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Optimization-based Formation Reconfiguration Planning For Autonomous Vehicles

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Abstract

Given a group of autonomous vehicles, an initial configuration, a final configuration, a set of inter- and intra-vehicle constraints, and a time for reconfiguration, the Formation Reconfiguration Planning problem is focused on determining a nominal input trajectory for each vehicle such that the group can start from the initial configuration and reach its final configuration at the specified time while satisfying the set of inter- and intra-vehicle constraints. In this paper, we are interested in solving the Formation Reconfiguration Planning problem for a specific class of systems and a particular form of input signals so that the problem can be reformulated as an optimization problem which can be solved more efficiently, especially for a large group of vehicles.

1 Introduction

Advances in sensing, communication and computation are revolutionizing the development of advanced control technologies for distributed, multi-vehicle systems. These advances also enable the conduct of missions deemed impossible in the recent past. Autonomous formations have applications anywhere there is a task to be done requiring a group effort with minimal human supervision. Space applications benefit from formation control of satellites to perform distributed observations. In automated highway systems (AHS), cars organize themselves in platoons to increase highway throughput. Groups of unmanned aerial vehicles (UAVs) perform search and rescue collectively in restricted areas where human intervention is dangerous. To perform deep sea exploration, autonomous underwater vehicles (AUVs) must maintain tight formations due to limited bandwidth for communication and low visibility. In order to perform a set of predetermined missions, each vehicle is equipped with the necessary sensing, communication, and computation capabilities.

Recent years have seen the emergence of autonomous formation planning and control as a topic of great interest. In [1] and [2] , by considering vehicles as linear systems, the problem of stabilizing a set of vehicles according to a given graph structure by utilizing only relative measurements is studied. The feasibility for keeping vehicles in such a given formation is studied in [1]. [4] also studies the feasibility for keeping vehicles in formation, while also considering a kinematical model of vehicle. Given a formation and vehicle dynamics, various control strategies have been developed based on information flow and group organization. In [2], a distributed control law for stabilizing the formation is derived for keeping a feasible formation around the group equilibrium point. A control design that preserves mesh stability of a group of vehicle is presented in [3] where a leader-follower organization is considered in the control design and only local information is used. In [5] a different control strategy based on virtual leaders and artificial potentials in order to keep a stable formation is considered. A method of formation reconfiguration planning and control for a group of vehicles in order to avoid obstacles is presented in [6]. The reconfiguration planning is based on enumeration of formation graphs to obtain next possible formation that can be used to navigate in the environment.

We are interested in the formation reconfiguration problem. In particular, we first consider if the problem is feasible assuming that all the information is accessible. Given a feasible problem, we will then be able to derive the necessary information flow and group organization for decentralized formation planning and control. Here, we are interested in solving the feasibility problem. The *Formation Reconfiguration Planning* (FRP) problem addressed in this paper is:

Problem 1.1 *Given a group of autonomous vehicles, an initial configuration, a final configuration, a set of inter- and intra- vehicle constraints, and a time for reconfiguration determine a nominal input trajectory for each vehicle such that the group can start from the initial configuration and reach its final configuration at the specified time while satisfying the set of inter- and intra-vehicle constraints.*

In this paper, we are interested in solving the FRP problem for a specific class of systems and a particular form of input signals so that we can represent the problem as an optimization problem that can be solved more efficiently especially for a large group of vehicles.

In particular, a point mass model is used to model dynamics of each autonomous vehicle. Although the simple dynamical model is used, the results can be naturally extended to systems that can be feedback linearized [7] such as S/VTOL aircraft [8], PVTOL aircraft [9] and helicopters[10].

The paper structure is as follows. We will begin by formulating the formation reconfiguration planning problem. Then we will discuss our approach to solving the problem. This will be followed by a design example using our solution and a presentation of some results gathered by simulating the design example. The paper will end with conclusions.

2 Problem Formulation

Consider a group of autonomous vehicles with the following dynamics

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t)) \qquad (1)$$

where the i^{th} vehicle state $x_i \in \mathbb{R}^n$, the i^{th} vehicle input $u_i \in \mathbb{R}^m$ and $i = 1, \cdots, N$. Each $f_i : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is assumed to be as smooth as needed. The admissible input for each vehicle is specified by an input constraint $b_i(u_i(t)) \leq \alpha_i$ such as $\|u_i(t)\| \leq \alpha_i$. Denoting the group state as $x = [x_1^T, \cdots, x_N^T]^T$ and the group input as $u = [u_1^T, \cdots, u_N^T]^T$, the group dynamics can be rewritten as

$$\dot{x}(t) = F(x(t), u(t)) \qquad (2)$$

where

$$F(x(t), u(t)) = \begin{bmatrix} f_1(x_1(t), u_1(t)) \\ \vdots \\ f_N(x_N(t), u_N(t)) \end{bmatrix}$$

with $x \in \mathbb{R}^{nN}$ and $u \in \mathbb{R}^{mN}$. Assume that all the inter- and intra-vehicle constraints are specified as a set of group state constraints $c_j(x(t)) \leq \beta_j$ for $j = 1, \cdots, M$. Especially, since we are interested in generating collision-free paths, a minimal separation requirement between vehicles is introduced such that each vehicle can keep a safe distance from any vehicle in the group. Thus, the minimal separation requirement for each pair of vehicles can be encoded as a group state constraint and hence there are $\frac{N(N-1)}{2}$ minimal separation constraints.

Define the group configuration at time t as $g(t) = [x^T(t), u^T(t)]^T$ which specifies the state and input conditions for all the vehicles in the group at time t.

In a mission, a cost function is given as a part of the mission specification and in general can be written as $J = \varphi(x(T), T) + \int_0^T L(x(t), u(t), t)dt$ where $\varphi(x(T), T)$ and $L(x(t), u(t), t)$ define the terminal cost and the running cost, respectively. Hence, if there are feasible solutions for the FRP problem as specified in Problem 1.1, it is desirable to find the optimal one with respect to the given cost function. Now, we restate our FRP problem as follows:

Problem 2.1 (FRP Problem) *Given a group dynamics, an initial group configuration g_s, a final group configuration g_f, a set of inter- and intra-vehicle constraints $b_i(u_i(t)) \leq \alpha_i$ for $i = 1, \ldots, N$ and $c_j(x(t)) \leq \beta_j$ for $j = 1, \ldots, M$, and the time for reconfiguration T, does there exit a group input $a(t)$ for $t \in [0, T]$ such that the group starting from $g(0) = g_s$ can reach $g(T) = g_f$ while satisfying the set of inter- and intra- vehicle constraints? If so, then select the group input $a(t)$ over $[0, T]$ which produces minimal value for a given cost function.*

The Formation Reconfiguration Planning (FRP) problem can be formulated as an optimal control problem[11, 12, 13] with dynamical and algebraic constraints as follows:

$$\min_{u(t)} J = \varphi(x(T), T) + \int_0^T L(x(t), u(t), t)dt \qquad (3)$$

subject to

$$\begin{aligned} \dot{x}(t) &= F(x(t), u(t)) && (4)\\ g(0) &= g_s && (5)\\ g(T) &= g_f && (6)\\ b_i(u_i(t)) &\leq \alpha_i \ \forall t \in [0, T] \ \forall i \in \{1, \ldots, N\} && (7)\\ c_j(x(t)) &\leq \beta_j \ \forall t \in [0, T] \ \forall j \in \{1, \ldots, M\}. && (8) \end{aligned}$$

The optimal control problem in principle can be solved by applying standard techniques described in [11, 12, 13] based on calculus of variations or on Pontryagin's maximum principle. However, for a large group of vehicles these techniques become computationally inefficient since the performance of these techniques scales poorly not only with the number of states but also with the number of inter- and intra- constraints which increase rapidly with the number of vehicles. For example, the number of minimal separation constraints grows in the order of $O(N^2)$.

In this paper, we are interested in solving the FRP problem for a specific class of systems and a particular form of input signals so that the problem can be reformulated as an optimization problem that can be solved more efficiently especially for a large group of vehicles.

3 Approach

In order to reduce the problem complexity, we parameterize each input signal $u_i(t)$ over the interval $[0, T]$ by a set of parameters $\theta_{ki} \in \mathbb{R}^m$ for $k = 1, \cdots, K$. Hence, for $t \in [0, T]$ we have

$$u_i(t) = \sum_{k=0}^{K} \theta_{ki}\omega_k(t)$$

where $\omega_k : \mathbb{R} \to \mathbb{R}$ are basis functions for input $u_i(t)$. Many families of basis functions such as B-splines can be chosen and each would provide different advantages on representation and efficiency. This approach is proposed in [14] for solving many motion planning problems. In this paper, polynomials are used as basis functions and hence

$$u_i(t) = \sum_{k=0}^{K} \theta_{ki} t^k \quad (9)$$

where $\omega_k(t) = t^k$ for $k = 1, \ldots, K$. However, the selection of the order of polynomials K is problem dependent. In the next section, we will show how to pick the order of polynomials for an application. Therefore, the FRP problem becomes:

$$\min_{\theta_{01},\ldots,\theta_{KN}} \; J = \varphi(x(T), T) + \int_0^T L(x(t), u(t), t)dt \quad (10)$$

subject to

$$\dot{x}(t) = F(x(t), u(t)) \quad (11)$$
$$g(0) = g_s \quad (12)$$
$$g(T) = g_f \quad (13)$$
$$b_i(u_i(t)) \leq \alpha_i \; \forall t \in [0, T] \; \forall i \in \{1, \ldots, N\} \quad (14)$$
$$c_j(x(t)) \leq \beta_j \; \forall t \in [0, T] \; \forall j \in \{1, \ldots, M\}. \quad (15)$$

Depending on applications, various cost functions could be considered. However, the same set of constraints specified by (10)-(15) has to be satisfied regardless of which cost function is chosen. Once the feasible parameter range is obtained, then one can solve the FRP problem by searching for minimal value of the cost function over the range. Here, we are interested in the existence of solution for the FRP problem.

Problem 3.1 (Existence of Solution for FRP) *Given the FRP problem specified by (10)-(15), does there exit a set of parameters $\theta_{11}, \ldots, \theta_{KN}$ such that all the constraints (11)-(15)can be satisfied? If so, then determine the feasible range of the parameters.*

For certain classes of systems, Problem 3.1 can be solved by using computational tools.

Theorem 3.2 *Given the FRP problem specified by (10)-(15) if $F(x(t), u(t)) = Ax(t) + Bu(t)$, where A is a $nN \times nN$ nilpotent matrix, B is a $nN \times mN$ matrix, and constraints specified by (12)-(15) are semi-algebraic constraints, there exits a computational procedure that decides whether there exits a set of parameters that satisfies (12)-(15).*

The system equation $\dot{x}(t) = Ax(t) + Bu(t)$ with nilpotent matrix A and polynomial input $u(t)$ belongs to a family of linear differential equations with decidable reachability problem [15]. Theorem 3.2 can be proved by posing the reachability computation as a quantifier elimination problem in the decidable theory of the reals. There are quantifier elimination tools that can perform symbolic computation and answer the existence problem. Since the problem is proved to be decidable for this class of systems, the computation is guaranteed to terminate in finite steps.

A clear illustration on how to apply the theory is provided in [15] for a single robot navigation problem. Furthermore, a feasible range of the parameters is also provided. The current algorithms for solving quantifier elimination are not able to handle problems with a large number of constraints or high order polynomials.

4 Design example

Now, we focus on the point-mass dynamics of N vehicles. The dynamics of each vehicle is then specified by a double integrator which is:

$$\begin{bmatrix} \dot{p}_i(t) \\ \dot{v}_i(t) \end{bmatrix} = \begin{bmatrix} v_i(t) \\ a_i(t) \end{bmatrix} \quad (16)$$

where $p_i, v_i, a_i \in \mathbb{R}^3$ and $i = 1, \cdots, N$. Define $x_i(t) = [p_i^T(t) \; v_i^T(t)]^T$, $u_i(t) = a_i(t)$. Hence, (16) can be written as $\dot{x_i}(t) = A_i x_i(t) + B_i u_i(t)$ with

$$A_i = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix}, \; B_i = \begin{bmatrix} 0_{3\times3} \\ I_{3\times3} \end{bmatrix}.$$

Thus, the group dynamics can be written as $\dot{x}(t) = Ax(t) + Bu(t)$ where $A \in \mathbb{R}^{nN\times nN}$ and $B \in \mathbb{R}^{nN\times nM}$ with $x = [x_1^T \cdots x_N^T]^T \in \mathbb{R}^{nN}$ and $u = [u_1^T \cdots u_N^T]^T \in \mathbb{R}^{nM}$.

Given the input signals, the state trajectories of each vehicle can be derived according to the vehicle dynamics. In particular, we have the following equations $v_i(t) = \int_0^t a_i(t)dt + v_i(0)$ and $p_i(t) = \int_0^t v_i(t)dt + p_i(0)$ Therefore, if the input trajectories and the initial conditions are provided, we can derive the state trajectories. The input signals are parameterized as polynomials of time. $a_i(t)$ is the acceleration vector of the i^{th} vehicle and it can represented as:

$$a_i(t) = \sum_{k=0}^{K} a_{ki} t^k \quad (17)$$

where K is the order of the polynomial, and $a_{0i} \cdots a_{Ki}$ are the parameter vectors for the i^{th} vehicle. As with $a_i(t)$, all of the a_{ki} parameter vectors are in $\mathbb{R}^3$.

Given the initial configuration g_s, the final configuration g_f, and the time for reconfiguration T, the state trajectories are constrained by the four vector equations $a_i(0) = a_{i0}$, $a_i(T) = \sum_{k=0}^{K} a_{ki}T^k$, $v_i(T) = \int_0^T a_i(t)dt + v_i(0)$, $p_i(T) = \int_0^T v_i(t)dt + p_i(0)$ for $i = 1, \ldots, N$. Therefore, in order to obtain feasible solutions for the FRP problem by considering only the dynamical and configuration constraints, $K \geq 4$. The necessary order of polynomials K thus depends on the remaining constraints.

Now, we are ready to formulate the FRP problem for autonomous vehicles. The objective is to determine the parameters for the input trajectories, minimizing the input energy, subject to dynamical, configuration, minimum vehicle proximity and maximum acceleration constraints. In general, other cost functions and constraints could be used, but we found energy, minimum proximity, and maximum acceleration to be most necessary to this problem.

$$\min_{a_{01}\cdots a_{KN}} J = \sum_{i=1}^{N} \frac{1}{T} \int_0^T a_i^T(t) W_i a_i(t) dt \tag{18}$$

subject to

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) && (19)\\ g(0) &= g_s && (20)\\ g(T) &= g_f && (21)\\ \|p_i(t) - p_j(t)\| &\geq \epsilon \; \forall t \in [0,T], \; \forall_{i \neq j} i,j && (22)\\ \|a_i(t)\| &\leq \alpha, \; \forall t \in [0,T], \; \forall i && (23) \end{aligned}$$

where $i, j = 1, \cdots, N$, ϵ is the minimum allowable distance between vehicles, and α is the maximum allowable acceleration input at any time. W_i is a diagonal matrix of weighting constants for the i^{th} vehicle.

W_i helps shape the cost function to different scenarios. For example if one wants to restrict movement of a vehicle in the z plane, one would make the diagonal entries of W_i associated with acceleration in the z direction higher than the rest. A higher weight can be given to all entries of W for any vehicle with special energy limitations due to low fuel and make it move slower than the other vehicles. In the following section we will show several examples of how Wi can be used to shape the result.

It can been easily shown that the FRP problem for autonomous vehicles satisfies the conditions specified in Theorem 3.2. Thus, given an order of the polynomials K, one can apply Theorem 3.2 to determine whether the K^{th} polynomials would be sufficient for solving the FRP problem.

5 Optimization results

In order to simplify the problem further, we have chosen to use 4^{th}-order polynomials for the parameterization of the acceleration trajectories. Hence, there is just one free vector for each vehicle. Without loss of generality, we choose this free parameter vector to be a_{1i}. Define the optimization vector q to be the composite of all free parameters a_{1i} given by

$$q = [a_{11}^T \cdots a_{1N}^T]^T. \tag{24}$$

Instead of writing the state and input trajectories as functions of q and t, we simply express them as $p_i(t), v_i(t)$, and $a_i(t)$ to avoid introducing new notations. We can now express the FRP related optimization problem as

$$\min_{q} J = \sum_{i=1}^{N} \frac{1}{T} \int_0^T a_i^T(t) W_i a_i(t) dt \tag{25}$$

subject to

$$\begin{aligned} \|p_i(t) - p_j(t)\| &\geq \epsilon \; \forall t \in [0,T], \; \forall_{i \neq j} i,j && (26)\\ \|a_i(t)\| &\leq \alpha, \; \forall t \in [0,T], \; \forall i && (27) \end{aligned}$$

where $i, j = 1, \cdots, N$. The FRP related optimization problem was solved using a constrained optimization algorithm. We will discuss further in the final version of this paper. In order to illustrate the effectiveness of the weighting matrix W_i, we perform the same FRP related optimization problem in three different cases varying only W_i. The given formation configurations and constraint constants are as follows.

$$\begin{aligned} a_i(0) = a_i(T) &= [0\ 0\ 0]^T \; \forall i \in \{1 \cdots 3\}\\ v_i(0) = v_i(T) &= [0\ 0\ 0]^T \; \forall i \in \{1 \cdots 3\}\\ \left[p_1^T(0)\ p_2^T(0)\ p_3^T(0)\right]^T &= [0\ 0\ 10\ 0\ 10\ 10\ 10\ 0\ 10]^T\\ \left[p_1^T(T)\ p_2^T(T)\ p_3^T(T)\right]^T &= [0\ 0\ 10\ 10\ 0\ 10\ 0\ 10\ 10]^T\\ T &= 30, \; \epsilon = 9.9, \; \alpha = 3 \end{aligned}$$

In case (a), all vehicles and directions are given equal weighting. $W_i = I_{3\times3}$ for all vehicles. Figure 1(a) shows a simulation of the position trajectory result for case (a). Since vehicle 1 was given the same desired final position as its initial position, and no other restrictions were made, it does not move. However, vehicles 2 and 3 curve their position trajectory to stay at least an ϵ distance away from vehicle 1. The q found for the solution in case (a) is $q = [-0.0012\ 0\ 0\ 0.0492\ 0.0065\ 0\ 0.0047\ 0.051\ 0]$.

In case (b), vehicle 2 and 3 are given higher weighting than vehicle one such that $W_1 = I_{3\times3}$ and $W_2 = W_3 = 5 \cdot I_{3\times3}$. Figure 1(b) shows a simulation of the position trajectory result for case (b). In Figure 1(b) it is more optimal for vehicles 2 and 3 to find a path that uses less energy than vehicle 1. Therefore, even though vehicle 1 ultimately ends up in the same position, it moves away from vehicle 2 and 3 during the formation reconfiguration in order to remain ϵ distance away. This illustrates the centralized nature of the optimization solution of the FRP problem by showing how one vehicle sacrifices for the good of the group. The q found for the solution in case (b) is $q = -0.0343\ 0\ 0\ 0.0303\ 0.002\ 0\ -0.0132\ 0.0444\ 0]$.

In case (c), the x and y directions of all vehicles are given a higher weighting than the z direction. $W_i = diag(5\ 5\ 1)$ for all vehicles. Figure 1(c) shows a simulation of the position trajectory result for case (c). In Figure 1(c) once again, since vehicle 2 and 3 must accelerate in x and y but are inhibited in the x and y directions, it is optimal for them to find a path that uses less energy than vehicle 1. It is still up to vehicle 1 to move away from vehicle 2 and 3 during the formation reconfiguration. However this time, it moves away in the z direction because it is more costly to move in either the x or y direction. Once again, this illustrates the centralized nature of the optimization solution of the FRP problem by showing how one vehicle sacrifices for the good of the group. The q found for the solution in case (c) is $q = [0\ 0\ 0.0767\ 0.404\ 0.004\ 0\ 0.004\ 0.0404\ 0]$.

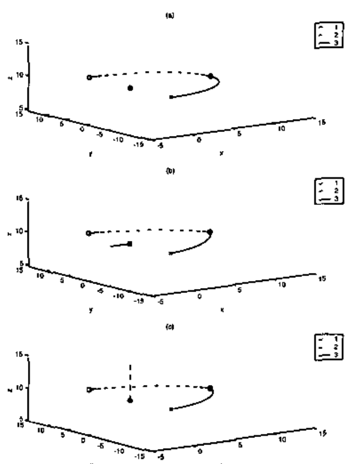

Figure 1: FRP solutions of the same problem using different W_i matrices. The circles and crosses represent the initial and final position of each vehicle respectively. (a). All vehicles and directions are given identical weighting. (b). Vehicles 2 and 3 are given higher weighting than vehicle 1. (c). The x and y directions are given higher weighting than the z direction for all vehicles.

The FRP optimization method can also be used to change formation in the presence of obstacles. All that is needed is extra constraints describing the obstacles. We assume that the space occupied in $\mathbb{R}^3$ by an obstacle can be described by an inequality as:

$$O_j = \{p_0 \in \mathbb{R}^3 : \ D_j(p_0) > \gamma_j\} \tag{28}$$

where $D_j : \mathbb{R}^3 \to \mathbb{R}$, $p_0 = [p_{x0} p_{y0} p_{z0}]^T \mathbb{R}^3$ and $\gamma_j \in \mathbb{R}$. Even though there is only one obstacle, the obstacle constraints apply to all vehicles. Therefore for each obstacle constraint, there are N additional constraints added to the FRP problem, *i.e.*

$$d_j(x_i) \le \gamma_j \quad \text{for } i = 1, \dots, N \tag{29}$$

where $d_j(x_i) = (D_j(\Pi x_i))$ with the projection matrix $\Pi = [I_{3\times3}\ 0_{3\times3}]$. Figure 2 shows how a formation of three vehicles can reconfigure around a circular obstacle in the x y plane. The two solutions have the same FRP

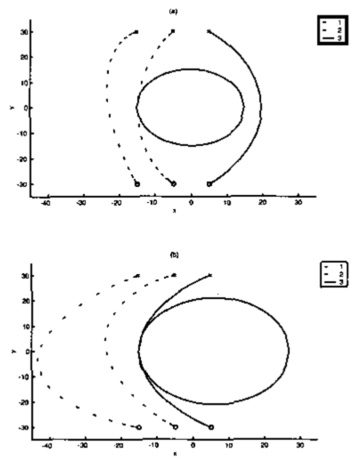

Figure 2: Two examples of a formation avoiding cylindrical obstacles. The circles and crosses represent the initial and final position of each vehicle respectively. (a). One vehicle breaks from the group to avoid the obstacle. (b) All three vehicles stay together when avoiding the obstacle.

specifications given by

$$\begin{aligned}
&a_i(0) = a_i(T) = v_i(0) = v_i(T) = [0\ 0\ 0]^T\ \forall i \\
&[p_1^T(0)\ p_2^T(0)\ p_3^T(0)] = \\
&\qquad [-15\ -30\ 10\ -5\ -30\ 10\ 5\ -30\ 10] \\
&[p_1^T(T)\ p_2^T(T)\ p_3^T(T)] = \\
&\qquad [-15\ 30\ 10\ -5\ 30\ 10\ 5\ 30\ 10] \\
&T = 30,\ \epsilon = 9.9,\ \alpha = 3,\ W_i = I^{3\times3},\ \forall i
\end{aligned}$$

Results shown in Figure 2(a) and 2(b) were given additional obstacle constraints. The obstacles considered in Figure 2(a) and (b)are given by

$$p_{x0}^2 + p_{y0}^2 < 15^2 \text{ and } (p_{x0} - 6)^2 + p_{y0}^2 < 21^2$$

respectively, each describing a circular obstacle of radius 15 and 21 respectively. In Figure 2(a) the obstacle is such that vehicle 3 breaks off from the formation in order to find a more optimal path around the obstacle. In 2(b) it is more optimal for vehicle 3 to stay with the group even though the trajectories of vehicle 1 and 2 increase in energy in order to accommodate the presence of vehicle 3. The FRP solution found for these two examples are (a)

$q = [-0.1199\ 0.2465\ 0\ -0.1402\ 0.104\ 0\ 0.2107\ 0.1341\ 0]$, and (b) $q = [-0.403\ 0.0772\ 0\ -0.2757\ 0.1912\ 0\ -0.2844\ 0.1315\ 0]$. Even though a formation may be capable of finding its way around a given obstacle, it may be more beneficial to avoid the obstacle in two FRP stages in order the use of high order of polynomials. Figure 3 shows a two stage example of how a formation can perform a sequence of reconfigurations in order to avoid a set of obstacles.

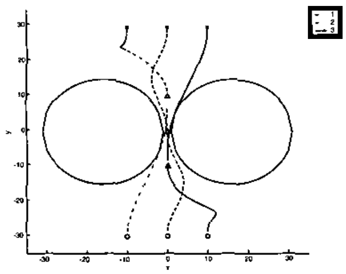

Figure 3: A formation performs a two stage FRP to move through a set of obstacles. The circles and crosses represent the initial and final configurations of the entire FRP problem respectively. The triangles represent the middle configuration as both the final configuration of the first stage and the initial configuration of the second stage of the FRP problem.

6 Conclusions

Optimization has proved to be a successful solution to the FRP problem. Our method of implementation is general and portable allowing for use in a wide range of applications for coordinated robots. For example, our method could easily be transported to two dimensions for ground robot coordination. This centralized control scheme has limitations in applications where formations are very large or communication is disrupted. For such applications, a decentralized control scheme is preferred. We are currently working on a decentralized approach to the FRP problem where each vehicle produces its own localized solution based on only local sensor information about its neighboring vehicles. As expected, this is proving to be a more complex problem. Therefore, centralized control is preferred in applications for smaller fully connected formations.

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