EECS 144/244: Fundamental Algorithms for System Modeling, Analysis, and Optimization

Discrete Systems

Lecture: Transition systems, Temporal logic

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TRANSITION SYSTEMS

Transition Systems

An even more basic model than automata and state machines:

transition system $=$ states $+$ transitions ($+$ labels)

But: possibly infinite sets of states/transitions.

- Can describe infinite-state systems (e.g., software).
- Can also be used in non-discrete systems (e.g., timed automata or dataflow graphs, as we will see later).
- Form the basis for the semantics of temporal logics (later in this lecture) and other equivalences between systems (e.g., bisimulation).

Many variants: Labeled Transition Systems, Kripke Structures, ...

Labeled Transition Systems

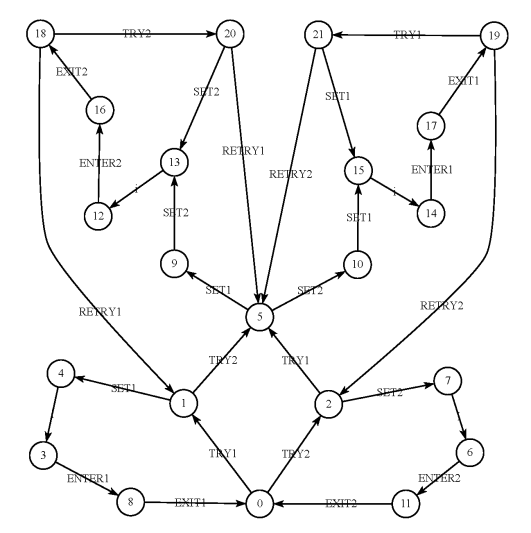
An LTS is a tuple:

$$(\Sigma, S, S_0, R)$$

- Σ: set of labels (modeling events, actions, ...)
- S: set of states (perhaps infinite)
- $S_0 \subseteq S$: set of initial states
- R: transition relation

$$R \subseteq S \times \Sigma \times S$$

Example: LTS


18
TRY2
20
21
TRY1
19
EXIT2
SET2
SET1
EXIT1
16
17
13
RETRY1
RETRY2
15
ENTER2
ENTER1
i
12
SET2
SET1
i
14
9
10
SET1
SET2
5
RETRY1
RETRY2
4
TRY2
TRY1
7
SET1
1
2
SET2
3
TRY1
TRY2
6
ENTER1
ENTER2
8
EXIT1
0
EXIT2
11

Kripke Structures

A Kripke structure is a tuple:

$$(P, S, S_0, L, R)$$

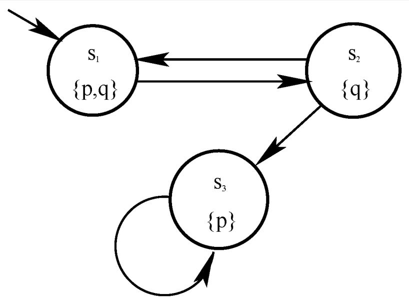
- P: set of atomic propositions (modeling state properties)
- S: set of states (perhaps infinite)
- $S_0 \subseteq S$: set of initial states
- L: labeling function on states

$$L : S \to 2^P$$

  For $p \in P$ and $s \in S$: "s has property p" iff $p \in L(s)$.
- R: transition relation

$$R \subseteq S \times S$$

Example: Kripke Structure



(Linear) Paths, Traces, ...

A *path* in a Kripke structure (P, S, S_0, L, R) is a (finite or infinite) sequence of states:

$$s_0, s_1, s_2, \cdots$$

such that

- $s_0 \in S_0$
- $\forall i : (s_i, s_{i+1}) \in R$

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An observable *trace* σ is the corresponding sequence of sets of properties:

$$\sigma = L(s_0), L(s_1), L(s_2), \cdots$$

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Note: we can look at a trace also as a sequence of *minterms*.
Example: if $P = \{p, q\}$ then a possible trace is:

$$\sigma = \{p\}, \{q\}, \{p, q\}, \{\}, \{p, q\}, ... = p\overline{q}, \overline{p}q, pq, \overline{pq}, pq, ...$$

Homework: Can we translate a KS to an "equivalent" LTS (and what does equivalent mean)? And vice-versa?

Reachable State Space

Given Kripke structure (P, S, S_0, L, R), a state $s \in S$ is called *reachable* if there exists a finite path:

$$s_0, s_1, s_2, \cdots, s_n$$

such that $s_n = s$.

Reachable state space: the sub-graph containing only reachable states and their transitions.

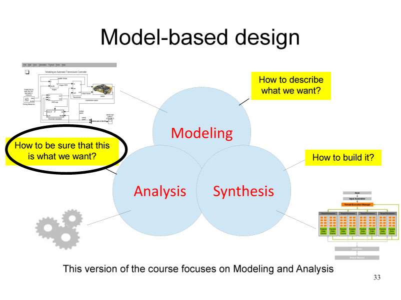
Deadlocks

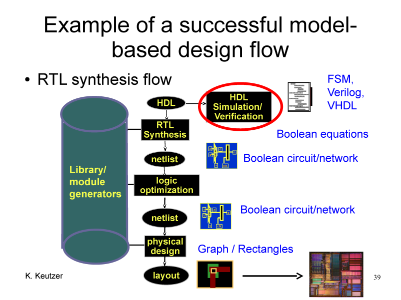
A state s is a *deadlock* if it has no outgoing transitions:

$$\nexists s' : (s, s') \in R$$

TEMPORAL LOGIC

Why temporal logic





a formal specification language

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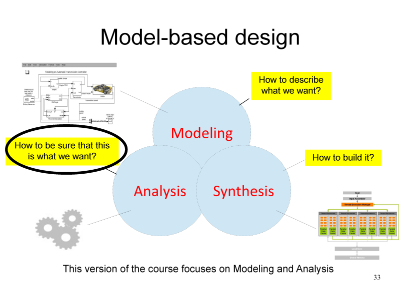
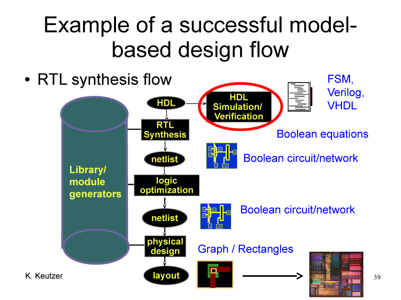
a way to specify what we want mathematically
(and unambiguously!)

Why temporal logic





a formal specification language

=

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Amir Pnueli (1941 - 2009) won the ACM Turing Award, in part for proposing to use temporal logic for specification.

Example: Specification of the SpaceWire Protocol (European Space Agency standard)

8.5.2.2 ErrorReset

a. The *ErrorReset* state shall be entered after a system reset, after link operation is terminated for any reason or if there is an error during link initialization.

b. In the *ErrorReset* state the Transmitter and Receiver shall all be reset.

c. When the reset signal is de-asserted the *ErrorReset* state shall be left unconditionally after a delay of 6,4 µs (nominal) and the state machine shall move to the *ErrorWait* state.

d. Whenever the reset signal is asserted the state machine shall move immediately to the *ErrorReset* state and remain there until the reset signal is de-asserted.

From Sanjit Seshia.

Current status of property specification in HW

Usage of formal property specification languages is becoming widespread

- **68% in 2007 (John Cooley, DVCon'07)**
- **Properties often called "assertions"**

Properties are used not just in formal verification, but also in simulation

- **"Assertion-Based Verification" (ABV)**

Some property specification languages: PSL/Sugar, System Verilog Assertions (SVA), OVA, OVL, etc.

All of these are just ways of writing variants of Temporal Logic

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From Sanjit Seshia.

Assertion-Based Verification

Monitor generated from assertions:

Temporal Logics

- Many variants: for linear, branching, timed, continuous, security, ..., properties
- We will look at LTL (linear) and CTL (branching).

LTL (Linear Temporal Logic) – Syntax

LTL[1] formulas are defined by the following grammar:

$$
\begin{array}{rcl}
\phi & ::= & p \mid q \mid \ldots \\
 & & \mid \phi_1 \wedge \phi_2 \mid \neg\phi_1 \\
 & & \mid G\phi_1 \\
 & & \mid F\phi_1 \\
 & & \mid X\phi_1 \\
 & & \mid \phi_1 U \phi_2
\end{array}
$$

$G\phi$ is also written $\Box\phi$.
$F\phi$ is also written $\Diamond\phi$.
$X\phi$ is also written $\bigcirc\phi$.

[1] In fact this is PLTL: Propositional LTL (vs. first-order LTL).

LTL (Linear Temporal Logic) – Syntax

Examples:

$$GFp$$

$$G(p \rightarrow Fq)$$

$$pU(qU(p \wedge r))$$

LTL (Linear Temporal Logic) – Syntax

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What do these formulas mean?

LTL – Semantics: Intuition

LTL formulas are evaluated over infinite sequences (traces).
Satisfaction relation looks like this – for LTL formula ϕ and infinite trace σ:

$$\sigma \models \phi$$

formula	mnemonic
p	proposition (must hold now)
$\mathbf{G}\phi$	always, globally
$\mathbf{F}\phi$	finally, future, eventually
$\mathbf{X}\phi$	next step
$\phi_1 \mathbf{U} \phi_2$	until

Intuitive semantics

LTL – Semantics: Formally

We want to define formally the satisfaction relation: $\sigma \models \phi$.

What kind of object is σ ?

An infinite trace, usually coming from a Kripke structure (P, S, S_0, L, R):

$$\sigma = \sigma_0, \sigma_1, \sigma_2, \cdots$$

where $\sigma_i \subseteq P$ for all i.

Example:

$$\sigma = \{p\}, \{q\}, \{p, q\}, \{\}, \{p, q\}, ...$$

LTL – Semantics: Formally

Let

$$\sigma = \sigma_0, \sigma_1, \sigma_2, \cdots$$

Notation: $\sigma[i..] = \sigma_i, \sigma_{i+1}, \sigma_{i+2}, \cdots$

Satisfaction relation defined recursively on the syntax of a formula:

$$
\begin{array}{lll}
\sigma \models p & \text{iff} & p \in \sigma_0 \\
\sigma \models \phi_1 \wedge \phi_2 & \text{iff} & \sigma \models \phi_1 \text{ and } \sigma \models \phi_2 \\
\sigma \models \neg\phi & \text{iff} & \sigma \not\models \phi \\
\sigma \models G\phi & \text{iff} & \forall i = 0, 1, \ldots : \sigma[i..] \models \phi \\
\sigma \models F\phi & \text{iff} & \exists i = 0, 1, \ldots : \sigma[i..] \models \phi \\
\sigma \models X\phi & \text{iff} & \sigma[1..] \models \phi \\
\sigma \models \phi_1 U \phi_2 & \text{iff} & \exists i = 0, 1, \ldots : \sigma[i..] \models \phi_2 \wedge \\
 & & \qquad \forall 0 \leq j < i : \sigma[j..] \models \phi_1
\end{array}
$$

LTL – Semantics: Formally

Let

$$\sigma = \sigma_0, \sigma_1, \sigma_2, \cdots$$

Notation: $\sigma[i..] = \sigma_i, \sigma_{i+1}, \sigma_{i+2}, \cdots$

Satisfaction relation defined recursively on the syntax of a formula:

$\sigma \models p$ iff $p \in \sigma_0$ — p holds at the first (current) step

$\sigma \models \phi_1 \wedge \phi_2$ iff $\sigma \models \phi_1$ and $\sigma \models \phi_2$

$\sigma \models \neg\phi$ iff $\sigma \not\models \phi$

$\sigma \models G\phi$ iff $\forall i = 0, 1, ... : \sigma[i..] \models \phi$ — ϕ holds for every suffix of σ

$\sigma \models F\phi$ iff $\exists i = 0, 1, ... : \sigma[i..] \models \phi$ — ϕ holds for some suffix of σ

$\sigma \models X\phi$ iff $\sigma[1..] \models \phi$ — ϕ holds for the suffix starting at the next step

$\sigma \models \phi_1 U \phi_2$ iff $\exists i = 0, 1, ... : \sigma[i..] \models \phi_2 \wedge \forall 0 \leq j < i : \sigma[j..] \models \phi_1$

ϕ_2 holds for some suffix of σ and
ϕ_1 holds for all previous suffixes

LTL – examples

See `http://embedded.eecs.berkeley.edu/eecsx44/fall2011/lectures/TemporalLogic.pdf`.

Errata:

- Slides 13-14: "if and only if it holds" should be "if and only if p holds".
- Slide 19: $F(p \Rightarrow (XXq))$ should be $G(p \Rightarrow (XXq))$.

Linear-Time vs. Branching-Time Semantics

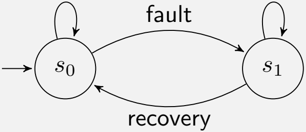
Some properties cannot be expressed on linear behaviors, e.g.:

> *"it is possible to recover from any fault"*

or

> *"there exists a way to get back to the initial state from any reachable state"*

Based on *one* (linear) behavior alone,[2] we cannot conclude whether our system satisfies the property.
E.g., the following system satisfies the property, although it contains a behavior that stays forever in state s_1:



[2]if we had *all* linear behaviors of a system, we could in principle reconstruct its branching behavior as well

Branching-Time Temporal Logics

The “solutions” (models) of a formula are states of a transition system (Kripke structure).

CTL (Computation Tree Logic) – Syntax

CTL formulas are defined by the following grammar:

$$
\begin{array}{rcl}
\phi & ::= & p \mid q \mid \ldots \\
 & & \mid \phi_1 \wedge \phi_2 \mid \neg\phi_1 \\
 & & \mid EG\phi_1 \mid AG\phi_1 \\
 & & \mid EX\phi_1 \mid AX\phi_1 \\
 & & \mid E(\phi_1 U \phi_2) \mid A(\phi_1 U \phi_2)
\end{array}
$$

E ("there exists a path") and A ("for all paths") are called *path quantifiers*. They are sometimes written $\exists$ and $\forall$.

CTL – Semantics: Intuition

Let s be a state of the Kripke structure.

$$s \models EG\phi$$

iff there exists a path starting from s such that the corresponding trace σ satisfies

$$\sigma \models G\phi$$

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Note:

- The 2nd $\models$ is the LTL satisfaction relation.
- The 1st $\models$ refers to the CTL satisfaction relation.
- To be more pedantic:

$$s \models_{CTL} EG\phi \qquad \text{iff} \qquad \sigma \models_{LTL} G\phi$$

CTL – Semantics: Intuition

Let s be a state of the Kripke structure.

$$s \models AG\phi$$

iff

CTL – Semantics: Intuition

Let s be a state of the Kripke structure.

$$s \models AG\phi$$

iff for every path starting from s, the corresponding trace σ satisfies

$$\sigma \models G\phi$$

CTL – Semantics: Intuition

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Quiz: do we need $EF\phi$? can we express it in terms of the CTL modalities shown earlier?

CTL – Semantics

Let (P, S, S_0, L, R) be a Kripke structure and let $s \in S$.
A trace starting from s is a sequence $\sigma = \sigma_0, \sigma_1, \cdots$, such that there is a path $s = s_0, s_1, \cdots$ starting from s, and $\sigma_i = L(s_i)$ for all i.

Satisfaction relation for CTL:

$s \models p$	iff	$p \in L(s)$
$s \models \phi_1 \wedge \phi_2$	iff	$s \models \phi_1$ and $s \models \phi_2$
$s \models \neg\phi$	iff	$s \not\models \phi$
$s \models EG\phi$	iff	$\exists$trace σ starting from s : $\sigma \models_{LTL} G\phi$
$s \models AG\phi$	iff	$\forall$traces σ starting from s : $\sigma \models_{LTL} G\phi$
$s \models EX\phi$	iff	$\exists$trace σ starting from s : $\sigma \models_{LTL} X\phi$
$s \models E(\phi_1 U \phi_2)$	iff	$\exists$trace σ starting from s : $\sigma \models_{LTL} \phi_1 U \phi_2$

...

CTL – Examples

How to express these properties in CTL?

"p *holds at all reachable states*"

CTL – Examples

How to express these properties in CTL?

"p *holds at all reachable states*" AGp

CTL – Examples

How to express these properties in CTL?

"p holds at all reachable states" AGp

"there exists a way to get back to the initial state from any reachable state"

CTL – Examples

How to express these properties in CTL?

"p holds at all reachable states" AGp

"there exists a way to get back to the initial state from any reachable state" $AG\ EF$ *init*

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