Time-Domain non-Monte Carlo Noise Simulation for Nonlinear Dynamic Circuits with Arbitrary Excitations

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Abstract*

A new, *time-domain*, *non-Monte Carlo* method for computer simulation of *electrical noise* in *nonlinear dynamic circuits* with *arbitrary excitations* is presented. This time-domain noise simulation method is based on the results from the theory of *stochastic differential equations.* The noise simulation method is general in the sense that any *nonlinear dynamic circuit* with *any kind of excitation*, which can be simulated by the *transient analysis* routine in a *circuit simulator,* can be simulated by our noise simulator in time-domain to produce the noise variances and covariances of circuit variables as a function of time, provided that noise models for the devices in the circuit are available. Noise *correlations* between circuit variables at *different time points* can also be calculated. Previous work on computer simulation of noise in integrated circuits is reviewed with comparisons to our method. Shot, thermal and flicker noise models for integrated-circuit devices, in the context of our time-domain noise simulation method, are described. The implementation of this noise simulation method in a circuit simulator (SPICE) is described. Two examples of noise simulation (a CMOS ring-oscillator and a BJT active mixer) are given.

1 Introduction

This paper presents a new, *time-domain*, *non-Monte Carlo* method for computer simulation of *electrical noise* in *nonlinear dynamic circuits* with *arbitrary excitations*. This time-domain noise simulation method is based on the results from the theory of *stochastic differential equations.* The noise phenomena considered in this work are caused by the small current and voltage fluctuations that are generated within the integrated-circuit devices themselves. The existence of noise is basically due to the fact that electrical charge is not continuous but is carried in discrete amounts equal to the electron charge. Electrical noise is associated with fundamental processes in integrated-circuit devices [1]. Noise represents a lower limit to the size of electrical signal that can be amplified by a circuit without significant deterioration in signal quantity. It also results in an upper limit to the useful gain of an amplifier, because if the gain is increased without limit, the output stages of the circuit will eventually begin to cut off or saturate on the amplified noise from the input stages [1]. The influence of noise on the performance is not limited to amplifier circuits. For instance, active integrated mixer circuits, which are widely used for down conversion in UHF and microwave receivers, add noise to their output. It is desirable to be able to predict the noise performance of a given mixer design [2,3]. Most of the time, amplifier circuits operate in small-signal conditions, that is, the operating point of the circuit does not change. For analysis and simulation, the amplifier circuit with a fixed operating-point can be modeled as a linear time-invariant network by making use of the small-signal models of the integrated-circuit devices. On the other hand, for a mixer circuit, the presence of a large local-oscillator signal causes substantial change in the active devices' operating points over time. So, a linear time-invariant network model is not accurate for a mixer circuit. There are many other kinds of circuits which do not operate in small-signal conditions, such as a voltage-controlled-oscillator (VCO) composed of delay cells in a ring configuration. Noise simulation of these circuits requires a method which can handle nonlinear dynamic circuits with arbitrary excitations. The three important types of noise in integrated circuits are *shot* noise, *thermal* noise and *flicker* noise which will all be considered in this work.

In Section 2 below, previous work on computer simulation of noise in integrated circuits is reviewed with comparisons to our method. In Section 3, shot, thermal and flicker noise models for integrated-circuit devices, in the context of our time-domain noise simulation method, are described. Section 4 describes our noise simulation method. In Section 5, the implementation of the noise simulation method, in the context of a nodal-analysis circuit simulation program (SPICE), is described. Two examples of noise simulation are presented in Section 6. Finally, future work is stated in Section 7.

2 Previous work

The electrical noise sources in passive elements and integrated-circuit devices have been investigated extensively. Small-signal equivalent circuits, including noise, for many integrated-circuit components have been constructed [1]. The noise performance of a circuit can be analyzed in terms of these *small-signal equivalent* circuits by performing sinusoidal circuit analysis in frequency domain in the usual fashion. This analysis is done separately for each of the uncorrelated noise sources, and for a range of frequencies. For a complicated circuit, the large number of noise sources and circuit complexity completely preclude hand calculation. In fact, even machine computation of the noise contributions from all noise sources can be time consuming. Fortunately, an extremely efficient computational technique, based on the interreciprocal adjoint network concept, was proposed [4][5]. This technique calculates the noise contribution from an arbitrarily large number of noise sources at a given frequency with little more computer time than is normally required for a single noise source. The noise analysis in SPICE is based on this method. Unfortunately, this method is only applicable to *linear time-invariant* circuits (e.g. the small-signal equivalent circuits corresponding to circuits with fixed operating points). It is not appropriate for noise simulation of circuits with changing bias conditions, or circuits which are not meant to operate in small-signal conditions.

[2,3] and [6] present noise analysis techniques for nonlinear circuits with a periodic large signal excitation. The noise analysis for a nonlinear circuit with a periodic large signal excitation reduces to the analysis of a *linear periodically time-varying* circuit with *cyclostationary* [2,3][6] noise sources. This is arrived by a first-order Taylor's expansion of the circuit equations around the *periodic steady-state* solution of the circuit without the noise sources and the small-signal excitations. This Taylor's approximation is similar to the one we will present in Section 4.1. The noise analysis methods described in [2,3] and [6] use frequency-domain methods based on manipulating *impulse responses* and *transfer functions* for a linear periodically time-varying system, and *spectral densities* for cyclostationary noise sources. These noise analysis techniques are applicable to only a limited class of nonlinear circuits with two excitations, where one of the excitations is large and periodic and the other is small (e.g., mixer circuits, switched capacitor circuits).

The previous work on noise simulation in time-domain is restricted to techniques which employ the Monte Carlo method [7]. This method has several drawbacks. Pseudo-random number generators often do not generate a large sequence of independent numbers, but reuse old random numbers instead. This becomes a problem if a circuit with many noise sources is simulated. This is usually the case, because every device has several noise sources associated with its model. In this method, the same circuit is simulated many times by obtaining "different" sample

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paths for each noise source. Then a statistical analysis is carried out to calculate averages and variances over these many simulations. The noise content in a waveform will be much smaller when compared with the magnitude of the waveform itself. As a result, the waveforms obtained for different sample paths of noise generators will be very close to each other. It is known that, in a simulator, these waveforms are only numerical approximations to the actual waveforms, therefore they contain *numerical noise*. The rms value of noise is calculated by taking a difference of these waveforms. That is, two large numbers, which have uncertainty in them, are being subtracted from each other. Consequently, the rms noise calculated with this method, in fact, includes the *noise* generated by the numerical algorithms. This furthermore degrades the accuracy of the results obtained by this method. This method has one advantage when compared with the frequency domain methods discussed above: It is not restricted to linear time-invariant, or to nonlinear circuits with a large signal periodic excitation. In theory, it is applicable to the general class of nonlinear dynamic circuits with any kind of excitation.

Our method, unlike the frequency domain methods, is not restricted to linear time-invariant or nonlinear circuits with a large signal periodic excitation. Our time-domain noise simulation method is based on the results from the theory of stochastic differential equations. There are no pseudo-random number generators involved in the simulation, therefore the problems associated with them do not exist. The simulation of the average waveforms (without noise in the circuit) and the simulation of noise are separated, even though they are done concurrently. Thus, the *numerical noise* problem that arises in Monte Carlo methods is avoided. Our method is capable of calculating *variances and covariances* (that is, the *covariance matrix*) for the noise content in the node voltages and other circuit variables in a circuit as a *function of time*. Furthermore, *correlations* between circuit variables at *different time points* can also be calculated. Finally, the implementation of our method fits naturally into a circuit simulator (such as SPICE) which is capable of doing time-domain transient simulations. Noise simulation is done along with the transient simulation over the time interval specified by the user.

3 Noise models

The electrical noise sources in passive elements and integrated-circuit devices have been investigated extensively, and appropriate models have been derived [1][8]. Traditionally, these noise models are presented as *stationary* noise sources in the *small-signal equivalent* (at an operating point) circuits of the devices [1]. In this section, we describe the *adaptation* of these noise models for use in our time-domain noise simulation method. In our method, the noise sources are inserted in the *large-signal models* of the integrated-circuit devices and they are, in general, *nonstationary*. In Section 3.1, the adaptation of *shot, thermal* and *flicker* noise models for resistors and junction diodes will be described. The noise models for these two simple devices are representative of noise models for all other integrated-circuit devices such as BJTs and MOSFETs, because all kinds of noise we consider (shot, thermal and flicker noise) exist in these devices [16]. The noise source models we use in our method are adapted from [1] and [8].

As it will become clear in Section 4, our noise simulation method requires that noise sources are *white*. The thermal and shot noise sources are modeled as white noise sources, hence they can be directly included in the simulation. However, the flicker noise sources can *not* be included in the simulation as they are. The inclusion of *flicker* noise sources into the noise simulation method will be described in Section 3.2.

3.1 Shot, thermal and flicker noise models

3.1.1 Resistors

Monolithic and thin-film resistors display thermal noise. The thermal noise in a resistor can be modeled by a white Gaussian noise current source with *intensity*

$$IN^{R}_{thermal} = \sqrt{2kT/R} \tag{3.1}$$

where k is Boltzmann's constant, T is the absolute temperature and R is the resistance [1]. The thermal noise source associated with a resistor is a *stationary* white noise process, assuming that the resistance value is a constant as a function of time. The *intensity* of a stationary white Gaussian noise process is equal to the square root of the power spectral density. For a stationary white Gaussian noise process, the power spectral density (a function of frequency) is a constant on the entire real axis.

3.1.2 Junction diodes

The series resistance r_s, in the model of a junction diode [1], is a physical resistor due to the resistivity of silicon, hence it exhibits thermal noise. The thermal noise in r_s can be modeled as in Section 3.1.1.

The pn-junction exhibits shot noise which is associated with the current flow through the diode. The *intensity* of the shot noise current, which is white Gaussian, is given by

$$IN^{D}_{shot}(t) = \sqrt{qI_D(t)} \tag{3.2}$$

where q is the electronic charge ($1.6\times10^{-19}C$), and $I_D(t)$ is the *noiseless* diode current. Note that, in this case, *intensity* is a function of time, hence this white noise source is not *stationary*. The square of the time-varying intensity for a *nonstationary* white noise source as above can be thought to be the *time-varying* power spectral density, which is a *constant* (as a function of frequency) on the entire real axis. During nonlinear operation, the current through the diode shows variations as a function of time, so does the intensity. In this way, shot noise associated with a time-varying current is modeled as a nonstationary white Gaussian noise, which is also the case for thermal noise associated with a time-varying resistance. (3.1) is also valid for a time-varying resistance [16].

The flicker noise source in a diode is modeled by a *nonstationary* noise process which has a *time-varying* power spectral density given by

$$S^{D}_{flicker}(f, t) = KF(I_D(t)^a)/f \tag{3.3}$$

where KF is a constant for a particular device, a is a constant in the range 0.5 to 2 and f is the frequency. This noise source can not be included in the noise simulation directly, because it is not white (i.e. the *time-varying* power spectral density is *not* a constant as a function of frequency). A way of synthesizing this source from white noise sources will be discussed in Section 3.2.

3.2 Flicker noise sources

In our noise simulation method, only *white* noise sources are allowed. Flicker noise sources have a power spectral density which is *not* a constant as a function of frequency. The natural way to include flicker noise sources into simulation is, somehow, to synthesize them using white noise sources. A promising approach for $1/f$ (flicker) noise generation is to use the summation of *Lorentzian spectra* which is defined by (3.4) [9]. It has been shown that a constant distribution of 1.4 poles per decade gives a $1/f$ spectrum with less than 1% error [9]. A sum of N Lorentzian spectra is given by

$$S(f) = \frac{2\sigma^2}{\pi}\sum_{h=1}^{N}\frac{\varphi_h}{\varphi_h^2+f^2} \tag{3.4}$$

where φ_hs designate the pole-frequencies and f is the frequency. It has been shown in [9] that $N = 20$ poles uniformly distributed over 14 decades are sufficient to generate $1/f$ noise over 10 decades with a maximum error less than 1%. Each Lorentzian spectrum in the summation in (3.4) can be easily obtained by using the *thermal* noise generator of a resistor R_h connected in parallel to a capacitance $C_h = C$, and their sum can be achieved by putting (Fig. 3.1) N of such $R_h - C_h$ groups in series [9].

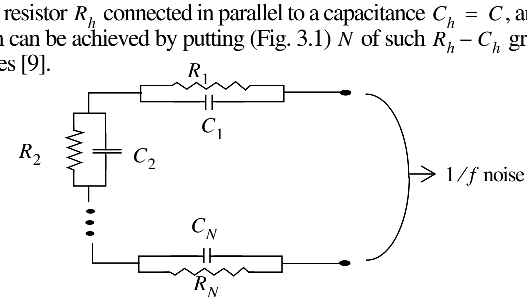

Figure 3.1: $1/f$ **Noise Synthesizing Circuit**

In the noise simulation, a flicker noise source in the model of an integrated-circuit device is built by using the circuit in Fig. 3.1 with an ideal *voltage-controlled current source*. This is illustrated in Fig. 3.2. The voltage-controlled current source is connected between the two nodes of a device where the flicker noise source is modeled.

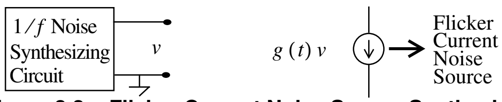

Figure 3.2: Flicker Current Noise Source Synthesis

The spectral density of the $1/f$ noise obtained from the circuit in Fig. 3.1 is approximately

$$S(f) = 2\sigma^2/\pi f \tag{3.5}$$

where $\sigma^2 = kT/(2C)$. This spectral density is time-invariant. The flicker noise model given in Section 3.1.2 requires a time-varying spectral density. This is achieved by having a time-varying *transconductance* ($g(t)$) for the voltage-controlled current source in Fig. 3.2. For instance, for a diode, we require that the flicker noise source spectral density is in the form given by (3.3). This is assured with

$$g(t) = \sqrt{\pi KF(I_D(t)^a)/(2\sigma^2)} \tag{3.6}$$

4 Development of the simulation method

The noise simulation method will be described assuming that modified nodal analysis (MNA) [10] is used for the formulation of circuit equations. MNA is the method for circuit equation formulation in most of the circuit simulators (such as SPICE) available. Translation of the noise simulation method into other ways of circuit equation formulation is straightforward.

4.1 Derivation of the stochastic differential equation for noise from MNA formulation of the nonlinear circuit equations

The MNA equations for any circuit, *without the noise sources*, can be written compactly as

$$F(\dot{x}, x, t) = 0 \qquad x(0) = x_0 \tag{4.1}$$

where x is the vector of the circuit variables with dimension n, $\dot{x}$ is the time derivative of x, t is time and F is mapping x, $\dot{x}$ and t into a vector of real numbers of dimension n. It is obvious that $x = x(t)$ and $\dot{x} = \dot{x}(t)$. The time dependence of x and $\dot{x}$ will not be written explicitly for notational simplicity. In MNA, the circuit variables consist of node voltages and branch currents for some elements (e.g. inductors and voltage sources). The circuit equations consist of the node equations (KCL) and branch equations of the elements for which branch currents are included in the circuit variables vector. Under some rather mild conditions (which are satisfied by well modeled circuits) on the continuity and differentiability of F, it can be proven that there exists a unique solution to (4.1) assuming that a fixed initial value $x(0) = x_0$ is given [10]. Let x_s be the solution to (4.1). The *transient analysis* in circuit simulators solves for x_s using numerical methods for solving ordinary differential equations (ODEs) [10]. The initial value vector $x(0) = x_0$ is obtained by a dc solution of the circuit before the transient simulation is started. For a circuit, there may be several different dc solutions.

The first-order Taylor's expansion of F around x_s is expressed as

$$F(\dot{x}, x, t) \cong F(\dot{x}_s, x_s, t) + \frac{\partial}{\partial x}F(\dot{x}, x, t)\bigg|_{\substack{x = x_s\\ \dot{x} = \dot{x}_s}}(x - x_s) + \frac{\partial}{\partial \dot{x}}F(\dot{x}, x, t)\bigg|_{\substack{x = x_s\\ \dot{x} = \dot{x}_s}}(\dot{x} - \dot{x}_s) \tag{4.2}$$

which will be used later.

If the noise sources are included in the circuit, the MNA formulation of the circuit equations can be written as

$$F(\dot{x}, x, t) + B(x, t)v = 0 \qquad x(0) = x_0 + x_{noise,0} \tag{4.3}$$

where $B(x, t)$ is an $n \times p$ matrix, the entries of which are a function of x, and v is a vector of p white Gaussian stochastic processes. A one-dimensional Gaussian white noise is a stationary Gaussian process $\xi(t)$, for $-\infty < t < \infty$, with mean $\varepsilon[\xi(t)] = 0$ and a constant spectral density on the entire real axis. The covariance function of $\xi(t)$ is given by $\varepsilon[\xi(s)\xi(t+s)] = \delta(t)$, where δ is Dirac's delta function [11]. The white Gaussian noise $\xi(t)$ is a very useful mathematical idealization for describing random influences that fluctuate rapidly and hence are virtually uncorrelated for different instants of time. A white Gaussian noise model is appropriate for *thermal* and *shot* noise in integrated circuits [1]. Flicker noise sources are taken care of in the way described in Section 3.2. v in (4.3) is simply a combination of p independent one-dimensional white Gaussian noise processes as defined above. These noise processes actually correspond to the current noise sources which are included in the models of the integrated-circuit devices. Since the noise models for the integrated-circuit devices are to be employed here in the context of an MNA circuit simulator (SPICE), noise sources in the devices are all modeled as *uncorrelated current* sources.

$B(x, t)$, in (4.3), contains the *intensities*, as described in Section 3.1, for the white noise sources in v. The *intensities* for these noise sources are, in general, a function of time (not a constant). Because of intensity variations, these noise sources are not *stationary*. Thus, the *nonstationarity* of the noise sources in the circuit are captured in $B(x, t)$. Every column in $B(x, t)$ corresponds to a noise source in v, and has either one or two nonzero entries [16].

(4.3) is a system of nonlinear stochastic differential equations (SDEs) where the forcing is an irregular stochastic process (white noise). This kind of SDEs require fundamentally different and complex methods of analysis and numerical solution [12]. Fortunately, some characteristics of our problem help us simplify the numerical solution of (4.3): The noise content in the signals in any useful circuit is, almost always, much smaller when compared with the signal itself.

Let x_{sn} be the solution of (4.3). x_{sn} is not deterministic, since it is the solution of the circuit equations with the noise sources included, and satisfies

$$F(\dot{x}_{sn}, x_{sn}, t) + B(x_{sn}, t)v = 0 \qquad x_{sn}(0) = x_0 + x_{noise,0} \tag{4.4}$$

where x_0 is deterministic, and $x_{noise,0}$ is a vector of n zero-mean random variables. We use (4.2) in (4.4) to approximate $F(\dot{x}_{sn}, x_{sn}, t)$, and we obtain

$$\begin{gathered} F(\dot{x}_s, x_s, t) + \frac{\partial}{\partial x}F(\dot{x}, x, t)\bigg|_{\substack{x = x_s\\ \dot{x} = \dot{x}_s}}(x_{sn} - x_s) + \\ \frac{\partial}{\partial \dot{x}}F(\dot{x}, x, t)\bigg|_{\substack{x = x_s\\ \dot{x} = \dot{x}_s}}(\dot{x}_{sn} - \dot{x}_s) + B(x_{sn}, t)v \cong 0 \\ x_{sn}(0) = x_0 + x_{noise,0} \end{gathered} \tag{4.5}$$

Defining

$$x_{noise} = x_{sn} - x_s \tag{4.6}$$

x_{noise} is, actually, the difference between the solutions of the circuit equations, with and without the noise sources. In other words, x_{noise} is the *noise content* in x_{sn}. x_{noise} is much smaller when compared with x_s, which validates the above approximation.

For notational simplicity, define

$$A(t) = \frac{\partial}{\partial x}F(\dot{x}, x, t)\bigg|_{\substack{x = x_s\\ \dot{x} = \dot{x}_s}} \qquad C(t) = \frac{\partial}{\partial \dot{x}}F(\dot{x}, x, t)\bigg|_{\substack{x = x_s\\ \dot{x} = \dot{x}_s}} \tag{4.7}$$

where $A(t)$ and $C(t)$ are $n \times n$ matrices with time-dependent entries.

Furthermore, we approximate

$$B(x_{sn}, t) \cong B(x_s, t) \tag{4.8}$$

and define

$$B(t) = B(x_s, t) \tag{4.9}$$

If (4.6), (4.7), (4.8) and (4.9) are substituted in (4.5) we obtain

$$\begin{gathered} F(\dot{x}_s, x_s, t) + A(t)x_{noise} + C(t)\dot{x}_{noise} + B(t)v \cong 0 \\ x_{noise}(0) = x_0 + x_{noise,0} - x_s(0) \end{gathered} \tag{4.10}$$

Since x_s is the solution of (4.1) we have

$$F(\dot{x}_s, x_s, t) = 0 \qquad x_s(0) = x_0 \tag{4.11}$$

and if we substitute (4.11) in (4.10), we obtain

$$\begin{gathered} A(t)x_{noise} + C(t)\dot{x}_{noise} + B(t)v = 0 \\ x_{noise}(0) = x_{noise,0} \end{gathered} \tag{4.12}$$

(4.12) is a *linear SDE* [11] in x_{noise} with time-varying coefficients. $A(t)$, $B(t)$ and $C(t)$ are functions of x_s, and they do not depend on x_{noise}. The solution of this equation will be discussed in the next four subsections.

4.2 Transformation of the stochastic differential equation for noise into state-equation form

To make use of some of the results from the theory of SDEs, (4.12) will be put into the form

$$\dot{y} = E(t)\,y + F(t)\,v \qquad y(0) = y_0 \tag{4.13}$$

If $C(t)$ is a full-rank matrix, this can be easily done by premultiplying both sides of (4.12) by the inverse of $C(t)$. However, this is not true in general; $C(t)$ may have zero columns. For instance, if a circuit variable is a node voltage, and if this node does not have any capacitors connected to it in the circuit, then all of the entries in the column of $C(t)$ corresponding to this circuit variable will be zero for all t. At this point, we should note that the zero-nonzero structure of $A(t)$, $B(t)$ and $C(t)$ is independent of t. So, some of the columns of $C(t)$ are structurally zero, independent of t. If we reorder the variables in x_{noise} in such a way that the zero columns of $C(t)$ are grouped at the right-hand side of the matrix, (4.12) becomes

$$\begin{gathered}\begin{bmatrix} A_1(t) & A_2(t) \end{bmatrix} \begin{bmatrix} x^1_{noise} \\ x^2_{noise} \end{bmatrix} + \begin{bmatrix} C_1(t) & 0 \end{bmatrix} \begin{bmatrix} \dot{x}^1_{noise} \\ \dot{x}^2_{noise} \end{bmatrix} + B(t)\,v = 0 \\ \begin{bmatrix} x^1_{noise}(0) \\ x^2_{noise}(0) \end{bmatrix} = \begin{bmatrix} x^1_{noise,0} \\ x^2_{noise,0} \end{bmatrix}\end{gathered} \tag{4.14}$$

where $A_1(t)$ and $C_1(t)$ are $n \times m$, $A_2(t)$ is $n \times k$, x^1_{noise} is an m-dimensional vector, x^2_{noise} is a k-dimensional vector, m is the number of nonzero columns in $C(t)$ and k is the number of zero columns. Naturally, $n = m + k$.

Then, expanding (4.14) and performing straightforward operations on this equation [16], we arrive at the SDE for noise in the state equation form, which is given by

$$\dot{x}^1_{noise} = E(t)\,x^1_{noise} + F(t)\,v \qquad x^1_{noise}(0) = x^1_{noise,0} \tag{4.15}$$

$$\begin{gathered} x^2_{noise} = D_1(t)\,x^1_{noise} + D_2(t)\,v \\ x^2_{noise,0} = D_1(0)\,x^1_{noise,0} + D_2(0)\,v(0) \end{gathered} \tag{4.16}$$

with

$$\begin{bmatrix} x^1_{noise} \\ x^2_{noise} \end{bmatrix} = x_{noise}(\text{reordered}) \tag{4.17}$$

Here, $E(t)$ is $m \times m$, $F(t)$ is $m \times p$, $D_1(t)$ is $k \times m$, $D_2(t)$ is $k \times p$, and they are obtained from $A_1(t)$, $A_2(t)$, $C_1(t)$ and $B(t)$ by performing some matrix algebra operations [16].

4.3 Solution of the stochastic differential equation for noise

(4.15) is a linear differential equation where the forcing is an irregular stochastic process which is *white noise.* A mathematically rigorous treatment of equations of this type requires a new theory. In 1951, Ito defined the *Ito* or *stochastic* integral and in doing so put the theory of SDEs on a solid foundation [11]. (4.15) is written symbolically as a linear SDE, but it is interpreted as an integral equation with *Ito* or *Stratonovich* stochastic integrals [11]. The solution of (4.15) obtained by the *Stratonovich* interpretation is equal to the one obtained by the *Ito* interpretation, because it is a *linear SDE in the narrow sense* [11]. A detailed explanation of Ito and Stratonovich stochastic integrals and stochastic differential equations can be found in [11], [12] and [13]. In the following development, we state and use some of the results from the theory of SDEs.

(4.15) is often written in the form

$$dx^1_{noise} = E(t)\,x^1_{noise}dt + F(t)\,dw \qquad x^1_{noise}(0) = x^1_{noise,0} \tag{4.18}$$

where w is a vector of p independent one-dimensional Wiener processes. A p-dimensional Wiener process can be defined as a process with independent and stationary, $N(0, (t_1 - t_2)\,I_p)$-distributed increments $w(t_1) - w(t_2)$, with initial value $w(0) = 0$. Here, $N(Mean, Cov)$ denotes the p-dimensional normal distribution with expectation vector $Mean$ and covariance matrix Cov [11]. A Wiener process can be thought to be the "integral" of a white noise, or, alternatively, white noise is the "derivative" of a Wiener process in the sense of coincidence of the covariance functionals [11]. In our case, we have

$$w(t) = \int_0^t v(\tau)\,d\tau \qquad v(t) = \dot{w}(t) \tag{4.19}$$

As with ordinary differential equations, the general solution of a linear SDE can be found explicitly. The method of solution also involves an integrating factor or, equivalently, a fundamental solution of an associated homogeneous differential equation. The solution of (4.15) is given by

$$x^1_{noise}(t) = \phi(t, t_0)\,x^1_{noise}(t_0) + \int_{t_0}^t \phi(t, \tau)\,F(\tau)\,dw(\tau) \tag{4.20}$$

where $\phi(t, \tau)$ is the matrix determined as a function of t by the homogeneous differential equation

$$d\phi/dt = E(t)\,\phi \qquad \phi(\tau, \tau) = I_m \tag{4.21}$$

(4.20) involves an *Ito* integral as opposed to a Riemann integral [11]. The integral in (4.20) can not be interpreted as an ordinary Riemann integral, because almost all sample functions of $w(t)$ are of unbounded variation. Ito's definition of the stochastic integral includes the ordinary Riemann integral as a special case [11]. If the functions $E(t)$ and $F(t)$ are "measurable" and bounded on the time interval of interest, there exists a unique solution for every initial value $x^1_{noise}(t_0)$ [11]. We are interested in the case where

$$x^1_{noise}(0) = x^1_{noise,0} \tag{4.22}$$

In our problem, it is sufficient to find the *probabilistic characteristics* of x^1_{noise} as a function of t. In other words, we would like to determine the *mean* and the *covariance matrix* of x^1_{noise} as a function of time in the time interval desired. If x^1_{noise} is a *Gaussian* stochastic process, then it is *completely* characterized by its mean and covariance function as a function of time. Further explanation on this topic will be given in Section 4.5. If we substitute (4.22) in (4.20) with $t_0 = 0$ we obtain

$$x^1_{noise}(t) = \phi(t, 0)\,x^1_{noise,0} + \int_0^t \phi(t, \tau)\,F(\tau)\,dw(\tau) \tag{4.23}$$

If we take the expectation of both sides of (4.23) we get the mean of x^1_{noise} which is a function of t. Considering that $\varepsilon[v(t)] = 0$ and $\varepsilon[x^1_{noise,0}] = 0$, we get

$$m^1(t) = \varepsilon[x^1_{noise}(t)] = 0 \tag{4.24}$$

Next, we would like to determine the covariance matrix of the components of x^1_{noise} as a function of t, which is given by

$$K^1(t) = \varepsilon[x^1_{noise}(t)\,x^1_{noise}(t)^T] \tag{4.25}$$

since mean is zero as given by (4.24). Consider

$$\begin{gathered} dx^1_{noise}x^1_{noise}{}^T = x^1_{noise}dx^1_{noise}{}^T + (dx^1_{noise})\,x^1_{noise}{}^T + \\ F(t)\,F(t)^T dt \end{gathered} \tag{4.26}$$

Notice that there is an extra term in (4.26) which would not be there if we were using ordinary calculus instead of *stochastic,* or *Ito* calculus. This equation is obtained from *Ito's Theorem* [11] using (4.18). We use (4.18) to expand (4.26) and obtain

$$\begin{gathered} dx^1_{noise}x^1_{noise}{}^T = (E(t)\,x^1_{noise}x^1_{noise}{}^T + x^1_{noise}x^1_{noise}{}^T E(t)^T)\,dt + \\ F(t)\,F(t)^T dt + x^1_{noise}(F(t)\,dw)^T + (F(t)\,dw)\,x^1_{noise}{}^T \end{gathered} \tag{4.27}$$

If we take the expectation of both sides of this equation, noting that x^1_{noise} and dw are uncorrelated and using (4.25), we get

$$\dot{K}^1(t) = E(t)\,K^1(t) + K^1(t)\,E(t)^T + F(t)\,F(t)^T \tag{4.28}$$

where $K^1(t)$ is the unique symmetric nonnegative-definite solution of the matrix equation (4.28) with the initial value $K^1(0) = \varepsilon[x^1_{noise,0}(x^1_{noise,0})^T] = K^1_0$. Calculation of the initial value K^1_0 will be described in Section 4.4. The differential equation for $K^1(t) = K^1(t)^T$, (4.28), satisfies the Lipschitz and boundedness conditions in the time interval of interest, so that a unique solution exists [11].

(4.28) represents (in view of symmetry of $K^1(t)$) a system of $m(m+1)/2$ linear differential equations. (4.28) can be solved for $K^1(t)$ using a numerical method (such as Backward Euler) for the solution of ODEs.

$K^1(t)$ represents the noise covariance matrix of circuit variables as a function of time. So, the information about the noise variances of circuit variables, or the noise correlations between circuit variables at a given time point are contained in $K^1(t)$. In some problems, one might be interested in the noise correlations of circuit variables at different time points, which can be expressed as

$$K^1(t_1, t_2) = \varepsilon[x^1_{noise}(t_1)\, x^1_{noise}(t_2)^T] \tag{4.29}$$

In a similar way to the derivation of (4.28), one can derive

$$\frac{\partial}{\partial t_2} K^1(t_1, t_2) = K^1(t_1, t_2)\, E(t_2)^T \tag{4.30}$$

with the initial condition $K^1(t_1, t_1) = K^1(t_1)$ [13]. Integrating (4.30) at various values of t_1, one can obtain a number of sections of the covariance function $K^1(t_1, t_2)$ at $t_2 > t_1$. Then, $K^1(t_1, t_2)$ at $t_2 < t_1$ is determined by

$$K^1(t_1, t_2) = K^1(t_2, t_1)^T \tag{4.31}$$

4.4 Calculation of the initial value for the linear ODE for the covariance matrix of the components of x^1_{noise}

In the last subsection, we have derived a linear ODE, (4.28), for the covariance matrix of x^1_{noise}. In order to be able to solve (4.28), we need to know the initial value K^1_0. We set K^1_0 to the solution of the following matrix equation in P

$$E(0)P + PE(0)^T + F(0)F(0)^T = 0 \tag{4.32}$$

The matrix equation (4.32) has a symmetric nonnegative-definite solution P, if the equation $\dot{z} = E(0)z$ is asymptotically stable (that is, if all the eigenvalues of $E(0)$ have negative real parts) [11]. (4.32) represents (in view of symmetry of P) a system of $m(m+1)/2$ linear equations.

It is interesting to analyze the special case of noise simulation when the circuit is *linear time-invariant*, or *nonlinear dynamic with dc excitations* [16]. In this case, noise simulation reduces to solving the linear equation system (4.32) [16].

4.5 The condition for x^1_{noise} to be Gaussian

The noise in the circuit (solution of (4.15)) is a *Gaussian* stochastic process if and only if the initial value $x^1_{noise,0}$ is normally distributed or constant [11]. Up to this point, we have characterized the initial value $x^1_{noise,0}$ as being an m-dimensional vector of zero-mean random variables with the covariance matrix given by the solution of (4.32). Here, we restrict $x^1_{noise,0}$ to be a vector of zero-mean *normally distributed* random variables with the covariance matrix given by the solution of (4.32). With this restriction on the initial value $x^1_{noise,0}$, x^1_{noise} (solution of (4.15)) is a *Gaussian* stochastic process, *nonstationary* in general, and it is completely characterized by its mean, (4.24), and covariance function (given as the solution of (4.28) and (4.30) as a function of time). For linear time-invariant, or nonlinear dynamic circuits with dc excitations, x^1_{noise} is a *stationary* (in the strict sense) *Gaussian* process, completely characterized by its covariance matrix (a constant function of time as given by the solution of (4.32)) [16].

5 Implementation in SPICE

The noise simulation method described in Section 4, along with the noise models described in Section 3, was implemented inside the circuit simulator SPICE3 [14]. Time-domain noise simulation is done along with the *transient simulation* in the time interval specified by the user.

The transient simulation in SPICE3 solves for x_s, which is the solution of (4.1), using numerical methods for solving ordinary algebraic-differential equations. The initial value vector $x(0) = x_0$ in (4.1) is obtained by a dc solution before the transient simulation is started. The numerical methods for solving (4.1) subdivide the time interval [0,T], in which the transient simulation is to be performed, into a finite set of distinct points:

$$t_0 = 0, t_R = T, t_{r+1} = t_r + h_{r+1} \qquad r = 0, 1, \ldots, R. \tag{5.1}$$

where h_{r+1}s are the time steps. At each time point t_r, the numerical methods compute an "approximation" $x_s[r]$ of the exact solution $x_s(t_r)$ [10].

The noise simulation (solution of (4.28) and (4.30)) is done *concurrently* with the transient simulation. (4.28) represents a system of $m(m+1)/2$ linear differential equations. We currently use the *Backward Euler* scheme to discretize these equations in time.

At each time point t_r, after the transient simulation routines have calculated $x_s[r]$, the matrices $A[r] \cong A(t_r)$, $C[r] \cong C(t_r)$ and $B[r] \cong B(t_r)$, as defined by (4.7) and (4.9), are calculated using the values in $x_s[r]$. The routines for loading these matrices have been written for each device. The routines for loading $B[r]$ contain the noise models for the devices, which are described in Section 3. Then the operations described in Section 4.2 are performed to calculate $E[r] \cong E(t_r)$ and $F[r] \cong F(t_r)$ from $A[r]$, $C[r]$ and $B[r]$, using *sparse* matrix data structures and routines. Then, $E[r]$ and $F[r]$ are used to calculate $K^1[r] \cong K^1(t_r)$ in the discretized solution of (4.28) with the Backward Euler scheme. This last operation requires the solution of $m(m+1)/2$ simultaneous linear equations, because Backward Euler is an *implicit* method [10]. Here, m is, roughly, the number of nodes to which a capacitor is connected. Simulations have shown that, for larger circuits, the CPU time spent for this last operation at a time point *heavily* dominates the CPU time required by the other operations. Most of the CPU time is used for solving systems of linear equations. We currently use a general-purpose, direct method, sparse matrix solver to solve systems of linear equations. With this direct method linear solver, the computational cost of noise simulation is still high for large-scale circuits. Experiments with several circuits have shown that significant speedup can be obtained by using a *parallel iterative* linear solver (running on a CM-5) [17], especially for larger circuits. CPU times obtained with this parallel iterative solver suggest that even using a sequential version of this iterative solver will reduce the computational cost of noise simulation considerably when compared with the CPU times obtained with the direct solver we currently use.

The operations described in the above paragraph are performed at every time point. Upon completion, $x_s[r] \cong x_s(t_r)$, $r = 0, \ldots, R$ contains the *mean* waveforms for the circuit variables as a function of time, which is the usual SPICE transient simulation output. And $K^1[r] \cong K^1(t_r)$, $r = 0, \ldots, R$ contains the waveforms for the covariance matrix of the noise contents in the circuit variables, as defined by (4.25) as a function of time, which is the noise simulation output.

6 Noise simulation examples

In this section, we present two examples of noise simulation. In particular, noise simulations for a CMOS ring-oscillator circuit and a BJT active mixer circuit will be presented. For both of these circuits, we have included only the shot and thermal noise sources in the simulation. One reason for this is that flicker noise has little effect on the noise performance of these circuits. Secondly, including the flicker noise sources increases the simulation time because of the extra nodes created for flicker noise source synthesis.

6.1 CMOS ring-oscillator

Three CMOS inverters loaded with 1 pF capacitors were connected in a ring-oscillator configuration and a noise simulation was done. In Fig. 6.1, the *mean* and *noise variance* of one of the taps of this ring-oscillator can be seen. As seen in Fig. 6.1, the noise at one of the taps of the ring-oscillator is *nonstationary*, that is, the noise variance is *not* a constant as a function of time. The noise variance is *highest* during low-to-high and high-to-low *transitions* of the tap voltage.

Ring-oscillator based VCOs and delay-lines are used in many phase/delay-locked systems such as clock generators and clock recovery circuits. Phase noise/jitter is a major concern in the design of such systems. Behavioral models which capture noise effects, and behavioral simulation is used to predict the phase noise/jitter performance of these systems [15]. Our transistor-level noise simulator can be used to simulate ring-oscillator VCOs and delay-lines to obtain the timing jitter at the outputs of the delay cells (as well as the correlations between the jitters.) This information is then used in behavioral simulation [15].

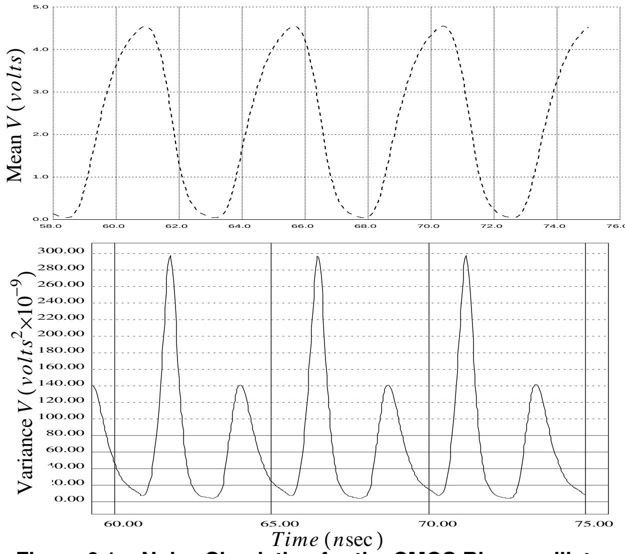

Figure 6.1: Noise Simulation for the CMOS Ring-oscillator

6.2 BJT Active Mixer

This circuit was obtained from industry sources. It contains 14 BJTs, 21 resistors, 5 capacitors, and 18 parasitic capacitors connected between some of the nodes and ground. The LO (local oscillator) input is a sinewave at 1.75 GHz with an amplitude of 178 mV. The RF input is a sinewave at 2 GHz with an amplitude of 31.6 mV. Thus, the IF frequency is 250 MHz. $1/f$ noise sources are not included in the simulation, because $1/f$ noise is rarely a factor at RF and microwave frequencies [2].

This circuit was simulated to calculate the noise variance at the output as a function of time. (Fig. 6.2: This waveform is periodic with a period of 4 nsecs: IF frequency is 250 MHz.) The noise at the output of this circuit is *not* stationary, because the signals applied to the circuit are large enough to change the operating point. The noise analysis of this circuit by assuming a small-signal equivalent circuit around a fixed operating point does not give correct results. Such an analysis would predict the noise at the output as stationary, i.e. a constant noise variance as a function of time.

The noise performance of a mixer circuit is commonly characterized by its *noise figure* which can be defined by [1]

$$NF = \frac{\text{total output noise}}{\text{that part of the output noise due to the source resistance}} \qquad (6.1)$$

This definition is intended for circuits in small-signal operation. For such circuits, noise figure is a scalar quantity. In our case, the noise at the output of the mixer circuit changes as a function of time over one period. We can generalize the noise figure definition such that noise figure is a quantity that is a function of time. For the mixer circuit we have simulated, the noise figure turns out to be a periodic function of time. To calculate the noise figure as defined, we simulate the mixer circuit again to calculate the noise variance at the output with all the noise sources turned off except the noise source for the source resistance $RS_{RF} = 50\Omega$ at the RF port. Then we can calculate the noise figure as below, and the result is shown in Fig. 6.3.

$$NF(t) = 10\log\left(\frac{\text{Total Noise Var. } V_{out}(t)}{\text{Noise Var. } V_{out}(t) \text{ due to the source res.}}\right) \qquad (6.2)$$

As observed in Fig. 6.3, the maximum and minimum value of the noise figure over one period differs by over 4 dB.

This BJT mixer circuit has 65 nodes (including the internal nodes for BJTs) which are connected to capacitors. The noise simulation requires the solution of 2145 $(65 \times 66/2)$ simultaneous linear equations at every time point, as it was explained in Section 5. The simulation (with 250 time points) took approximately 17 hours on a DECstation 5900/260 with our current implementation (with the direct method linear solver).

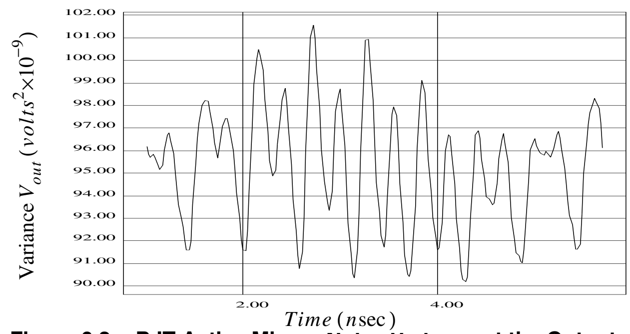

Figure 6.2: BJT Active Mixer - Noise Variance at the Output

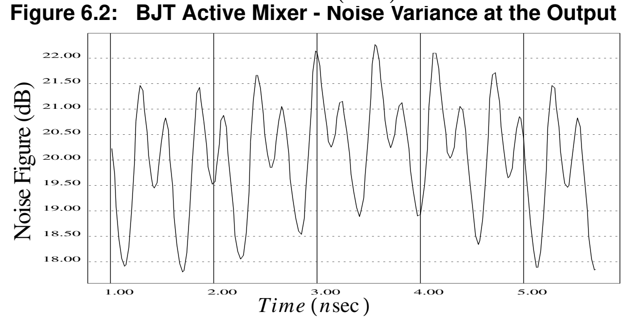

Figure 6.3: BJT Active Mixer - Noise Figure

7 Future Work

We plan to compare the results from this noise simulator with noise measurements on actual circuits. The numerical methods used in the noise simulator will be modified to make it more efficient (as explained in Section 5). We will be using our transistor-level noise simulator in the top-down constraint-driven design of a clock generator circuit for a RAMDAC. The noise simulator will be used to extract *noise parameters* in the *behavioral modeling* of phase/delay-locked loops [15].

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