Concurrent Models of Computation

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EECS 219D
Concurrent Models of Computation
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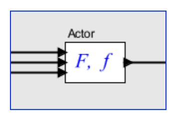
Week 10: Consistency

Recall: Execution Policy for a Dataflow Actor



Suppose $s \in S^n$ is a concatenation of firing rules,

$$s = u_1 . u_2 . u_3 \quad \ldots$$

Then the output of the actor is the concatenation of the results of a sequence of applications of the firing function:

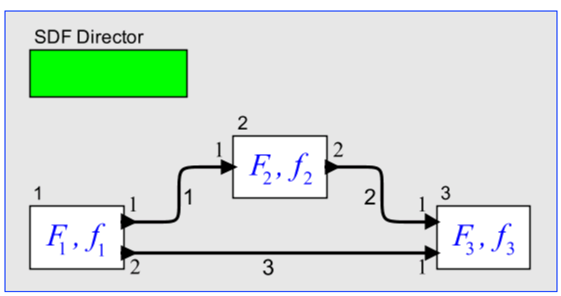
$$F_0(s) = \perp_n$$
$$F_1(s) = (\phi(F_0))(s) = f(u_1)$$
$$F_2(s) = (\phi(F_1))(s) = f(u_1).f(u_2)$$
$$\ldots$$

The problem we address now is *scheduling*: how to choose which actor to fire when there are choices.

Dataflow variants constrain firing rules and trade off expressiveness and analyzability

- Computation graphs [Karp & Miller - 1966]
- Process networks [Kahn - 1974]
- Static dataflow [Dennis - 1974]
- Dynamic dataflow [Arvind, 1981]
- K-bounded loops [Culler, 1986] today
- Synchronous dataflow [Lee & Messerschmitt, 1986]
- Structured dataflow [Kodosky, 1986]
- PGM: Processing Graph Method [Kaplan, 1987]
- Synchronous languages [Lustre, Signal, 1980's]
- Well-behaved dataflow [Gao, 1992]
- Boolean dataflow [Buck and Lee, 1993]
- Multidimensional SDF [Lee, 1993]
- Cyclo-static dataflow [Lauwereins, 1994]
- Integer dataflow [Buck, 1994]
- Bounded dynamic dataflow [Lee and Parks, 1995]
- Heterochronous dataflow [Girault, Lee, & Lee, 1997]
- Parameterized dataflow [Bhattacharya and Bhattacharyya 2001]
- Structured dataflow (again) [Thies et al. 2002]
- …

Recall: Synchronous Dataflow (SDF)



production/consumption matrix

Connector 1

$$\Gamma = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & -1 \\ 2 & 0 & -1 \end{bmatrix} \qquad q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Actor 1

firing vector

balance equations

$$\Gamma q = \vec{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Consistent Models

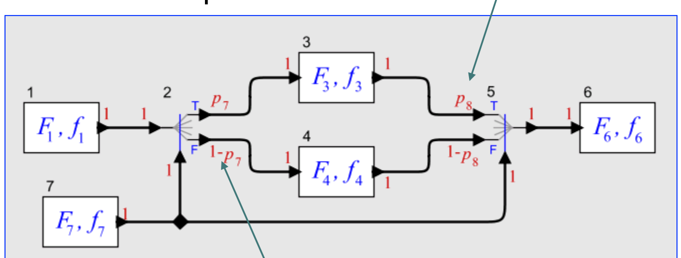
Let a be the number of actors in a connected model. The model is *consistent* if Γ has rank $a - 1$.

If the rank is a, then the balance equations have only a trivial solution (zero firings).

When Γ has rank $a - 1$, then the balance equations always have a non-trivial solution.

Recall: Boolean Dataflow uses Symbolic Production/Consumption Rates

Symbolic consumption rate.



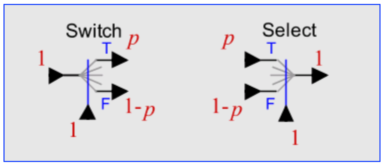
Symbolic production rate.

Imperative equivalent:

```
while (true) {
   x = f1();
   b = f7();
   if (b) {
     y = f3(x);
   } else {
     y = f4(x);
   }
   f6(y);
}
```

Production and consumption rates are given symbolically in terms of the values of the Boolean control signals consumed at the control port.

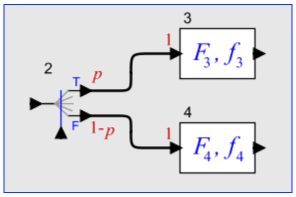
Interpretations of Symbolic Rates



- General interpretation: p is a symbolic placeholder for an unknown.
- Probabilistic interpretation: p is the probability that a Boolean control input is *true.*
- Proportion interpretation: p is the proportion of *true* values at the control input in one *complete cycle*.

NOTE: We do not need numeric values for p. We always manipulate it symbolically.

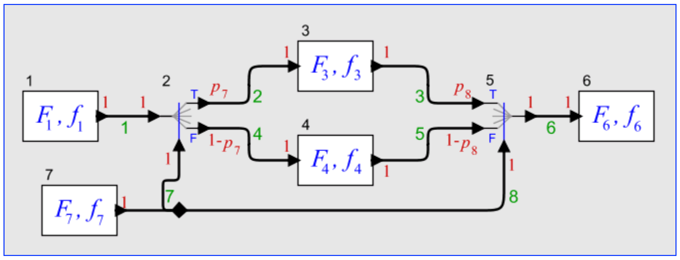
Symbolic Balance Equations



The two connections above imply the following balance equations:

$$q_2 p = q_3$$
$$q_2 (1 - p) = q_4$$

Production/Consumption Matrix for If-Then-Else



Symbolic variables:

$$\vec{p} = \begin{bmatrix} p_7 \\ p_8 \end{bmatrix}$$

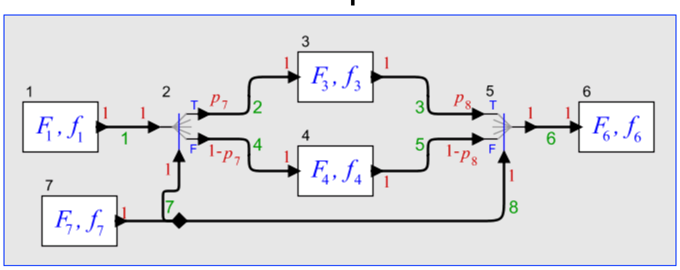
$$\Gamma(\vec{p}) = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & p_7 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -p_8 & 0 & 0 \\ 0 & 1-p_7 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -(1-p_8) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

Balance equations:

$$\Gamma(\vec{p})q(\vec{p}) = \vec{0}$$

Note that the solution $q(\vec{p})$ now depends on the symbolic variables $\vec{p}$

Production/Consumption Matrix for If-Then-Else



The balance equations have a solution $q(\vec{p})$ if an only if $\Gamma(\vec{p})$ has rank 6. This occurs if and only if $p_7 = p_8$, which happens to be true by construction because signals 7 and 8 come from the same source. The solution is given at the right.

$$q(\vec{p}) = \begin{bmatrix} 1 \\ 1 \\ p_7 \\ 1-p_7 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Strong and Weak Consistency

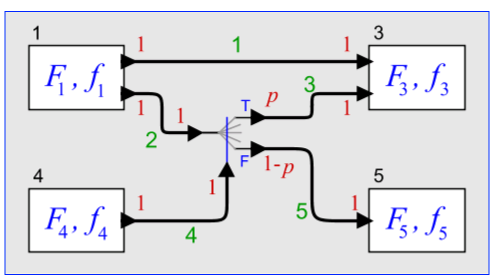
A *strongly consistent* dataflow model is one where the balance equations have a solution that is provably valid without concern for the values of the symbolic variables.

- The if-then-else dataflow model is strongly consistent.

A *weakly consistent* dataflow model is one where the balance equations cannot be proved to have a solution without constraints on the symbolic variables that cannot be proved.

- Note that whether a model is strongly or weakly consistent depends on how much you know about the model.

Weakly Consistent Model

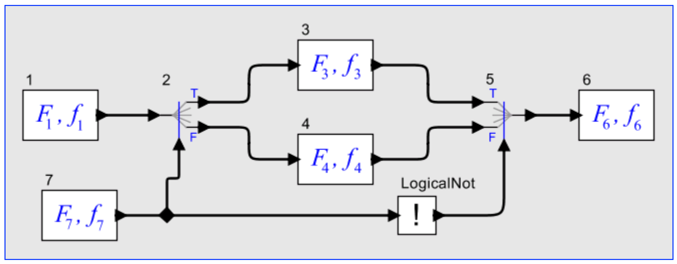


$$\Gamma(\vec{p}) = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & p & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 1-p & 0 & 0 & -1 \end{bmatrix}$$

This production/consumption matrix has full rank unless $p = 1$.

Unless we know f_4 , this cannot be verified at compile time.

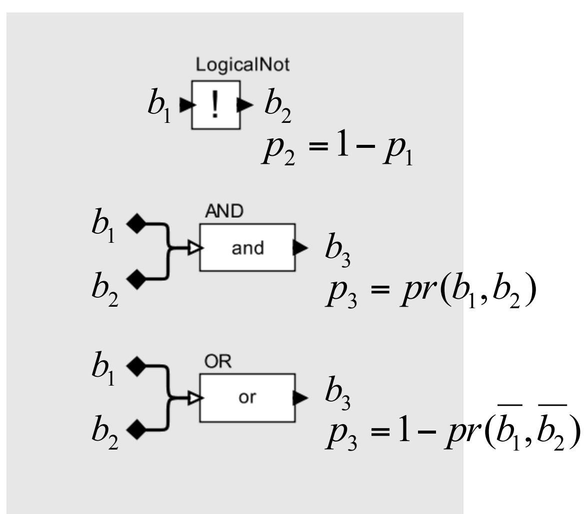
Another Example of a Weakly Consistent Model



This one requires that actor 7 produce half true and half false (that $p = 0.5$) to be consistent. This fact is derived automatically from solving the balance equations.

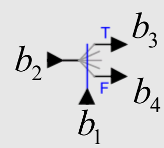
Use Boolean Relations

Symbolic variables across logical operators can be related as shown.



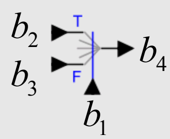
Routing of Boolean Tokens

Symbolic variables across switch and select can be related as shown.



$$p_3 = pr(b_2 \mid b_1)$$

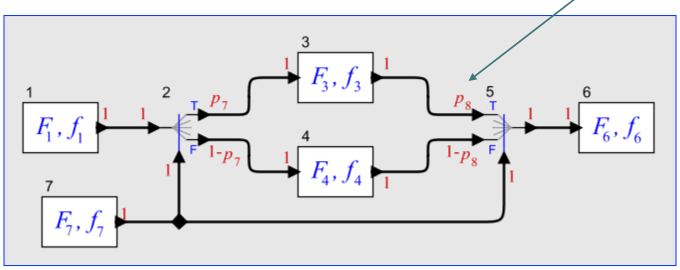
$$p_4 = pr(b_2 \mid \overline{b_1})$$



$$p_4 = pr(b_2 \mid b_1) + pr(b_3 \mid \overline{b_1})$$

Recall If-Then-Else Pattern

Symbolic consumption rate.



Solution to the symbolic balance equations:

$$q(\vec{p}) = \begin{bmatrix} 1 \\ 1 \\ p_7 \\ 1 - p_7 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

The if-then-else model is strongly consistent and we can give a *quasi-static* schedule for it:

(1, 7, 2, b?3, !b?4, 5, 6)

guard

$$\vec{p} = \begin{bmatrix} p_7 \\ p_8 \end{bmatrix}$$

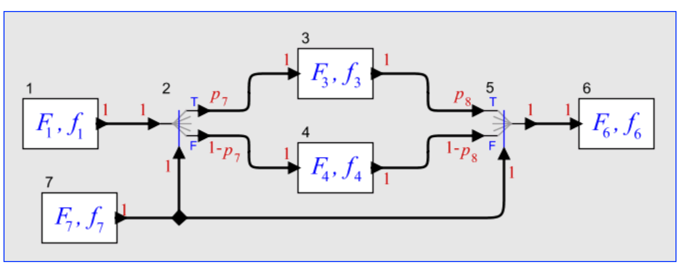
Quasi-Static Schedules & Traces

A *quasi-static schedule* is a finite list of guarded firings where:

- The number of tokens on each arc after executing the schedule is the same as before, regardless of the outcome of the Booleans.
- If any arc has a Boolean token prior to the execution of the schedule, then it will have a Boolean token with the same value after execution of the schedule.
- Firing rules are satisfied at every point in the schedule.

A *trace* is a particular execution sequence.

Quasi-Static Schedules & Traces

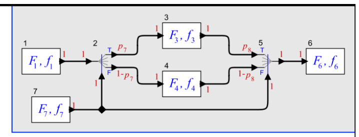


Solution to the symbolic balance equations:

$$q(\vec{p}) = [1 \quad 1 \quad p_7 \quad 1-p_7 \quad 1 \quad 1 \quad 1]^T$$

Quasi-static schedule: (1, 7, 2, b?3, !b?4, 5, 6)
Possible trace: (1, 7, 2, 3, 5, 6)
Another possible trace: (1, 7, 2, 4, 5, 6)

Proportion Vectors



- Let S be a trace. E.g. (1, 7, 2, 3, 5, 6)
- Let q_S be a repetitions vector for S. E.g.

$$q_S = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}^T$$

- Let $t_{i,S}$ be the number of TRUEs consumed from Boolean stream b_i in S. E.g. $t_{7,S} = 1$, $t_{8,S} = 1$.
- Let $n_{i,S}$ be the number of tokens consumed from Boolean stream b_i in S. E.g. $n_{7,S} = 1$, $n_{8,S} = 1$.
- Let

$$\vec{p}_S = \begin{bmatrix} t_{7,S} / n_{7,S} \\ t_{8,S} / n_{8,S} \end{bmatrix}$$

← proportion vector

- We want a quasi-static schedule s.t. for every trace S we have $\Gamma(\vec{p}_S) q_S = \vec{0}$.

Proportion Interpretation

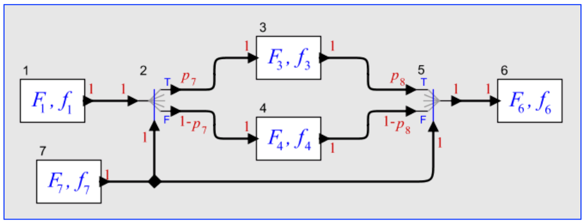
Recall the balance equations depend on $\vec{p}$, a vector with one symbolic variable for each Boolean stream that affects consumption production rates:

$$\Gamma(\vec{p}) q(\vec{p}) = \vec{0}$$

Under a proportion interpretation, for a trace S, $\vec{p}_S$ represents the *proportion* of TRUEs in S. We seek a schedule that always yields traces that satisfy

$$\Gamma(\vec{p}_S) q_S = \vec{0}$$

Proportion Interpretation for If-Then-Else



$$\Gamma(\vec{p}) = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & p_7 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -p_8 & 0 & 0 \\ 0 & 1-p_7 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -(1-p_8) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

Quasi-static schedule: (1, 7, 2, b?3, !b?4, 5, 6)

Possible trace: S = (1, 7, 2, 3, 5, 6)

$$\vec{p} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T \qquad q_S = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}^T$$

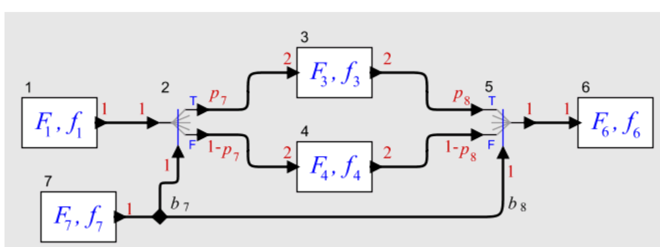
Another possible trace: (1, 7, 2, 4, 5, 6)

$$\vec{p} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T \qquad q_S = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}^T$$

Both satisfy the balance equations.

Limitations of Consistency

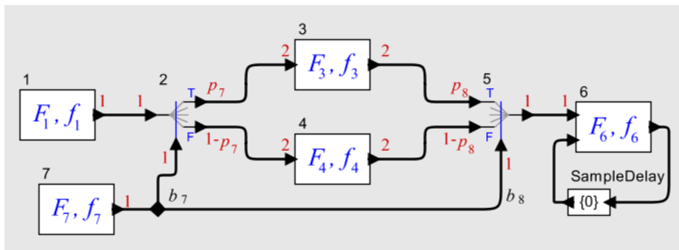
Consistency is necessary but not sufficient for a dataflow graph to have a bounded-memory schedule. Consider:



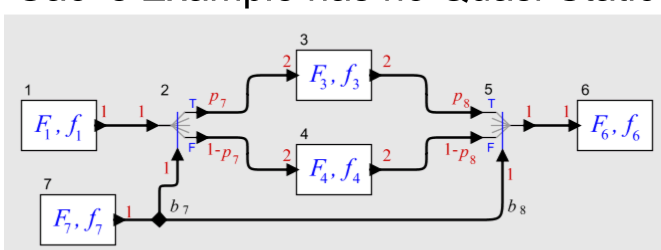
[Gao et al. '92]. This model is strongly consistent. But there is no bounded schedule (e.g., suppose $b_7 = (F, T, T, T, \ldots)$).

Limitations of Consistency

Even out-of-order execution (as supported by tagged-token scheduling [Arvind et al.] doesn't solve the problem:



Gao's Example has no Quasi-Static Schedule



Solution to the symbolic balance equations is

$$q(\vec{p}) = [2 \quad 2 \quad p_7 \quad 1-p_7 \quad 2 \quad 2 \quad 2]^T$$

A trace S with N firings (N even) of actor 1 must have

$$q_S = [N \quad N \quad t_{7,S}/2 \quad (N-t_{7,S})/2 \quad N \quad N \quad N]^T$$

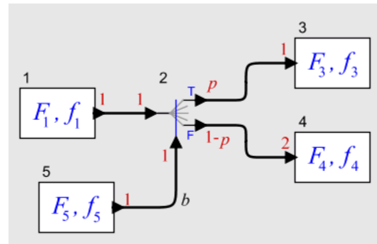
But this cannot be unless $t_{7,S}$ is even. There is no assurance of this.

Another Example

The model is strongly consistent.

Solution to symbolic equations:

$$q(\vec{p}) = [2 \quad 2 \quad 2p \quad 1-p \quad 2]^T$$



A trace S with N firings (N even) of actor 1 must have:

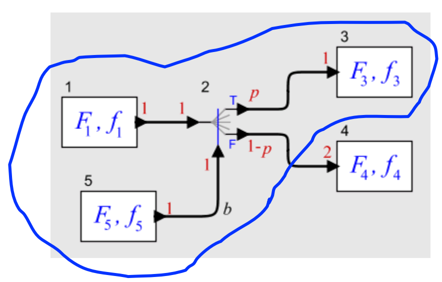
$$q_S = [N \quad N \quad t \quad (N-t)/2 \quad N]^T$$

where t is the number of TRUEs consumed. There is no finite N where this is assured of being an integer vector.

Clustered Quasi-Static Schedules

Consider the *clustered schedule:*

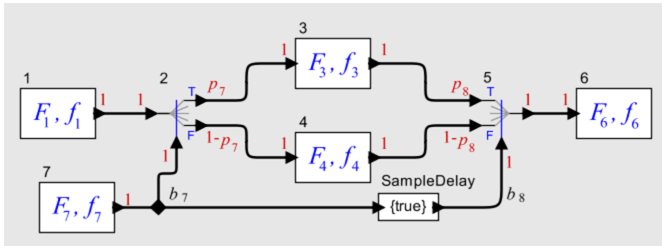
```
n = 0;
do {
    fire 1;
    fire 5;
    fire 2;
    if (b) {
        fire 3;
    } else {
        n += 1;
    }
} while (n < 2);
fire 4;
```



This schedule either fails to terminate or yields an integer vector of the form:

$$q_S = [N \quad N \quad t \quad (N-t)/2 \quad N]^T$$

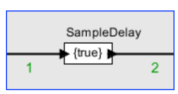
Delays Can Also Cause Trouble



This model is weakly consistent, where the balance equations have a non-trivial solution only if $p_7 = p_8$, in which case the solution is:

$$q(\vec{p}) = [1 \quad 1 \quad p_7 \quad 1-p_7 \quad 1 \quad 1 \quad 1]^T$$

Relating Symbolic Variables Across Delays

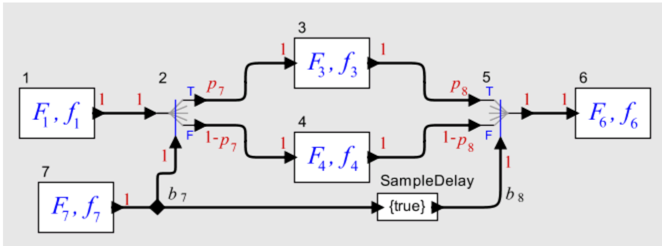
For the sample delay: 

What is the relationship between p_1 and p_2?

Since consistency is about behavior in the limit, under the probabilistic of the interpretation for the symbolic variables, it is reasonable to assume $p_1 = p_2$.

Is this reasonable under the proportion interpretation?

Delays Cause Trouble with the Proportion Interpretation



Solution to the symbolic balance equations is

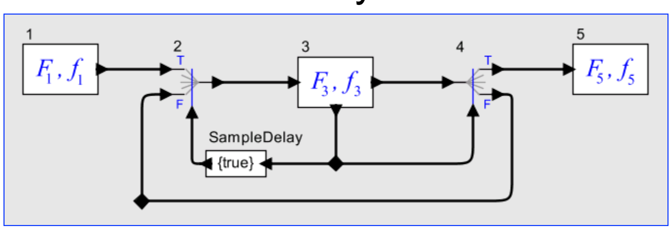
$$q(\vec{p}) = [1 \quad 1 \quad p_7 \quad 1-p_7 \quad 1 \quad 1 \quad 1]^T$$

A trace S with N firings of actor 1 must have

$$q_S = [N \quad N \quad t_{7,S} \quad (N-t_{7,S}) \quad N \quad N \quad N]^T$$

But for no value of N is there any assurance of being able to fire actor 5 N times. This schedule won't work.

Do-While Relies on a Delay

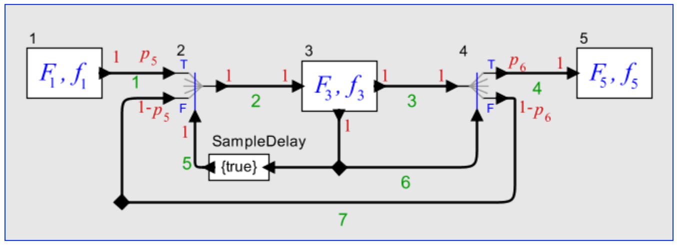


Imperative equivalent:

```
while (true) {
   x = f1();
   b = false;
   while(!b) {
      (x, b) = f3(x);
   }
   f5(x);
}
```

Is this model strongly consistent? Weakly consistent? Inconsistent?

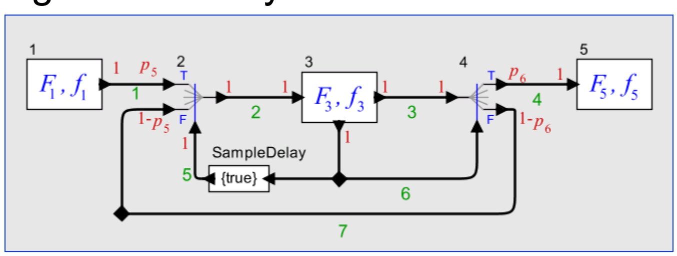
Checking Consistency of Do-While



$$\Gamma(\vec{p}) = \begin{bmatrix} 1 & -p_5 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & p_6 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & -(1-p_5) & 0 & 1-p_6 & 0 \end{bmatrix}$$

This model is consistent if and only if $p_5 = p_6$, which is true under the probabilistic interpretation, but not under the proportion interpretation.

Checking Consistency of Do-While

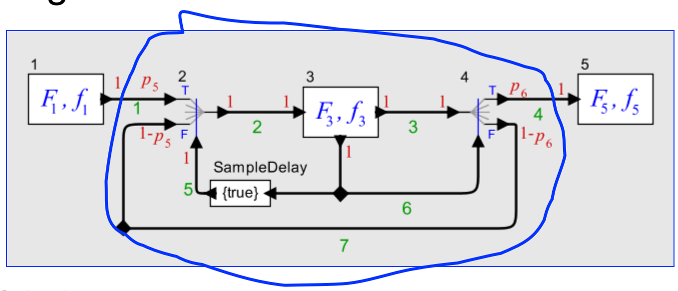


$$\Gamma(\vec{p}) = \begin{bmatrix} 1 & -p_5 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & p_6 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & -(1-p_5) & 0 & 1-p_6 & 0 \end{bmatrix}$$

Let $p = p_5 = p_6$, then the solution to the balance equations is:

$$q(\vec{p}) = \begin{bmatrix} 1 & 1/p & 1/p & 1/p & 1 \end{bmatrix}^T$$

Clustering Solution for Do-While



Clustered Schedule:

```
fire 1;
do {
   fire 2;
   fire 3;
   fire 4;
} while(!b);
fire 5;
```

This schedule yields traces S for which $p_5 = p_6 = 1/N$ and

$$q_S = \begin{bmatrix} 1 & N & N & N & 1 \end{bmatrix}^T$$

compare:

$$q(\vec{p}) = \begin{bmatrix} 1 & 1/p & 1/p & 1/p & 1 \end{bmatrix}^T$$

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Extensions

- State enumeration scheduling approach: Seek a finite set of finite guarded schedules that leave the model in a finite set of states (buffer states), and for which there is a schedule starting from each state.
- Integer dataflow (IDF [Buck '94]): Allow symbolic variables to have integer values, not just Boolean values. Extension is straightforward in concept, but reasoning about consistency becomes harder.

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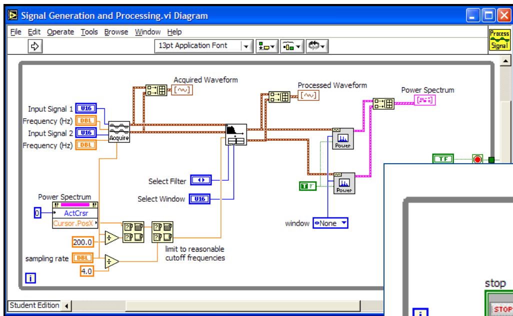
Taking Stock

- BDF and IDF generalize the idea of balance equations and introduce *quasi-static scheduling.*
- BDF and IDF are Turing complete, so existence of quasi-static schedules is undecidable.
- Can often construct quasi-static schedules anyway.
- Tricks like clustered schedules make the set of manageable models larger.
- Are Switch and Select like unrestricted GOTO?

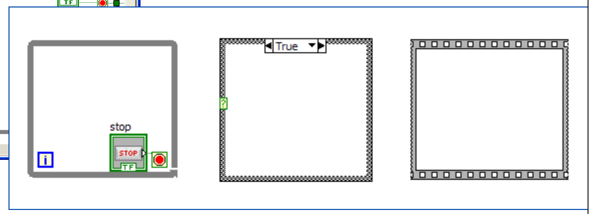
Extensions of SDF that Improve Expressiveness

Structured Dataflow [Kodosky 86, Thies et al. 02]
Boolean dataflow [Buck and Lee, 93]
Cyclostatic Dataflow [Lauwereins 94]
Multidimensional SDF [Lee & Murthy 96]
Heterochronous Dataflow [Girault, Lee, and Lee, 97]
Parameterized Dataflow [Bhattacharya et al. 00]
Teleport Messages [Thies et al. 05]

Many of these remain decidable



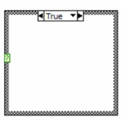
Structured Dataflow [Kodosky 86]



LabVIEW uses homogeneous SDF augmented with syntactically constrained forms of feedback and rate changes:

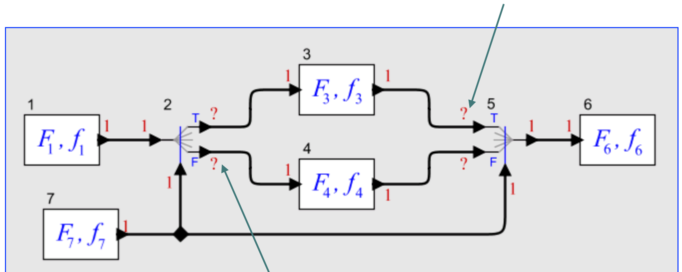
- While loops
- Conditionals
- Sequences

LabVIEW models are decidable.



vs. Dynamic Dataflow, which uses token routing for control flow

What consumption rate?



What production rate?

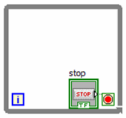
Imperative equivalent:

```
while (true) {
    x = f1();
    b = f7();
    if (b) {
      y = f3(x);
    } else {
      y = f4(x);
    }
    f6(y);
}
```

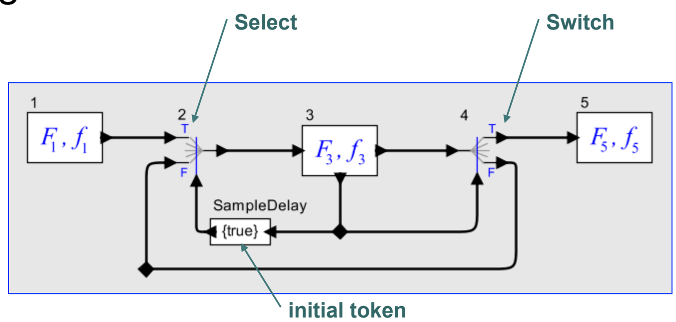
The if-then-else model is not SDF. But we can clearly give a bounded quasi-static schedule for it:

(1, 7, 2, b?3, !b?4, 5, 6)

guard



vs. Dynamic Dataflow, which uses token routing for control flow



Imperative equivalent:

```
while (true) {
    x = f1();
    b = false;
    while(!b) {
        (x, b) = f3(x);
    }
    f5(x);
}
```

This model uses conditional routing of tokens to iterate a function a data-dependent number of times.

Syntax: Graphical or Textual?

component technologies

- Sutherland (66)
- Prograph (85)
- LabVIEW (86)
- Gabriel (86)
- Show and Tell (86)
- Cantata (91)
- Ptolemy Classic (94)
- Ptolemy II (00)
- Scade (05)
- …

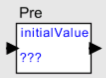
- Lucid (77)
- Id (78)
- VAL (79)
- Sisal (83)
- Lustre (86)
- Signal (90)
- Granular Lucid (95)
- StreamIT (02)
- Cal (03)
- …

The graphical vs. textual debate obscures a more important question:

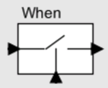
Are actors and streams a programming language technology or a software component technology?

Consistency in Synchronous/Reactive Models

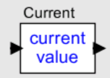
Key SR Actors



Pre: When the input is present, the output is the previous present input value.

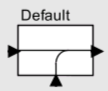


When: When the bottom input is present and true, the output equals the input. Otherwise, the output is absent.

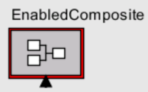


Current: The output equals the most recent present input value.

NonStrictDelay: The output is equal to the input in the previous clock tick.



Default: The output equals the left input, if it is present, and the bottom input otherwise.



EnabledComposite: Composite actor whose internal clock ticks only when the bottom input is present and true.

SR models are intrinsically bounded, but have related consistency issue: *Clock consistency*.

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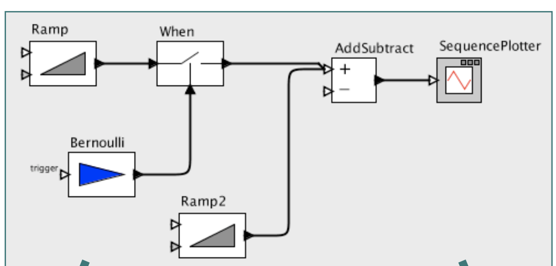
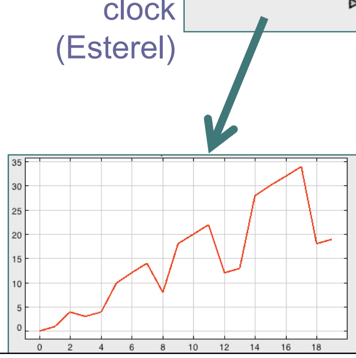
Two Interpretations of Clocks

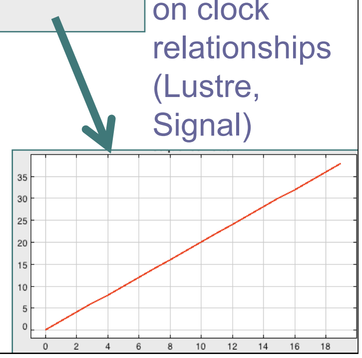
A clock is a property of a model, and signals may be *absent* at ticks of the clock (Esterel)



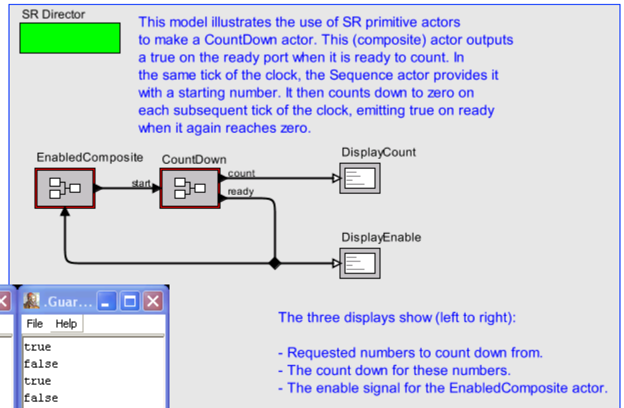
A clock is a property of a signal, and components impose constraints on clock relationships (Lustre, Signal)

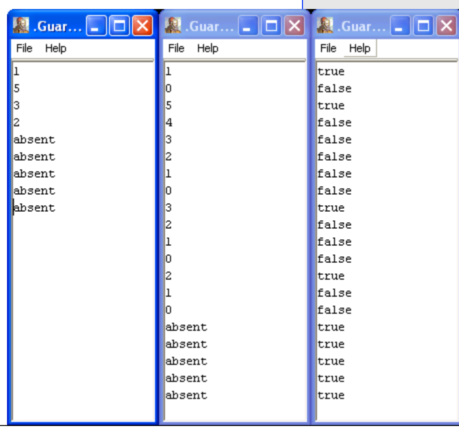


The choice between these has profound consequences



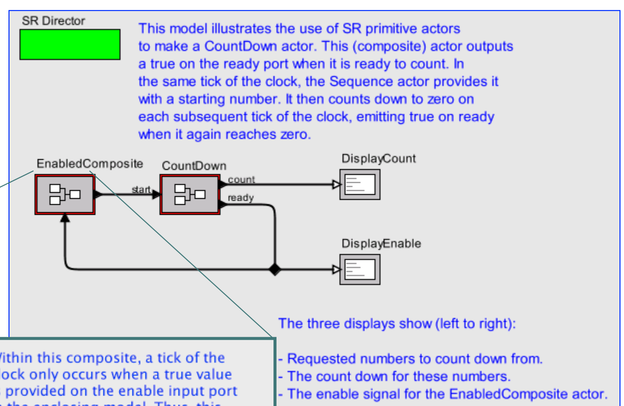
The Ptolemy II SR Director realizes Esterel-style clocks with hierarchical clock domains.

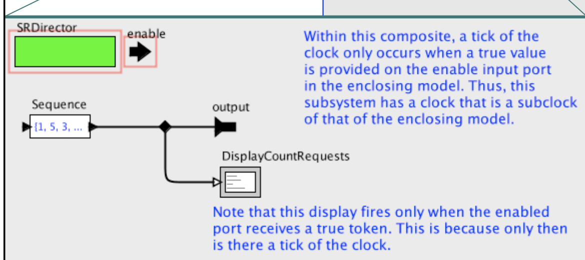




In this example, the CountDown composite issues a "ready" signal to the EnabledComposite, which then issues a number. The CountDown composite counts down from that number to 0, then issues another ready.

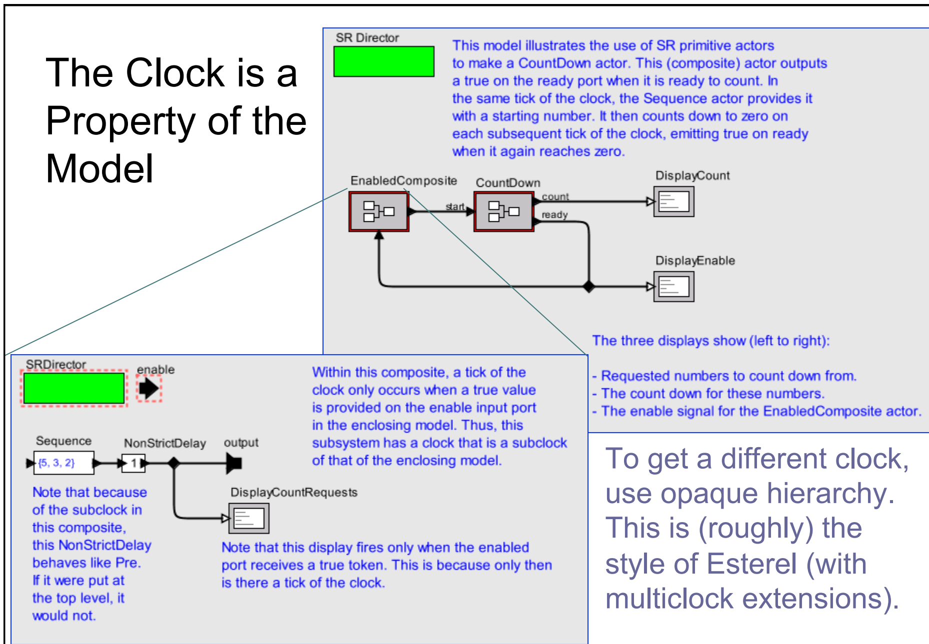
The Ptolemy II SR Director realizes Esterel-style clocks with hierarchical clock domains.





The EnabledComposite has a clock that ticks only when the enable input is present and true. It issues the sequence 1, 5, 3, 2, followed by absent henceforth.

The Clock is a Property of the Model



To get a different clock, use opaque hierarchy. This is (roughly) the style of Esterel (with multiclock extensions).

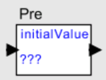
Hierarchical clock domains bear some resemblance to structured dataflow

Opaque hierarchy can do:

- Conditioning an internal tick on an external signal
  - Like a conditional
  - If the internal component is an instance of the external, then this amounts to recursion
- Multiple internal ticks per external tick
  - Like a do-while
- Iterated internal ticks over a data structure (use IterateOverArray higher-order actor)
  - Like a for

A Consequence: Pre and NonStrictDelay have different behaviors!

Key SR Actors



Pre: When the input is present, the output is the previous present input value.

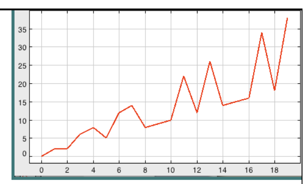
NonStrictDelay: The output is equal to the input in the previous clock tick.

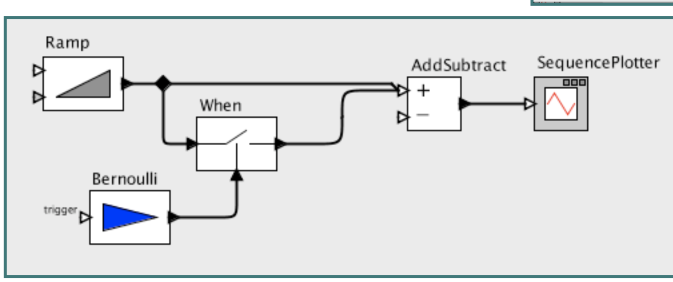
Alternative Semantics:
The Clock is a Property of the Signal

In Lustre and Signal, a clock is a property of a signal, and Pre and NonStrictDelay behave identically by constraining the input clock to be the same as the output clock. They only fire when the clock of the input signal ticks.

This leads to a clock consistency problem, which is in general undecidable.

Inconsistent clocks





In Lustre-style clock systems, the AddSubtract actor imposes the constraint that all its input signals have the same clock. The above model becomes *inconsistent* and will not execute.

Clock Calculus

- Let T be a totally ordered set of tags.
- Let s: $T \rightarrow V \cup \{\varepsilon\}$ be a signal of type V, where ε means "absent."
- Let c: $T \rightarrow \{-1, 0, 1\}$ be a *clock* associated with s where

  $$s(t) = \varepsilon \Rightarrow c(t) = 0$$
  $$s(t) = true \Rightarrow c(t) = 1$$
  $$s(t) = false \Rightarrow c(t) = -1$$

  If V is not boolean, then when $s(t)$ is present, $c(t)$ has value or 1 or -1 (we will make no distinction).

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Operations on Clocks

Arithmetic on clocks is in GF-3 (a Galois field with 3 elements), as follows:

$0 + x = x$	$0 \cdot x = 0$
$1 + 1 = -1$	$1 \cdot x = x$
$-1 + -1 = 1$	$-1 \cdot x = -x$
$-1 + 1 = 0$	

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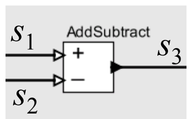
Clock Relations: Simple Synchrony

Most actors require that the clocks on all signals be the same. For example:

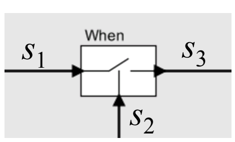


$$\forall t \in T, \quad c_1^2(t) = c_2^2(t) = c_3^2(t)$$

This means that either all are present, or all are absent.

Clock Relations: When Operator

Assuming that s_1 is a boolean-valued signal (which it must be), the clocks on signals interacting through the when operator are related as follows:



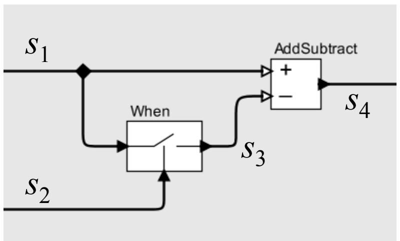
$$\forall t \in T, \quad c_3(t) = c_1(t)(-c_2(t) - c_2^2(t))$$

This means:

If s_1 is absent, then s_3 is absent.
If s_2 is false, then s_3 is absent.
If s_2 is true, then s_3 is the same as s_1.

Consistency Checking

Consider the following model:



$$\forall t \in T, \quad c_1^2(t) = c_4^2(t) = c_3^2(t)$$

$$\forall t \in T, \quad c_3(t) = c_1(t)(-c_2(t) - c_2^2(t))$$

These two together imply that:

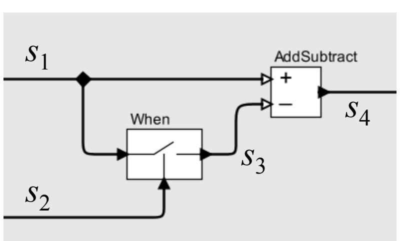
$$\forall t \in T, \quad c_1^2(t)(1 + c_2^2(t)) = -c_2(t)c_1^2(t)$$

where we have used the fact that:

$$(-c_2(t) - c_2^2(t))^2 = (-c_2(t) - c_2^2(t))$$

Interpretation of Consistency Result

Consistency check implies that:



$$\forall t \in T, \quad c_1^2(t)(1 + c_2^2(t)) = -c_2(t)c_1^2(t)$$

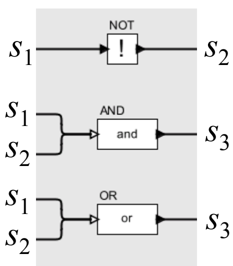
This means:

s_1 is absent if and only if s_2 is absent or false.

Logic Operators Affect Clocks

The output of the When actor has a clock that depends on the Boolean control signal. Clocks of Boolean-valued signals reflect the signal value as follows:

$$\forall t \in T,$$



$$c_2(t) = -c_1(t)$$

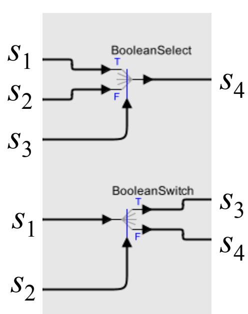
$$c_3(t) = (c_1(t)c_2(t))^2(-(c_1(t)+1)(c_2(t)+1)-1)$$

$$c_3(t) = (c_1(t)c_2(t))^2((c_1(t)-1)(c_2(t)-1)+1)$$

Token Routing Also Affects Clocks

Switch and Select affect the clocks as follows:

$$\forall t \in T,$$



$$c_4(t) = c_3(t)(c_2(t)(1-c_3(t)) - c_1(t)(1+c_3(t)))$$

$$-(c_3(t)+1)c_3(t) = c_1^2(t)$$

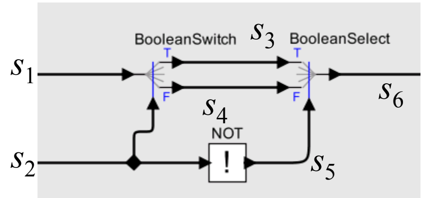
$$-(c_3(t)-1)c_3(t) = c_2^2(t)$$

$$c_3(t) = -c_2(t)(c_2(t)+1)c_1(t)$$

$$c_4(t) = c_2(t)(1-c_2(t))c_1(t)$$

$$c_2^2(t) = c_1^2(t)$$

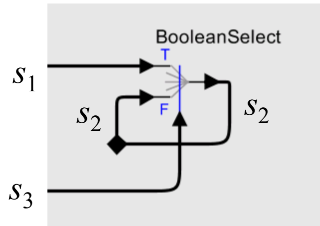
Example 1 Using Switch and Select



What can you infer about the clock of s_6 ?

$$c_6(t) = 0$$

Example 2 Using Switch and Select



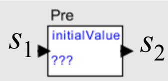
What can you infer about the clocks?

$$c_1(t) = 0 \quad \text{and}$$

$$\text{either} \quad c_3(t) = 0 \quad \text{or} \quad 1 + c_3(t) = 0$$

This means that s_1 is absent and s_3 is either absent or false.

What About Delays?



Pre: When the input is present, the output is the previous present input value.

NonStrictDelay: The output is equal to the input in the previous clock tick.

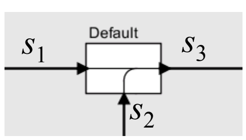
Clock relations across the delays become dependent on the tags. E.g., if T is the natural numbers, then we get a nonlinear dynamical system:

$$c_1^2(t) = c_2^2(t) \quad \text{and}$$
$$c(0) = \text{initial state}$$
$$c(t+1) = (1 - c_1^2(t))c(t) + c_1(t)$$
$$c_2(t) = c_1^2(t)c(t)$$

This makes clock analysis very difficult, in general.

Default Operator

Default: The output equals the left input, if it is present, and the bottom input otherwise:

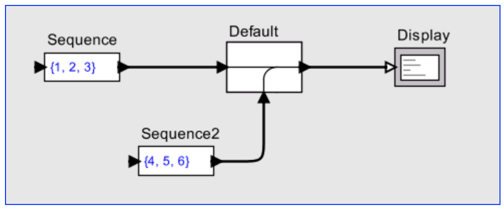


$$\forall t \in T, \quad c_3(t) = c_1(t) + c_2(t)(1 - c_1^2(t))$$

This means the clock of s_3 is equal to the clock of s_1, if it is present, and to the clock of s_2 otherwise.

Default Operator in SIGNAL is Nondeterministic

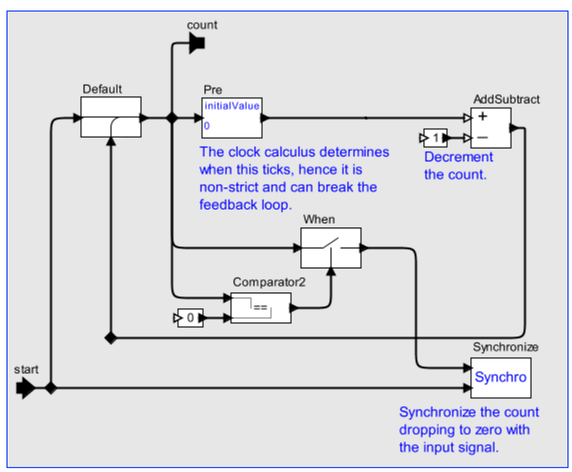
In SIGNAL semantics, the following model has many behaviors:



The two generated sequences have independent clocks (defined over incomparable values of $t \in T$), and the output sequence is any interleaving that preserves the ordering.

Guarded Count in SIGNAL

Instead of generating a “ready” signal, in SIGNAL, the count hitting zero can be synchronized with the input being present.



Conclusion and Open Issues

- When clocks are a property of the model, the result is structured synchronous models, where differences between clocks are explicit and no consistency checks are necessary (and signals may be *absent* at ticks of the clock).
- When clocks are a property of a signal, the result is similar to Boolean Dataflow (BDF). It is arguable that clock operators like "when," "default," "switch," and "select" become analogous to unstructured gotos. Clock consistency checking becomes undecidable.
- When further extended as in SIGNAL to partially ordered clock ticks, models easily become nondeterministic.