Exercises

1. Given a set A and the CPO $(A^{**}, \sqsubseteq)$, for each of the following functions, state whether it is monotonic, continuous, both, or neither. Assume the domain and codomain of every function is A^{**}. Assume the period operator represents concatenation of sequences.

   (a) The **unit delay** function d given by $\forall\, s \in A^{**}$, $d(s) = (a).s$ where $a \in A$.

   (b) The **trailer function** t given by, $\forall\, s \in A^{**}$,

   $$t(s) = \begin{cases} s.a & \text{if } s \text{ is finite} \\ s & \text{otherwise} \end{cases}$$

   where $a \in A$.

   (c) The **is finite function** f given by, $\forall\, s \in A^{**}$,

   $$f(s) = \begin{cases} (a) & \text{if } s \text{ is finite} \\ (a') & \text{otherwise} \end{cases}$$

   where $a, a' \in A$, $a \neq a'$.

   (d) An alternative **is finite function** f' given by, $\forall\, s \in A^{**}$,

   $$f(s) = \begin{cases} (a) & \text{if } s \text{ is finite} \\ (a,a) & \text{otherwise} \end{cases}$$

   where $a \in A$.

   (e) Let $m\colon (A^{**})^2 \to A^{**}$ be the **fair alternating merge** function, defined as follows. Given two infinite sequences $s_1 = (a_0, a_1, \cdots)$ and $s_2 = (b_0, b_1, \cdots)$ it outputs the infinite sequence $m(s_1, s_2) = (a_0, b_0, a_1, b_1, \cdots)$. That is, it alternates the elements of the sequences. If either or both of the inputs is finite, then it alternates their elements until the shorter of the two runs out of elements, and then it outputs the remaining values from the longer of the two. For example, if $s_1 = (a_0, a_1, \cdots, a_n)$ is finite, but s_2 is infinite, then it produces

   $$m(s_1, s_2) = (a_0, b_0, a_1, b_1, \cdots, a_n, b_n, b_{n+1}, b_{n+2}, \cdots)\ .$$

   If s_2 is also finite, but longer than s_1, then the result will be similar to the above, but finite, ending with the last value of s_2