EE 144/244: Fundamental Algorithms for System Modeling, Analysis, and Optimization Fall 2014

State-Space Exploration

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State-Space Exploration

- Goal: explore **state-space** of a system (typically a transition system).
  - E.g., **reachability analysis**: visit all states **reachable** from the initial states.
- For finite-state systems, it can be done exhaustively and fully automatically! (in principle)
- Basic method for solving the **model checking** problem.
  - Turing award 2007: Clarke, Emerson, Sifakis.
- Established practice in the industry (mainly hardware, but increasingly also software).

The Model Checking Problem

Does a given system M (the *implementation*, e.g., a state machine or a transition system) satisfy a given temporal logic formula ϕ (the *specification*, e.g., an LTL or CTL formula) ?

$$M \stackrel{?}{\models} \phi$$

Meaning:

- If ϕ is LTL: **all** execution traces of the system must satisfy ϕ.
- If ϕ is CTL: the initial state of the system must satisfy ϕ.

Invariants

Suppose ϕ is of the form

$$\mathsf{G}\psi \qquad \text{or} \qquad \mathsf{AG}\psi$$

where ψ is a propositional formula (boolean expression on atomic propositions).

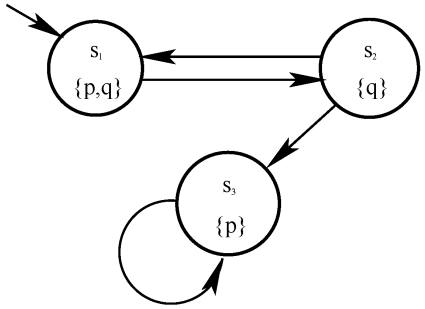
E.g.,

$$\mathsf{G}(p \vee q), \qquad \mathsf{G}(p \Rightarrow q), \qquad \cdots$$

Then ψ is called an **invariant**: it's a property that must hold at all **reachable** states.

Recall: Transition System (Kripke Structure)

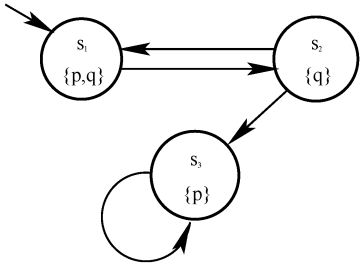
A tuple (P, S, S_0, L, R).



- P: set of atomic propositions, e.g., $P = \{p, q\}$.
- S: set of states, e.g., $S = \{s_1, s_2, s_3\}$.
- S_0: set of initial states, could be more than one, in this example just one: $S_0 = \{s_1\}$.
- $L : S \to 2^P$: labeling function, e.g., $L(s_1) = \{p, q\}$, $L(s_2) = \{q\}$, ...
- $R \subseteq S \times S$: transition relation, e.g., $R = \{(s_1, s_2), (s_2, s_1), (s_2, s_3), (s_3, s_3)\}$.

Reachable States

Given transition system (P, S, S_0, L, R).



A state $s \in S$ is called reachable if there exists a finite sequence of states

$$s_0, s_1, s_2, ..., s_k$$

such that:

1. $s_0 \in S_0$.
2. $\forall i = 0, ..., k-1 : (s_i, s_{i+1}) \in R$. We also write $s_i \to s_{i+1}$.
3. $s_k = s$.

Reachability Analysis

Visit all reachable states of a (typically finite) transition system.

At the same time, we can check whether every reachable state satisfies a given invariant ψ ...

... and therefore check that the system satisfies Gψ.

Caveat: Deadlocks

This assumes our system is **deadlock-free**, since only infinite paths count for the verification of Gψ.

Formally, s a deadlock state if $\nexists s' : s \rightarrow s'$.

How can we check that a given system is deadlock-free?

Use reachability analysis!

State-Space Exploration: Summary

- Reachability analysis: Check that system is never in an "incorrect" state, e.g.,
  - deadlock state
  - state which violates an invariant
  - e.g., "train is at intersection but gate is not lowered"
  - "autopilot is off but pilot thinks it is on"
  - ...
- Also the basis for checking *liveness* properties: every so often system does something useful.

State-Space Exploration Algorithms

- Enumerative (also called "explicit state").
  - These are basically search algorithms on directed graphs.
- Symbolic
  - Bounded model-checking using SAT/SMT solvers.
  - Symbolic reachability.

An Enumerative Algorithm: Depth-First Search

Assume given: Kripke structure (P, S, S_0, L, R).

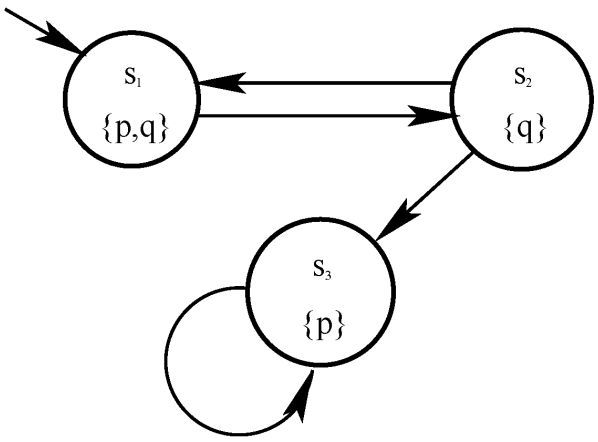
main:

1: $V := \emptyset$; /* V: set of visited states */
2: **for all** $s \in S_0$ **do**
3: DFS(s);
4: **end for**

DFS(s):

1: check s; /* is s a deadlock? is given $p \in L(s)$? ... */
2: $V := V \cup \{s\}$;
3: **for all** s' such that $(s, s') \in R$ **do**
4: **if** $s' \notin V$ **then**
5: DFS(s'); /* recursive call */
6: **end if**
7: **end for**

An Enumerative Algorithm: Depth-First Search



Let's simulate the algorithm on this graph.

An Enumerative Algorithm: Depth-First Search

Quiz:

- Does the algorithm terminate?
- Does it visit all reachable states?
- Does it visit any unreachable states?
- What is the complexity of the algorithm?

Enumerative Methods

Many algorithms: DFS, BFS, A*, ...

Many approaches to combat state-space explosion: partial-order reduction, symmetry reduction, bit-state hashing, ...

Lots of literature on the topic, including research papers [Godefroid and Wolper, 1991, Valmari, 1990, Holzmann, 1998] and textbooks [Clarke et al., 2000, Baier and Katoen, 2008].

In-depth discussion: Computer-Aided Verification course by Sanjit Seshia.

SYMBOLIC METHODS

Symbolic Methods: Why?

The plague of exhaustive verification: *state explosion*.

- A chip with 100 flip-flops: 2^{100} (potentially reachable) states.
- That is $1267650600228229401496703205376$ states.
- Even if each state costs 1 bit to store, this still makes $2^{100-60-8} = 2^{32} = 4,294,967,296$ exabytes ...
- Even if only $\frac{1}{32}$ states are reachable, this still makes $2^{100-5} = 2^{95}$ states.

Symbolic methods aim to improve this.

A seminal paper: "*Symbolic model checking:* 10^{20} *states and beyond.*" [Burch et al., 1990].

10^{20} is less than 2^{67}, but a great leap forward at that time.

Symbolic Representation of State Spaces

Key idea:

Instead of reasoning about individual states, reason about **sets** *of states.*

How do we represent a set of states?

Symbolic representation:

Set = predicate.

Set of states = predicate on state variables.

Symbolic Representation of Sets of States

Examples:

1. Assume 3 state variables, p, q, r, of type boolean.

$$S_1 : \quad p \vee q = \{p\overline{q}r, p\overline{q}\overline{r}, \overline{p}qr, \overline{p}q\overline{r}, pqr, pq\overline{r}\}$$

2. Assume 3 state variables, x, i, b, of types real, integer, boolean.

$$S_2 : \quad x > 0 \wedge (b \rightarrow i \geq 0)$$

How many states are in S_2?

Symbolic Representation of Transition Relations

Key idea:

> *Use a predicate on* **two copies** *of the state variables: unprimed (current state) + primed (next state).*

If $\vec{x}$ is the vector of state variables, then the transition relation R is a predicate on $\vec{x}$ and $\vec{x}'$:

$$R(\vec{x}, \vec{x}')$$

e.g., for three state variables, x, i, b:

$$R(x, i, b, x', i', b')$$

Symbolic Representation of Transition Relations

Examples:

1. Assume one state variable, p, of type boolean.

$$R_1 : \quad (p \rightarrow \neg p') \wedge (\neg p \rightarrow p')$$

   Which transition relation does this represent? Is it a relation or a function (deterministic)?

2. Assume one state variable, n, of type integer.

$$R_2 : \quad n' = n + 1 \vee n' = n$$

   Which transition relation does this represent? Is it a relation or a function (deterministic)?

Symbolic Representation of Kripke Structures

Kripke structure:

$$(P, S, S_0, L, R)$$

Symbolic representation:

$$(P, Init, Trans)$$

where

- $P = \{x_1, x_2, ..., x_n\}$: set of (boolean) state variables, also taken to be the atomic propositions.[1]
- Predicate $Init(\vec{x})$ on vector $\vec{x} = (x_1, ..., x_n)$ represents the set S_0 of initial states.
- Predicate $Trans(\vec{x}, \vec{x}')$ represents the transition relation R.

Basis of the language of NuSMV.

[1] this is done for simplicity, the two could be separated

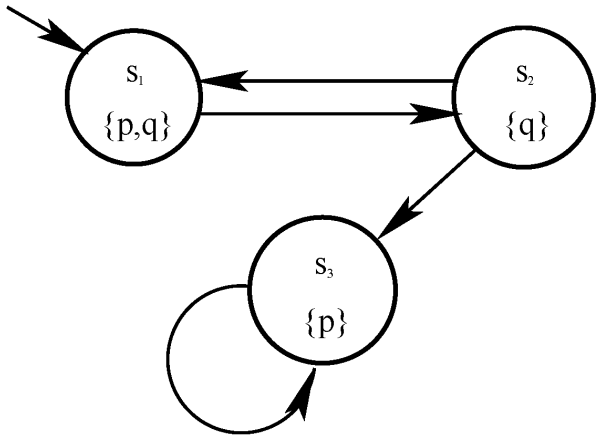
Example: NuSMV model

```
MODULE inverter(input)
VAR
  output : boolean;
INIT
  output = FALSE
TRANS
  next(output) = !input | next(output) = output
```

What is the Kripke structure defined by this NuSMV program?

What about P and L?

Example: Kripke Structure



Represent this symbolically.

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