n$, suppose that $Q(s) \subset S^n$ is a joinable set where for all $q \in Q(s)$, $q \sqsubseteq s$. Then show that there is an s' such that $s = (\bigvee Q(s)).s'$.

2. Consider the model shown in Figure 3.1. Assume that data types are all $V = \{0, 1\}$. Assume f is a dataflow actor that implements an identity function and that Const is an actor that produces an infinite sequence $(0, 0, 0, \cdots)$. Obviously, the overall output of this model should be this same infinite sequence. The box labeled g indicates a composite actor. Find firing rules and a firing function g for the composite actor to satisfy the generalized firing rules. Note that the composite actor has one input and two outputs.

3. Consider the SDF graph shown in Figure 3.2. In this figure, A, B, and C are actors. Adjacent to each port is the number of tokens consumed or produced by a firing

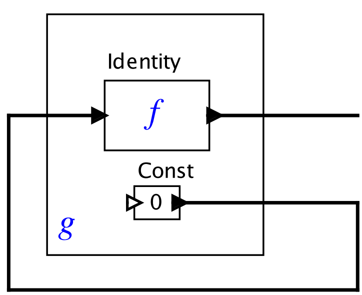

Figure 3.1: A model.

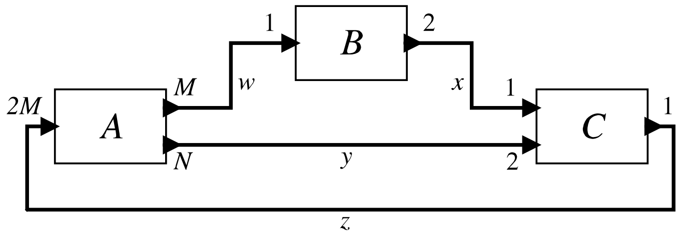

Figure 3.2: A model.

of the actor on that port, where N and M are variables with positive integer values. Assume the variables w, x, y, and z represent the number of initial tokens on the connection where these variables appear in the diagram. These variables have non-negative integer values.

(a) Derive a simple relationship between N and M such that the model is consistent, or show that no positive integer values of N and M yield a consistent model.

(b) Assume that $w = x = y = 0$ and that the model is consistent and find the minimum value of z (as a function N and M) such that the model does not deadlock.

(c) Assume that $z = 0$ and that the model is consistent. Find values for w, x, and y such that the model does not deadlock and $w + x + y$ is minimized.

(d) Assume that $w = x = y = 0$ and z is whatever value you found in part (b). Let b_w, b_x, b_y, and b_z be the buffer sizes for connections w, x, y, and z, respectively. What is the minimum for these bus sizes?

4. This problem considers parallel and concurrent scheduling of the SDF model in figure 3.3. Assume that actors a and c have execution times of 1 and actor b has an execution time of 4 and that the production and consumption rates are as shown in the figure.

(a) Write the balance equations for this model in matrix form.

(b) Find the least positive integer solution to the balance equations.

(c) An **acyclic precedence graph** (**APG**) is a graph that shows the dependencies of actor firings. Specifically, it contains one node for each firing of an actor, and a directed arc from one firing to the next if the first firing must occur before the second. Give the APG for the firings represented by your solution in (b). You should assume that each firing of an actor depends on previous firings of the actor (so that the actor can have state).

(d) The **latency** between actor a and actor c is defined as the maximum time between the start of a firing of a and the start of a firing of actor c that depends directly on that firing of a. Find a single processor, non-preemptive schedule that minimizes this latency. What is the latency?

(e) Suppose that actors a and c represent real-time interactions via sensors and actuators and that they must execute at regular time intervals. That is, actor a must fire every τ time units, like clockwork. What is the minimum value for τ given the schedule in the previous part? Given this value, what is the processor utilization, for a single processor, non-preemptive schedule?

(f) Consider a single-processor schedule that allows preemption. Is a smaller value for τ achievable? What is the minimum? What is the processor utilization? Assume the overhead associated with a preempting an execution is zero.

(g) The **makespan** of a set of firings is the time it takes to complete the entire set. Give a two-processor schedule that minimizes the makespan for the set of firings in your solution to part (b). If you assume no overlap between executions of successive iterations, what is the processor utilization?

5. Consider the dataflow graph shown in Figure 3.4. The production and consumption parameters are shown next to each port, and initial tokens on each connection are shown with dots.

   (a) Write down the balance equations in matrix form.

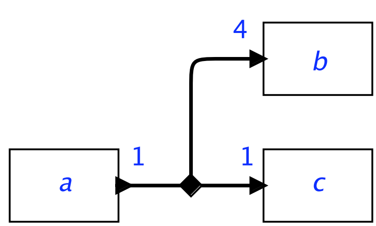

Figure 3.3: An SDF model with production and consumption rates shown.

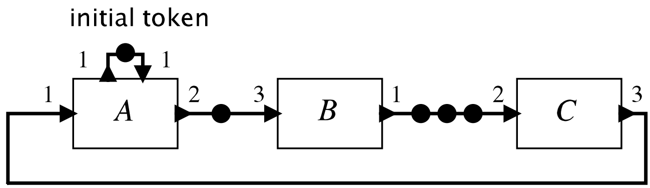

Figure 3.4: A cyclic SDF model.

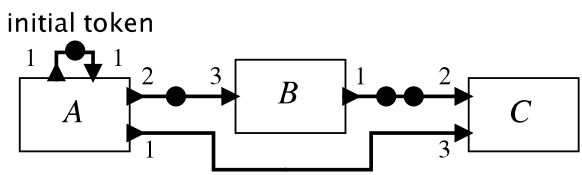

Figure 3.5: A parallelizable SDF model.

(b) Solve the balance equations using the procedure outlined in Section 3.3.1, where you start assuming A executes $q_A = 1$ time and find rational solutions for all other actors. Then find the least positive integer solution. Is the graph consistent? Show your work.

(c) Use symbolic execution to construct an acyclic precedence graph.

(d) Assume that the execution times are all one time unit. Construct the minimum makespan schedule for one iteration of the minimal balanced schedule. What is the makespan? How many processors are required to achieve this makespan?

(e) Define the throughput to be the number of iterations of C per unit time. Would a vectorization factor larger than one improve the throughput?

6. Consider the dataflow model shown in Figure 3.5. The production and consumption parameters are shown next to each port, and initial tokens on each connection are shown with dots.

   (a) Write down the balance equations in matrix form.

   (b) Is the graph consistent? If so, give the least positive integer solution.

(c) Use symbolic execution to construct an acyclic precedence graph.

(d) Assume that the execution times are all one time unit. Construct the minimum makespan schedule for one iteration of the minimal balanced schedule. What is the makespan? How many processors are required to achieve this makespan?