1. A solution is shown below:

The parameter for the Scale block is “- stiffness/momentOfInertia”.

2. A solution is shown below:

A simple model of friction subtracts a quantity proportional to speed from the acceleration. The proportionality constant, $frictionCoefficient$, is the parameter of the second Scale actor. A result of running this model is shown below:
3. A solution is shown below:

where the modal model refines to:

where the running state refines to:
This model can be found at http://gigascale.org/concurrency/homework/tuningFork/bouncing.xml. Note that the self transition on the running state needs to have the reset parameter set to true.

A result of running this model is shown below:

Note that the tine bounces off the surface for the first few cycles, and then is free of it.

4. We want to show that for any well-posed initial value ODE problem (i.e. $f$ piecewise continuous and Lipschitz) that

$$\dot{x} = f(x, t), x(0) = a, t \in [0, T]$$

with a unique solution $x(t)$, the forward Euler method with step size $h$,

$$x_{n+1} = x_n + h \cdot f(x_n), x_0 = x(0),$$

(1)
converges. (Note: \( x(t_n) \) is the exact solution at \( t_n \), while \( x_n \) is the numerical solution at \( t_n \).) I.e. we want to show that

\[
\lim_{h \to 0} \max_{0 \leq n \leq T/h} |x(t_n) - x_n| = 0. \tag{3}
\]

proof:

Use a Taylor series expansion for \( x(t_{n+1}) \) based on \( x(t_n) \) for any \( 0 \leq n \leq (T/h - 1) \):

\[
x(t_{n+1}) = x(t_n) + h \cdot f(x(t_n), t_n) + \frac{h^2}{2} \cdot \ddot{x}(\xi_n)
\]

(4)

for some \( \xi_n \in [t_n, t_{n+1}] \). Comparing with the numerical solution (2) and defining \( e_n = x(t_n) - x_n \), we have,

\[
e_{n+1} = e_n + h(f(x(t_n), t_n) - f(x_n, t_n)) + \frac{h^2}{2} \ddot{x}(\xi_n)
\]

(5)

Let \( L \leq \infty \) be the Lipschitz constant, we have,

\[
|f(x(t_n), t_n) - f(x_n, t_n)| \leq L \cdot |x(t_n) - x_n|.
\]

(6)

Let \( C = \max_{\xi \in [0,T]} |\dddot{x}(\xi)| \). (Note that we actually need second order smoothness of \( x(t) \) in order for \( C \) to be finite.) By applying the triangular inequality of norms, and iterating on the sequence index \( n \), we get,

\[
|e_{n+1}| \leq |e_n| + hL|e_n| + \frac{h^2}{2} |\ddot{x}(\xi_n)|
\]

\[
\leq (1 + hL)|e_n| + \frac{Ch^2}{2}
\]

\[
\leq (1 + hL)^2|e_{n-1}| + \frac{Ch^2}{2}(1 + (1 + hL))
\]

\[
\leq (1 + hL)^3|e_{n-2}| + \frac{Ch^2}{2} (1 + (1 + hL) + (1 + hL)^2)
\]

\[
\leq \ldots
\]

\[
\leq (1 + hL)^{n+1}|e_0| + \frac{Ch^2}{2} (1 + (1 + hL) + \ldots + (1 + hL)^n), \text{ (note } e_0 = 0 \text{)}
\]

\[
= \frac{Ch^2}{2} \left( \frac{(1 + hL)^{n+1} - 1}{(1 + hL) - 1} \right)
\]

\[
= \frac{Ch}{2L} ((1 + hL)^{n+1} - 1)
\]

Since \( (1 + hL)^n < e^{nhL} = e^{Ln} \), for any \( h > 0 \), we have that for any \( 0 \leq n \leq T/h \),

\[
|e_n| < \frac{Ch}{2L} (e^{L_{n+1}} - 1) \leq \frac{Ch}{2L} (e^{LT} - 1) \tag{7}
\]

But, since \( \frac{Ch}{2L} (e^{LT} - 1) \) is a constant independent of \( h \), so (3) holds.

QED.