Delay Analysis of Distribution Coordination Function in IEEE 802.11

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***Abstract*— Distributed Coordination Function is the crucial component of IEEE 802.11 standard and has been examined over the past years. We extended the Markov Model to formulate the delay characteristic of the network. Our approach is unique in terms of finding the probability mass function of the time interval between successful transmissions of a station. These results are essential for predicting the performance of the system for QoS implementations.**

I. Introduction

Wireless multimedia is emerging with the proliferation of WLANs. It is important to study the delay performance of IEEE 802.11 [1]-[4] because the quality of service of any implementation depends on a tight delay budget [5]-[8]. Distributed Coordination Function (DCF) is the highly implemented access mechanism of IEEE 802.11. DCF implements CSMA/CA which requires a waiting time which is picked uniformly from [0,CW] after each transmission. If the transmission is unsuccessful, contention window (CW) is doubled and otherwise it is reset as seen in Figure 1. In recent years, the DCF has been analyzed extensively by the research community [8]-[30]. The famous analytical analysis for DCF is called Bianchi's model [11], which we call 802.11^b from now on. Our model 802.11^+ is taken from [10] which is an extended version of [11].

In DCF, the inter-arrival time (*jitter*) between successful packet transmission of a station is a i.i.d random variable. Previous works for delay analysis are presented in [31]-[37]. In this paper, we focus to formulate expected value and variance of the *jitter* as well as its *probability-mass-function* with respect to increasing number of stations and increasing load.

Next Section talks about the 802.11^+ and in Section III, we introduce the delay analysis. We discuss the performance in Section IV and we conclude the paper in Section V.

II. Markov Model

802.11^b model assumes *saturated* traffic, i.e., every station has a packet to transmit all the time. In this case, medium is fully utilized by transmissions and *empty-slots* [8]. If we define *virtual-slot* as a variable length slot which is either an *empty-slot* or a transmission or a reception for a station then a *virtual-slot* of a station in a given time is the same length for all others in a fully connected network. This leads to obtain the medium utilization by considering only one station and if wireless channel is modeled as two state Markov chain, which

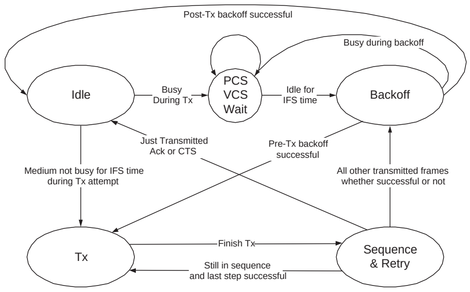

Fig. 1. DCF finite state machine

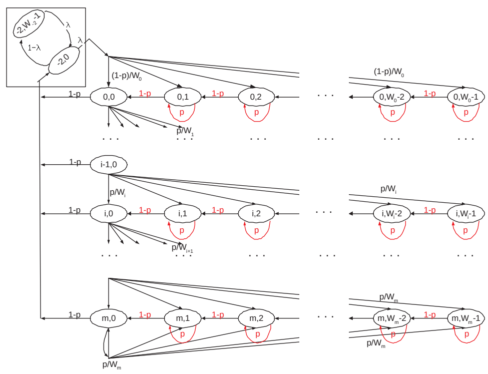

Fig. 2. Markov model for the IEEE 802.11 DCF in normal traffic condition 802.11^+

is *busy* with probability p and *idle* with probability $(1-p)$: i.e., a station's transmission is successful with probability $(1-p)$, we can find the probability that how many stations are transmitting in a given *virtual-slot*.

To model normal, *unsaturated* traffic, 802.11^+ introduce additional states, represented in Figure 2. The best way of introducing *idle* state into the Markov Model is to decrease the transmission probability [8].

Under *unsaturated* conditions, a station may now wait in the idle state for a packet from upper layers. This corresponds to a delay in the idle state, represented by the box in Figure 2. The delay in the idle state is geometric random variable with parameter λ. The transition probabilities are straightforward modifications of those previously obtained for the saturated case [10], [11].

If (i,j) is the state and $b_{i,j}$ is the stationary probability, the

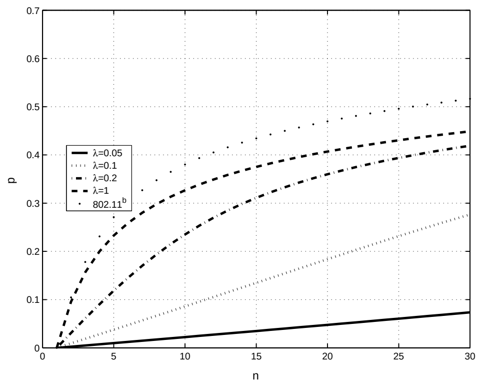

Fig. 3. Probability of collision (p)

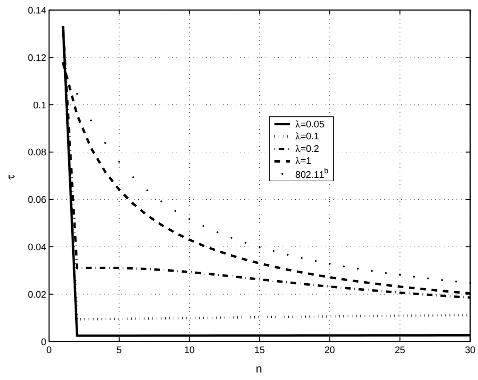

Fig. 4. Probability of transmission (τ)

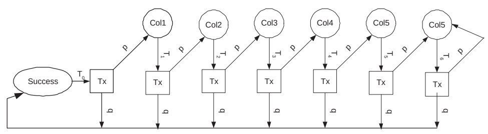

Fig. 5. Delay Analysis

following equation holds;

$$1 = \sum_{i=0}^{m} \sum_{k=0}^{W_i - 1} b_{i,k} + \sum_{k=0}^{W_{-2}-1} b_{-2,k} = \textit{backoff} + \textit{idle}. \quad (1)$$

The probabilities $b_{-2,0}$ and $b_{-2,1}$ can be expressed in terms of $b_{0,0}$ by:

$$idle = \sum_{k=0}^{W_{-2}-1} b_{-2,k} - 1 = \frac{b_{0,0}}{\lambda^2} - 1.$$

Then, the transmission probability of a station τ^+ is given by

$$\tau^+ = \frac{1}{\frac{(1-2p)(W+1)+pW(1-(2p)^m)}{2(1-2p)(1-p)} + (1-p)(\frac{1}{\lambda^2} - 1)}, \quad (2)$$

which reduces to τ^+ for the *saturated* case ($\lambda = 1$) [10]. From (2), we see that $\tau^+ = \tau^+(p, m, W, \lambda)$ depends on the unknown p [11],

$$p = 1 - (1 - \tau^+)^{n-1} \text{ or } \tau^+(p) = 1 - (1-p)^{\frac{1}{(n-1)}}. \quad (3)$$

Equations (2) and (3) together determine τ and p. Figure 3 and 4 plot the collision probability p and transmission probability τ as the number of stations increases: the 802.11^b gives higher values of p and τ than our model for $\lambda = 1$. In general, as expected, p increases and τ decreases with n. Also, τ increases with load λ, which is readily appreciated by taking $n \to 1$, $p \to 0$, for which

$$\lim_{p \to 0} \tau = \frac{1}{\frac{(W+1)}{2} + (\frac{1}{\lambda^2} - 1)}. \quad (4)$$

For the saturated case, $\lambda = 1$, and $m = 0$ (no exponential backoff), we can compare τ^+ with τ^b in [11],

$$\tau^+(p, 0, W, 1) = \frac{2(1-p)}{W+1} < \tau^b = \frac{2}{W+1}. \quad (5)$$

Unlike in 802.11^b, τ^+ depends on the collision probability p (and hence on n). Intuitively of course, τ should depend on n: if there are more stations, the medium will be busy more often, and a station will transmit less frequently.

III. Delay Model

We can easily infer from the analytical Markov Model of the DCF that if we define t_i as the i^{th} time when a station sends a packet successfully, the random variable $(t_i - t_{i-1})$ and $(t_{i+1} - t_i)$ are independent. Observing only one interval can give the delay characteristic of the network. As stated earlier, the interval starts and ends with a successful packet transmission of the same station. After a successful transmission, a station resets its backoff counter to CW_{min} and selects a backoff counter value in random from $[0, CW_{min}]$. Then it waits in the backoff process and decrements the counter whenever there is idle slot time (σ). It suspends decrementing the counter when there is activity in the medium. Let's call random variable $\top_i$ the time taken to come to backoff counter 0 from the selected backoff counter value in backoff level i. When the backoff counter is 0, the station transmits the packet. The transmission may be successful with probability q, or it can experience a collision with probability p, where $p+q = 1$. We know that the transmission time is fixed and given by T_s and T_c, for successful and unsuccessful transmission, respectively. If the transmission is unsuccessful, then the station doubles its contention window and selects another backoff counter. Of course after the 6^{th} try the contention window reaches maximum CW_{max} and stays there [1]. Therefore, the random variable for waiting time in backoff level is $\top_6$ for $i > 6$.

Figure 5 illustrates the delay structure. From the successful transmission to another, all possibilities are presented. Note that in the 802.11^+ model (See Figure 2), the virtual slot is defined differently from 802.11^b [10], [11]. Transmission times must be included since we know that when backoff counter is 0, a transmission time is definitely consumed. When $\lambda = 1$, the 802.11^+ model operates in the saturated traffic condition, otherwise the traffic is unsaturated [8], [10].

If station transmits successfully after its X^{th} unsuccessful attempt the total delay Γ is

$$\Gamma = \sum_{i=0}^{\infty} 1(X = i)\Gamma_i \tag{6}$$

and Γ_i is

$$\Gamma_i = T_s + iT_c + \sum_{k=0}^{i} \top_k,\ \top_k = \top_6\ for\ k > 6. \tag{7}$$

Probability mass function of delay $P(\Gamma)$ is

$$P(\Gamma) = \sum_{i=0}^{\infty} Pr(X = i)P(\Gamma|X = i) \tag{8}$$

where $Pr(X = i)$ is qp^i and $P(\Gamma|X = i)$ requires convolution of $P(\top)$s because they are independent. Expected value $E(\Gamma)$ can be represented by

$$E[\Gamma] = \sum_{i=0}^{\infty} Pr(X = i)E[\Gamma_i] \tag{9}$$

where $E[\Gamma_i]$ is

$$E[\Gamma_i] = T_s + iT_c + \sum_{k=0}^{i} E[\top_k],\ \top_k = \top_6\ for\ k > 6 \tag{10}$$

and $Var[\Gamma]$ is

$$Var[\Gamma] = \sum_{i=0}^{\infty} Pr(X = i)E[\Gamma_i^2] - (E[\Gamma])^2. \tag{11}$$

The main element of these equations is finding the $\top$ for each backoff level. We look at $\top$ for 802.11^+ and 802.11^b models. As it can be inferred easily $\top$ has a uniform distribution for the 802.11^b model since average length of a virtual event is used in order to identify the time that is passed for a station to decrement its backoff counter by one. On the other hand for 802.11^+ the time to decrement the backoff counter by one can range from a slot time to forever. This is because of the self loop in the backoff states.

If we consider the backoff level probabilities of 802.11^+ [8], we can approximate that they resemble a uniform distribution except at the head and tail. We can find from the first order statistics that they both give approximately similar results.

IV. Performance Analysis

The following results are obtained with IEEE 802.11b PHY parameters where stations are with 1Mbps data rate and transmission time is $T_s = T_c = 0.0012sec.$

Figure 6 shows the average delay for a successful transmission. As one can infer from the figure, average delay is increasing with the number of stations. This is expected since probability of successful transmission decreases with the number of stations which affects the collision probability. As a result stations wait longer and experience more collisions than before.

Figure 7 illustrates the variance of the delay. As expected variance increases with the number of stations. The main reason why 802.11^+ model shows a higher delay is because of the self-loop in backoff since it takes consecutive transmission into consideration which introduces more randomness than before. It is not guaranteed that the station decrements the backoff counter in every event.

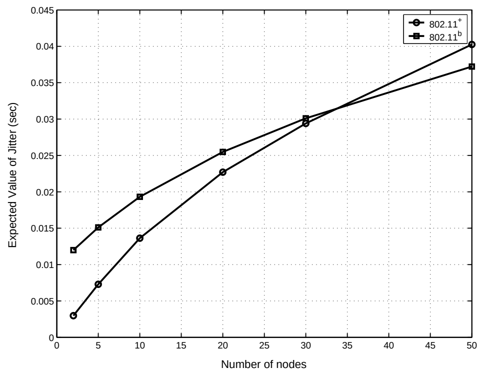

Fig. 6. Average delay with respect to number of stations ($E[\Gamma]$)

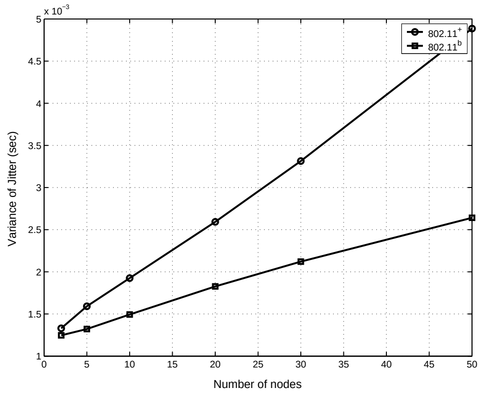

Fig. 7. Variance of the delay with respect to number of stations ($Var[\Gamma]$)

Figure 8 shows the probability mass function for a network with 30 stations. The result is semi-averaged meaning that average time for a virtual event is used. The actual length of a virtual event is a random variable and has a pmf as follows:

$$\begin{aligned} P(virtual - event) &= (1 - P_{tr})\delta(t - \sigma) \\ &+ P_s\delta(t - T_s) \\ &+ (P_{tr} - P_s)\delta(t - T_c). \end{aligned} \tag{12}$$

One can modify the formula for unsaturated traffic (US), $\lambda \leq 1$. The first order statistics are as follows: expected value $E[\Gamma_{US}]$ can be represented by

$$E[\Gamma_{US}] = \frac{1-\lambda}{\lambda}\sigma + E[\Gamma] \tag{13}$$

and $Var[\Gamma_{US}]$ is

$$Var[\Gamma_{US}] = \frac{1-\lambda}{\lambda^2}\sigma^2 + Var[\Gamma] \tag{14}$$

Figures 9 and 10 show the delay characteristics with respect to increasing load for different network sizes. One can see that delay increases with the load and with the number of stations.

V. Conclusion

We introduced a unique formulation to obtain the delay characteristic of distributed coordination function of IEEE

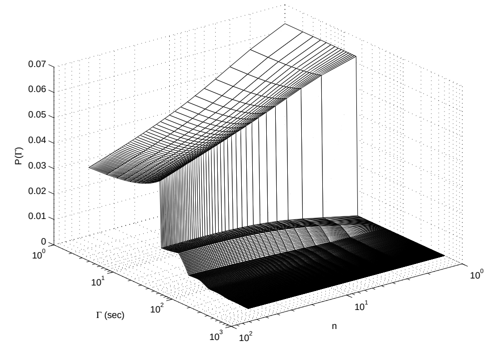

Fig. 8. Probability Mass Function of the Delay ($P(\Gamma)$)

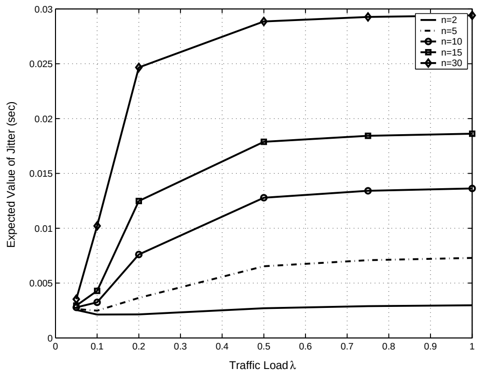

Fig. 9. Average delay with respect to load ($E[\Gamma_{US}]$)

802.11. We used analytical Markov model to find the delay characteristics of the time interval between two successful transmissions of a station. We formulated the average delay and standard deviation as well as the the probability mass function with respect to number of stations and finite load.

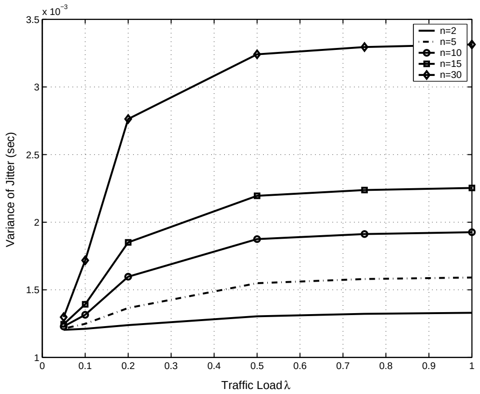

Fig. 10. Variance delay with respect to load ($Var[\Gamma_{US}]$)

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