A Distributed Localization System for Sensor Networks

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***Abstract*— We propose a distributed algorithm for single acoustic source localization. The algorithm is based on the local estimate of time-difference-of-arrival (TDOA) at each pair of sensors, and the distributed implementation is based on the projection onto convex sets (POCS) principle. Convergence of the algorithm in the is discussed and an approximate closed form expression is given in the projection problem. Simulation results show the performance of the algorithm compared to a centralized approach. We have implemented and tested this algorithm on a wireless sensor network test-bed. The test-bed is based on the measurements of a set of microphones, pairwise synchronized and connected via an *ad-hoc* wireless network, and performs real-time single source localization in a low reverberant room with no need for offline processing.**

I. Introduction

Acoustic localization with microphone arrays has been an active research area for many years. However, it is still a challenging task to develop efficient algorithms that approach the performance of standard methods but perform the data processing in a distributed, robust and scalable way. In this paper we propose a distributed algorithm for single-source acoustic localization based on the TDOA. Distributed localization is a research area that is still largely unexplored. In this class of algorithms there is no notion of fusion center that collects information from other sensors. Instead, sensors exchange information to localize the source in a distributed way. A distributed implementation of the incremental gradient algorithm for energy based localization is proposed in [1]. In [2], energy based localization problem is formulated as a POCS problem, and a distributed implementation of the POCS problem is shown. In this paper, we follow the same POCS approach and show a decentralized implementation of the localization algorithm based on the TDOA. Our algorithm is characterized by very low complexity and can be used for quickly providing a rough estimate of the source location, that can be possibly used for initiating a centralized localization algorithm. Our distributed approach is scalable, since the localization error decreases with the number of nodes in the network, while the computation requirement at each sensor node remains constant. Thus, this approach is well suited for performing acoustic localization in large scale wireless networks.

Various methods can be used for localizing an acoustic source, e.g. the angle-of-arrival, the energy of the received signal or the TDOA. TDOA based localization methods require sensor synchronization. When nodes are spatially separated and connected by a wireless network, achieving precise synchronization is a challenging task. In general synchronization algorithms require a heavy signaling payload in the network, which implies a significant energy consumption at each node and a consequent reduction in node lifetime. The scheme proposed in this paper does not require full synchronization rather only synchronized sensor pairs. This can be achieved by connecting two microphones to the same sampling board. The localization algorithm was implemented in a testbed used for video camera steering. Experimental results show that the algorithm quickly identifies the rough location of the source.

The paper is organized as follows. In Section II, we define the TDOA based localization problem and introduce a centralized non-linear least-squares (LS) estimator. In Section III, we formulate the localization problem as a convex feasability problem and propose a distributed localization algorithm. In Section IV, we present simulation results comparing our scheme to the LS centralized method. Finally, in Section V we present a distributed Windows-based testbed we implemented for single source localization and camera steering applications. Experimental results are presented.

II. Centralized source localization problem

In this section we introduce a centralized LS-based localization estimator. We consider an acoustic source located in at some unknown location $\mathbf{r}_s = [r_{s_x} \ r_{s_y} \ r_{s_z}]^T$. We assume to have $2R$ microphones divided into pairs, where microphones in a pair are separated by a fixed distance $2L$. The received signals at each pair of microphones are sampled synchronously at frequency f_c. Each pair is located at some known global location $\mathbf{r}_i = [r_{i_x} \ r_{i_y} \ r_{i_z}]^T, \quad i = 1, \dots, 2R$. We denote as j_1 and j_2 the two sensors in the *j*th pair. We are interested in estimating the location of the acoustic source, given the received samples and the location of each sensor $\mathbf{r}_i, \quad i = 1, \dots, 2R$. We assume that an estimate D_i of TDOA is available at the i-th microphone pair. The estimate can be computed, for instance, with algorithms based on the Generalized Cross Correlation function (GCC) [3], defined as the time cross-correlation function of filtered versions of the sensors' received signals. The TDOA estimates of each pair of sensors are sent to a fusion center. The central node uses the

TDOA estimates along with knowledge of the sensor positions to arrive at a source location estimate. Given D_j, the sound source $\mathbf{r}_s$ must lie in a three dimensional hyperboloid with two sheets given by $d(\mathbf{r}_{j_1} - \mathbf{r}_s) - d(\mathbf{r}_{j_2} - \mathbf{r}_s) = D_j$, where $d(\mathbf{x}, \mathbf{y})$ indicates the Euclidian distance between vectors **x** and **y**. Let us denote with $\mathbf{D} = [D_1 \ \ldots \ D_R]^T$ the set of distances from the source to every sensor pair. The vector **D** identifies a set of hyperboloids and, as a consequence, the location of the source must be the unique intersection point of this set of hyperboloids. However, errors in the TDOA estimation may introduce errors in **D**, and there might not exist a point which interpolates all the hyperboloids. So, given R sensor pairs, we estimate the source location as the vector $\hat{\mathbf{r}}_s$ which minimizes the LS error with the vector **D**. Thus, the source location is estimated as

$$\hat{\mathbf{r}}_s = \arg \min_{\mathbf{r} \in \mathcal{R}^3} \sum_{i=1}^{R} |d(\mathbf{r}_{i_1} - \mathbf{r}) - d(\mathbf{r}_{i_2} - \mathbf{r}) - D_i|^2 \qquad (1)$$

The non-linear LS minimization problem in (1) is not convex, and standard methods like the incremental gradient are not guaranteed to converge to the global minimum. Depending on the initial condition in the iterative algorithm used, a method can converge to a local optimum or a saddle point.

III. Distributed localization algorithm

The algorithm proposed in this section solves the localization problem based on TDAO in a distributed manner. A pair of microphones computes locally an estimate of the source location using the TDOA measurement and the knowledge of its global coordinates in the network. The source location estimate is passed from pair to pair. Each pair updates the location estimate using its local TDOA measurement and then passes the updated estimate to the next pair. We formulate the acoustic localization problem based on TDOA as a convex feasibility problem, i.e. as the problem of finding a point in the intersection of a family of convex sets. Consider the localization problem in the absence of measurement noise: each TDOA estimate identifies a locus of possible source location, a hyperboloid. Then, the localization problem can be solved by finding the intersection point of a set of hyperboloids. Let us define the *convex* hyperboloid identified by the *j*th pair as

$$\mathcal{C}_j = \{\mathbf{r} \in \mathcal{R}^3 : d(\mathbf{r}_{j_1} - \mathbf{r}) - d(\mathbf{r}_{j_2} - \mathbf{r}) \leq D_j\}. \qquad (2)$$

Then, the convex feasibility problem consists of finding the point

$$\hat{\mathbf{r}} \in \mathcal{C} = \bigcap_{j=1}^{R} \mathcal{C}_j. \qquad (3)$$

If $\mathcal{C} \neq \emptyset$ the convex feasibility problem is called consistent, otherwise it is inconsistent. The problem can be solved using standard techniques, e.g. the method of successive orthogonal POCS [4]. The POCS method is given by the algorithmic iterative step

$$\mathbf{r}_{s,k+1} = \mathbf{r}_{s,k} + \sigma_{j(k)} \left(\mathcal{P}_{\mathcal{C}_{j(k)}}(\mathbf{r}_{s,k}) - \mathbf{r}_{s,k} \right). \qquad (4)$$

where $\{\sigma_{j(k)}\}_{k \geq 0}$ is a sequence or relaxation parameters (a sequence of numbers $\in (0;2)$), $j(k) = k \mod R$ is a cyclic control sequence over the index set $0, 1, \ldots, R$, and $\mathcal{P}_{\mathcal{C}_j}(\mathbf{r})$ is the orthogonal projection of the point **r** onto the convex set $\mathcal{C}_j$:

$$\mathcal{P}_{\mathcal{C}_j}(\mathbf{r}) = \arg \min_{\mathbf{p} \in \mathcal{R}^3} |\mathbf{r} - \mathbf{p}|. \qquad (5)$$

In [5], it is shown that in the consistent case the POCS methods converges to a point in $\mathcal{C}$, while in the inconsistent case it converges to a cyclic subsequence. In [6], a convergence result for the inconsistent case is shown, under some conditions on the POCS parameters $\sigma_{j(k)}$. If the TDOA estimates are perfect, the set of hyperboloids intersects in a point corresponding to the source location, and the convex feasibility problem is consistent. Fig. 1(a) shows a two-dimensional view of the algorithm, where three pairs of sensors are located on the walls of a 3m x 3m room with the sound source located at [-2.1m,-1.7m]. Each pair identifies an hyperbola, the set of possible source locations given the measured TDOA. The hyperbolas (solid line) are well approximated by the asymptote lines (dashed line). Starting from a random initial estimate $R_{s,0}$, pair number 3 computes $R_{s,1}$ as the orthogonal projection of $R_{s,0}$ onto the corresponding hyperbola, then sends the updated estimate to pair number 2. Similarly, pair number 2 uses its local measurements to compute $R_{s,2}$. The location estimate is passed from pair to pair and the estimate eventually converges to the intersection of the three hyperbolas.

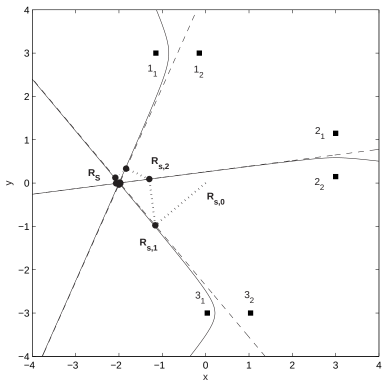
However, the TDOA estimates are always subject to estimation errors, due to quantization, reverberation etc. In general the hyperboloids do not intersect. Thus, the POCS problem can be inconsistent and converge to a cycle, i.e. each sensor's source estimate converges to a different value. In this case, we can use the result in [6, Theorem 18] to guarantee the convergence. If $\sigma_{j(k)}$ goes to 0 as k increases, in a way that

$$\lim_{k \to \infty} \sigma_{j(k)} / \sigma_{j(k+1)} = 1 \text{ and } \sum_k \sigma_{j(k)} = \infty, \qquad (6)$$

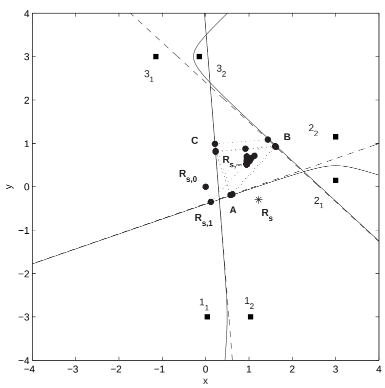
then iterative algorithm converges to a point that minimizes the sum of the distances to the convex sets $\mathcal{C}_j, j = 1, \ldots, R$. Hence, in the inconsistent case our source location estimator is defined as

$$\hat{\mathbf{r}}_s = \arg \min_{\mathbf{r} \in \mathcal{R}^3} \sum_{j=1}^{R} |\mathbf{r} - \mathcal{P}_{\mathcal{C}_j}(\mathbf{r})|. \qquad (7)$$

The relaxation parameters can be set as follow. Initially all the POCS parameters are set equal to one. At each iteration, consistency of the POCS is checked. If convergence to a cycle is detected, then the relaxation parameters are decreased at a rate of $1/k$. This relaxation sequence satisfies (6) and thus leads to convergence to the point defined in (7). Conditions 6 imply a slowdown in the convergence rate of the algorithm. An alternative estimator consists in computing the average of the points in the cyclic convergence. An example of the convergence of the POCS is shown in Fig. 1(b). Initially, the algorithm converges to the cycle A-B-C. Then, the POCS



(a) Noiseless case.



(b) Noisy case.

Fig. 1. 2D Example of convergence to a fixed point.

parameters are modified to guarantee convergence to $R_{s,\infty}$, the point that minimizes the distance from the three convex sets identified by the three hyperbolas.

A. Conic approximation

The POCS method as formulated in (4) requires, at each iterative step, the solution of a minimization problem associated with the projection (5). The projection of a point onto the set in (2) does not have closed form solution. In this section, we provide an approximate close form expression for the projection operator in the POCS. The idea consists of approximating the hyperboloid identified by the TDOA with a cone, and computing the projection of a point onto a cone. In the application we are targeting, the source can be assumed to lie at a distance which is great compared to the microphones spacing, i.e. $d(\mathbf{r}_{i_1} - \mathbf{r}) >> D,\ i = 1, 2, \ldots, R$. Under such an assumption, the direction of the source is given by the hyperboloid asymptote. Fig. 1(a) illustrates the 2D case, where the hyperbolas identified by the TDOA are approximated by lines, the hyperbola asymptotes. In the 3D case, the hyperboloid can be approximated with the cone generated by rotating the asymptote line about the axis of each pair of microphones. In this case, the localization problem reduces to finding the intersection of a set of cones. Let us consider the *j*th pair of microphones at the *k*th iteration of the distributed algorithm. Assume that the microphones j_1 and j_2 of the pair are located at $(-L, 0, 0)$ and $(L, 0, 0)$ respectively, according to a local coordinate system $(X, Y, Z)_j$. The pair's global coordinates are supposed known, as well as the mapping for expressing the local coordinates in terms of global coordinates. Denote this mapping as T_j, and denote as T_j^{-1} the inverse mapping from global coordinates to local coordinates. Given the distance measurement D_j, we saw that the source lies on a hyperboloid. The hyperboloid can be expressed in the local coordinates of the *j*th pair as the set (X, Y, Z) satisfying $\frac{X^2}{c_1} - \frac{Y^2+Z^2}{c_2} = 1$, where $c_1 = \frac{D_j^2}{4}$ and $c_2 = \frac{L^2 - D_j^2}{4}$. The hyperboloid is well approximated by the cone $\frac{X^2}{c_1} = \frac{Y^2+Z^2}{c_2}$. Thus, the convex set in (2) can be approximated as

$$\mathcal{C}_j^c = \left\{ \mathbf{r} \in \mathcal{R}^3 : \frac{X^2}{c_1} \le \frac{Y^2 + Z^2}{c_2} \right\}. \tag{8}$$

The equation of the cone can also be expressed in local parametric equations as $[X, Y, Z] = [c_1 u, c_2 u \cos t, c_2 u \sin t]$. From the sign of D_j and the location in the room, the local node knows whether u is positive and whether t belongs to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ or $[\frac{\pi}{2}, \frac{3\pi}{2}]$. For simplicity, we assume here that $D_j > 0$, $u > 0$ and $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ The pair receives an estimate $\mathbf{r}_k = [x_k, y_k, z_k]^T$ of the source location. This is first converted into local coordinates as $\mathbf{R}_k = T_j \mathbf{r}_k = [X_k, Y_k, Z_k]^T$. Next, the new estimate is computed as the projection of $\mathbf{R}_k$ on the cone $\mathcal{C}_j^c$. This problem has a closed form solution: $\mathbf{R}_{k+1} = [u^*, u^* \cos t^*, u^* \sin t^*]^T$, where

$$\begin{aligned} t^* &= \arctan \frac{Z_k}{Y_k} \\ u^* &= \frac{1}{c_1^2 + c_2^2} \left(c_1 X_k + c_2 \sqrt{Y_k^2 + Z_k^2} \right). \end{aligned}$$

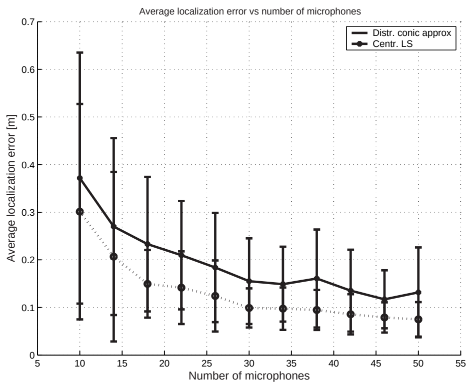
Finally,

$$\mathbf{r}_{k+1} = T_j^{-1} \mathbf{R}_{k+1}. \tag{9}$$

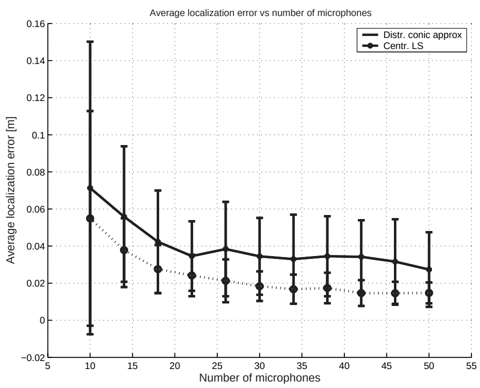
Given this closed form expression, it is clear that the computation requirement at each step of the iterative algorithm is extremely small.

IV. Simulation results

The main objective of the simulations presented in this section is to evaluate the performance of the distributed algorithm under noisy TDOA estimate. We simulate a distributed microphone network in a 6 m long, 2.5 m wide, and 3m room. The dimensions of the room are set to match the experimental setup used in Section V. There are R pairs of microphones located randomly on the walls of the room. One acoustic source is located randomly in the room. Each pair samples the received signal at frequency f_c. Each TDOA estimate is subject to a quantization error due to the finite sampling frequency. Hence, each pair's measurement is the true TDOA plus a quantization error term that is uniformly distributed in $[-\frac{v}{f_c}; \frac{v}{f_c}]$. The performance of the distributed algorithm described in Section III is compared to that of a centralized approach, where the location estimate is obtained solving (1). This is solved using the steepest descent method initiated from the middle point of the room. The projection

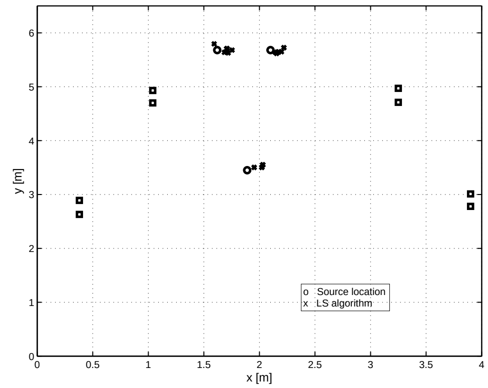


(a) $f_c = 8KHz$.

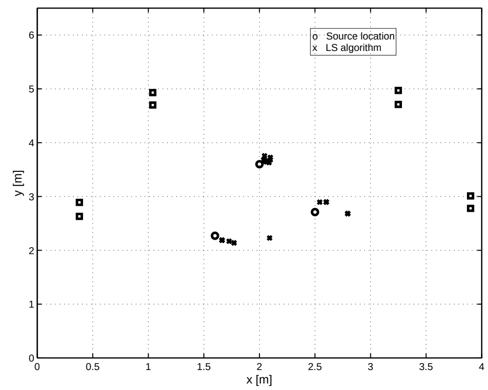


(b) $f_c = 44KHz$.

Fig. 2. Localization error performance with error in TDOA.



(a) Centralized LS algorithm (1).



(b) Centralized LS algorithm (2).

Fig. 3. experimental results using distributed algorithm.

problem in the distributed algorithm is solved using the closed form expression (9). We select the order of the sensors in the cyclic control sequence randomly. We increment the number of sensors R from 10 to 50, running 200 Monte Carlo simulations at each step. Figs. 2(a) and 2(b) summarize the results of the simulations. The average localization errors of the nonlinear LS estimator and the POCS estimator are shown for sampling frequency of 8KHz and 44KHz. As shown in the figures, performance of the LS estimator cannot be achieved by the POCS algorithm. However, the estimation error of the POCS method decreases with the number of sensors. Since the complexity in each node (evaluation of (4) and (9)) does not increase with the number of nodes in the network, the scheme is well suited for deployment in large scale sensor networks.

V. Real-time Testbed

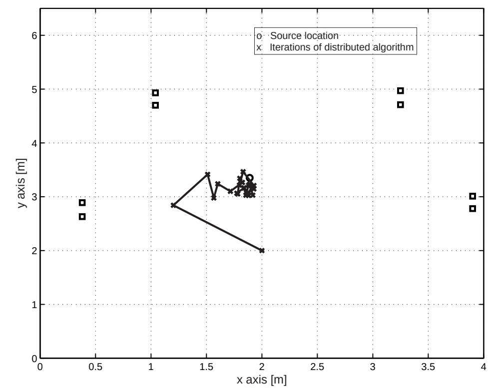
In this section, we describe the implementation of a real-time acoustic distributed localization system for a single-source in moderate reverberant environments. The application we are targeting is the localization of a speaker in a conference room [7]. A video camera is steered according the position information. The testbed we developed is completely automated and performs real-time signal processing, i.e no offline processing is necessary for localization. At this stage, we only assume that the sensor locations are known.

A. Experimental setup

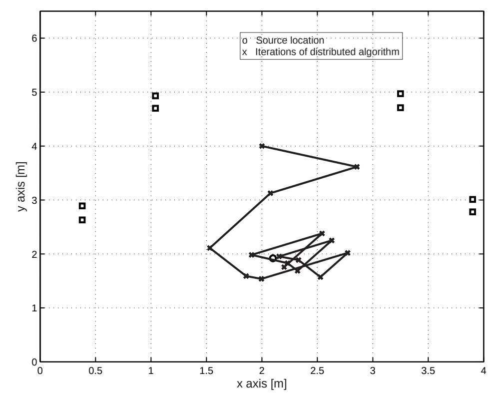
We implemented the testbed as a Windows-based wireless sensor network. The hardware system consists of eight microphones, five laptops and one video camera. The testbed has been used for a real-time camera steering application, a distributed version of [9]. Each pair of microphones is connected to the line-in of a laptop running Windows XP. The microphones in each pair are placed $0.25m$ away from each other. Microphone locations are pre-measured and stored in each laptop. All the signal processing is implemented in Matlab. The remaining laptop is used as an information sink for the centralized implementation of the localization algorithm. The central node is connected to the video camera, which is steered towards the speaker. When the algorithm is distributed, the central node is only used for steering the camera. The camera we tested is a Sony EVI-D71. Nodes communicate through a TCP/IP connection, via Wireless LAN. We connected the laptops through an *ad-hoc* 802.11a wireless network. The testbed was implemented in a typical office room which is 6 m long, 2.5 m wide, and 3 m high, with various encased and protruding spaces. The level of noise in the room was comparable to typical office noise level.

B. Experimental results

We conducted several experiments to demonstrate the effectiveness of the testbed in real-time localization. Two types of algorithm were considered, namely a centralized scheme that solves the non-linear LS problem in (1) and the distributed algorithm described in Section III using the approximate closed-form expression for the projection problem (9). In our experiments a single acoustic source (a loud speaker) playing pre-recorded music is placed somewhere in the room. Each node periodically (period of 250ms) samples the received sound signals at the two microphones with sampling frequency of 44KHz and performs the TDOA estimation using the GCC/PHAT method. If the centralized localization algorithm is used, one estimate per period is sent to the central node. The central node solves the LS problem whenever an updated TDOA is received, and sends a steering command to the video camera. If the distributed algorithm is used, then each node updates the location estimate using its local TDOA measurement and then passes the updated estimate to the next pair using the *ad-hoc* network. Each pair of microphones is assigned a number and the cyclic control for the POCS method is selected in accordance to the order of the nodes. The experimental performance of the centralized source localization results are shown in Figs. 3(a) and 3(b). The plots collect several experimental results, where the speaker is placed in six distinct locations in the room. For each speaker position, we play $10s$ of the pre-recorded sound and plot fifteen consecutive localization estimates produced solving the LS problem at the central node. We note that since the microphones are placed on the walls of the room, the source is always inside the convex hull of the microphones location. We are considering a three-dimensional localization problem, yet the plots only represent the points on the (x, y) plane. We note that the localization error does not change significantly with the different speaker positions. Within a few seconds, the camera steers in the right direction. From these results, the LS algorithm seems promising for the implementation of a distributed localization system for video camera steering. Figs. 4(a) and 4(b) show the convergence path of the first 15 iterations for two different positions of the speaker. The starting point was set to some random location in the room. The accuracy of the distributed algorithm is sufficient for video camera steering applications. The POCS algorithm quickly approaches the area where the source is located, but very high location accuracy might require many iterations. So, this method is well suited for providing the starting point for a centralized localization algorithm.



(a) Distributed algorithm (1).



(b) Distributed algorithm (2).

Fig. 4. experimental results using distributed algorithm.

VI. Conclusion

In this paper, we propose a distributed algorithm for localizing an acoustic source measuring the TDOA at pairs of microphones. We described a wireless sensor network testbed and presented some results using this testbed for localizing a single source in a reverberant room where the position of the microphones is known. In the future, we aim to implement the distributed algorithm in a real wireless sensor network. Future works also include an extension to multiple source localization.

VII. Acknowledgment

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