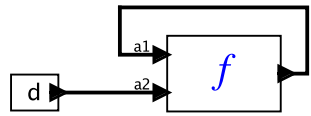
Exercises

1. Suppose $A = \{\perp\} \cup \mathbb{N}$ is a flat CPO with $\perp$ at the bottom and each element of $\mathbb{N}$ immediately above $\perp$. Each element of $\mathbb{N}$ is incomparable with all other elements of $\mathbb{N}$. Consider a function $f: A^2 \to A$ given by, for all $a_1, a_2 \in A$,

$$f(a_1, a_2) = \begin{cases} \perp & \text{if } a_1 = a_2 = \perp \\ a_1 & \text{if } a_1 \in \mathbb{N} \\ a_2 + 1 & \text{otherwise} \end{cases}$$

   (a) Is this function strict?

   (b) Suppose this function is used in the following SR model:



   where $d = 1$ is produced on every tick by the left actor. Find all possible outputs of f. Is there a least fixed point?

   (c) Show that this function is not monotonic.

2. Suppose D is a set of data values and $A = \{\perp, absent\} \cup D$ is a flat CPO with $\perp$ at the bottom. Consider a function $g: A^2 \to A$ given by, for all $a_1, a_2 \in A$,

$$g(a_1, a_2) = \begin{cases} \perp & \text{if } a_1 = a_2 = \perp \\ d & \text{if } a_1 \in D \text{ or } a_2 \in D \\ absent & \text{otherwise} \end{cases}$$

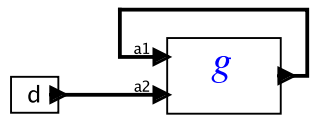
   for some $d \in D$.

   (a) Is this function strict?

   (b) Is this monotonic? continuous?

   (c) Is this function sequential?

   (d) Suppose this function is used in the following SR model:



where $d \in D$ is produced on every tick by the left actor. What is the output of f on each tick? What if d is replaced with *absent*?