Geometric Motion Estimation and Control for Robotic-Assisted Beating-Heart Surgery

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Abstract— One of the potential benefits of robotic systems in cardiac surgery is that their use can increase the number of possible off-pump (beating heart) coronary artery bypass grafting procedures. Robotic systems can actively synchronize the motion of surgical tools to the motion of the surface of the heart, with the surgeon specifying only the relative motion of the tool with respect to the heart. Accurate prediction of the motion of the heart surface is obviously of crucial importance for the safety and robustness of such a system.

This paper presents a novel approach to predict the motion of the surface of a beating heart. We show how ECG and respiratory information can be used to extract two periodic components from the quasi-periodic motion of the heart surface. Contrary to most existing literature, we consider the full geometric motion, including rotation due to respiration. We then show how to combine the periodic components to accurately predict future motion of the heart surface, and how this information can be used to design an explicit controller that asymptotically stabilizes the relative motion of the surgical tool to a desired relative distance and orientation.

I. INTRODUCTION

Robotic assisted surgery is becoming increasingly common, and advanced robotic systems are more widely available in hospitals. One of the advantages of robotic tools is the potential to facilitate more advanced minimally invasive surgery procedures, which have the benefits of shorter recovery time and less invasive scarring. Although current robotic tools are usually programmed to only follow the surgeon's movements, their true potential lies in the possibility to control them in a more active way.

One of the applications in which this could make a difference is in cardiac operations such as coronary artery bypass grafting (CABG). Currently, many cardiac procedures require the heart to be stopped (and a heart-lung machine to be used) to allow surgery on a motionless surface. However, the use of a heart-lung machine can have damaging effects for the patient and should therefore be used as little as possible. Actively controlled robotic tools could be employed instead, and be synchronized to move at a fixed relative distance and orientation with respect to the heart surface (see [1], [2], [3] for more details on this idea). Through this relative stabilization, together with stabilized visual feedback [4], [5] from the operating area to the surgeon, a surgeon only needs to provide the desired relative motion of the tool with respect to the heart, thus possibly eliminating the need to stop the heart.
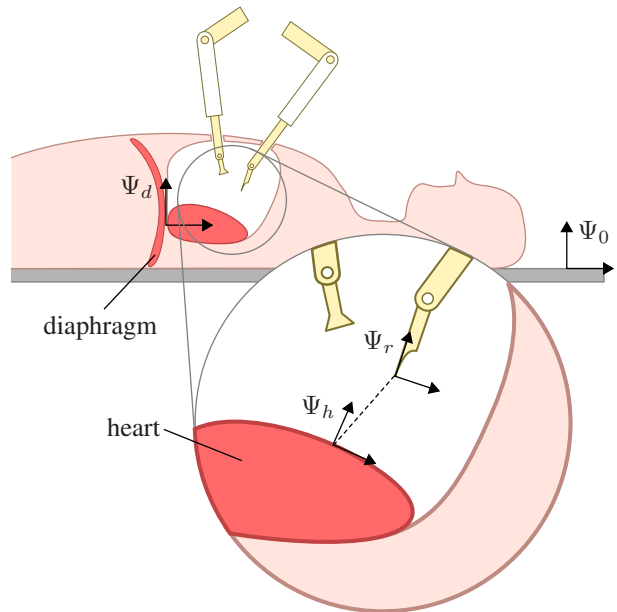

Fig. 1. Schematic system representation and definition of various coordinate frames. The motion of a robotic tool (with attached frame Ψ_r) is to be synchronized to the motion of a certain small area on the heart surface (frame Ψ_h). The motion of the heart surface is caused by a combination of motion of the diaphragm due to breathing (frame Ψ_d) and the beating motion of the heart itself.

One of the most crucial (and in some sense most difficult) aspects of such a motion synchronization system is the accurate measuring and prediction of the motion of the heart surface. First, it requires accurate sensors to measure the motion of the surface, and several different technologies have been proposed in literature, including high-speed cameras [6] and piezoelectric crystals [7]. But secondly, and this is the topic of the present paper, the motion of the heart surface is quasi-periodic, which means it is the combination of two periodic motions, as illustrated in Fig. 1. One periodic motion is caused by the diaphragm, which moves at the frequency of respiration, and the other periodic motion is caused by the beating of the heart itself.

The approach in literature to predict this quasi-periodic motion is to assume that it is simply the sum of two periodic components, and then to separate these two components using *e.g.* a low-pass filter [2], an adaptive harmonic filter bank [1], or direct time series analysis [8]. As shown in Section III, however, this implicitly assumes that the respi-

ration component does not contain a rotation, which may not be accurate enough for high-precision applications [9]. In addition, the methods only consider the motion of points, discarding rotational information, even though the orientation of the heart surface needs to be known in order to properly orient the surgical tool.

The purpose of this paper is to present a general method to separate a quasi-periodic rigid 3-D motion into its two periodic components, and to use the result in the design of an asymptotically synchronizing controller. We assume the frequencies of the two motion components to be known, for example based on an ECG signal and information from the mechanical ventilator used during surgery. The estimated components are then combined to predict future motion of the heart surface. This information, in turn, is used in the design of an explicit controller that stabilizes the relative motion of a surgical tool to a desired distance and orientation with respect to the heart surface.

After the necessary mathematical preliminaries in Section II, we discuss in Section III that the total motion of the heart surface is a (nonlinear) geometric phenomenon, and show how the periodic respiratory and cardiac motions can be extracted. We then present a control law (Section IV) that uses the predicted motion to asymptotically stabilize the motion of the surgical tool to a desired distance and orientation with respect to the heart surface. We illustrate the results in simulations.

II. MATHEMATICAL PRELIMINARIES

We first briefly summarize the theory and notation used to represent rigid 3-D motions and velocities. We refer to [10] and [11] for a more extensive background.

The relative *configuration* of a right-handed coordinate frame Ψ_a with respect to a right-handed frame Ψ_b, meaning their relative position and orientation, can be described by an element of the Lie group $SE(3)$, the Special Euclidean group. Numerically, this configuration can be expressed by a time-varying 4×4 homogeneous matrix $H_b^a(t)$ of the form

$$H_b^a(t) := \begin{bmatrix} R_b^a(t) & p_b^a(t) \\ 0 & 1 \end{bmatrix} \qquad (1)$$

where $R_b^a(t)$ is a 3×3 orthogonal matrix with $\det(R_b^a) = 1$ that describes the relative orientation, and $p_b^a(t)$ is a 3×1 vector that describes the relative position of the origins.

A rotation can not only be described by an appropriate 3×3 matrix, but also by so-called unit quaternions, which can be thought of as a set of four real numbers

$$(q_0, q) := (q_0, q_1, q_2, q_3) \in \mathbb{R}^4 \qquad (2)$$

satisfying $(q_0, q) \circ (q_0, -q) = (1, 0)$, with $\circ$ the (non-commutative) Grassman product defined as

$$(q_0, q) \circ (p_0, p) := (q_0 p_0 - q^T p, q_0 p + p_0 q + \hat{p} q) \qquad (3)$$

with $\hat{x} := x \wedge$ for $x \in \mathbb{R}^3$. Unit quaternion representations can be converted to matrix representations (and back) through relatively straight-forward computations.

The relative *velocity* of two frames, meaning both the linear and angular velocity, can be concisely represented by an element of $se(3)$, the Lie algebra of $SE(3)$. Numerically, it can be expressed as a 6×1 vector $T_b^{c,a}$ or 4×4 matrix $\hat{T}_b^{c,a}$ of the form

$$T_b^{c,a} := \begin{bmatrix} \omega_b^{c,a} \\ v_b^{c,a} \end{bmatrix} \quad \text{or} \quad \hat{T}_b^{c,a} := \begin{bmatrix} \hat{\omega}_b^{c,a} & v_b^{c,a} \\ 0 & 0 \end{bmatrix} \qquad (4)$$

with $\omega_b^{c,a}, v_b^{c,a} \in \mathbb{R}^3$ the relative angular and linear velocity. The vector $T_b^{c,a}$ is called a twist and describes the relative velocity of frame Ψ_b with respect to Ψ_a, expressed in frame Ψ_c. It is related to the relative configuration (1) as

$$\hat{T}_b^{c,a} = H_a^c \dot{H}_b^a H_c^b \qquad (5)$$

with $\dot{H}$ denoting the time-derivative of H. Finally, the following two relations are used in Section IV.

$$T_b^{d,a} = \text{Ad}_{H_c^d} T_b^{c,a} \text{ with } \text{Ad}_{H_c^d} := \begin{bmatrix} R_c^d & 0 \\ \hat{p}_c^d R_c^d & R_c^d \end{bmatrix}$$

$$\frac{d}{dt}\left(\text{Ad}_{H_c^d}\right) = \text{ad}_{T_c^{d,d}} \text{Ad}_{H_c^d} \text{ with } \text{ad}_{T_c^{d,d}} := \begin{bmatrix} \hat{\omega}_c^{d,d} & 0 \\ \hat{v}_c^{d,d} & \hat{\omega}_c^{d,d} \end{bmatrix}$$

III. QUASI-PERIODIC MOTION PREDICTION IN $SE(3)$

The motion of the heart surface is caused by the combined dynamics of respiration (mainly affecting the heart through motion of the diaphragm) and cardiac muscle contractions. The full dynamics are very complex and hard to predict, and require many patient-specific parameters to be fitted [12]. To simplify the analysis and allow real-time prediction, we do not take the cause of the motion into consideration, and only consider the kinematics of a local area of interest on the heart surface. Its motion is assumed to be a combination of two periodic motions: one due to respiration (with period T_d, with d for 'diaphragm'), and one due to heart beating (with period T_h, with h for 'heart'). The combination of two periodic motions is called a quasi-periodic motion.

Since the two periods T_d and T_h are generally not integer multiples of each other, the period of the total motion (the time after which it repeats itself) can be arbitrarily long. As the frequencies of the two periodic motions are different (typically $T_d \approx 4$ s and $T_h \approx 1$ s), a low-pass filter may seem a relatively simple solution to separate the respiratory component [2]. However, this assumes that the respiration is a purely sinusoidal motion without higher harmonics, and moreover, that the two signals mix linearly. As shown in Section III-A, this second assumption does not hold when rotation is taken into account. In addition, a (causal) low-pass filter that cleanly distinguishes between two signals with T_d and T_h in this close range will generally suffer from a large phase shift that can cause stability and performance problems in feedback loops.

Fortunately, in this application, we have a reasonably good estimate of the phase[1] and frequency of the two motion

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[1]The precise definition of phase is not important in this context; we can choose any number that indicates the 'part' of the cycle as a function of time. For example, it is sufficient to choose a linear interpolation from zero to one between every two consecutive QRS-complexes in the ECG cycle.

components: the respiratory phase and frequency can be obtained from the mechanical ventilator, and the cardiac phase and frequency are detected by an ECG monitor. We can exploit this prior knowledge to extract the two periodic components from measurements of the combined motion.

A. Heart motion as a group product

As the first step to characterize the motion of the heart surface, we define what we mean exactly by 'combination' of two motions. Since both the respiratory motion and the cardiac motion generally contain translation as well as rotation, we can write the relative displacement of the heart surface as a group product of two homogeneous matrices H_d^0 and H_h^d, as illustrated in Fig. 1. The matrix H_d^0 describes the relative motion of the diaphragm Ψ_d with respect to an inertial frame Ψ_0, and H_h^d describes the relative motion of the area of interest Ψ_h on the heart surface with respect to Ψ_d. Each component is assumed to have a (roughly constant) period but unknown shape, i.e.

$$H_d^0(t + T_d) = H_d^0(t) \qquad H_h^d(t + T_h) = H_h^d(t) \quad (6)$$

The 'combination' of the two motions is the product of the two matrices, i.e.

$$H_h^0 = H_d^0 H_h^d = \begin{bmatrix} R_d^0 R_h^d & p_d^0 + R_d^0 p_h^d \\ 0 & 1 \end{bmatrix} \quad (7)$$

It is important to note that, even if we are only interested in the relative positions, the combination of the positions is *not* a simple linear sum of the vectors p_d^0 and p_h^d, but also nonlinearly depends on the rotation of the diaphragm R_d^0. As shown in research on cardiac image registration [9], even though the rotation of the diaphragm is generally only a few degrees, it can still cause substantial deviations if not properly taken into account. For that reason, we consider in this paper both rotation and translation components. The orientation of the heart surface in itself is also a useful signal to predict, since generally we want to stabilize both the relative position and the relative orientation of the surgical tool with respect to the heart surface.

Note that the physical location of the intermediate frame Ψ_d is not clearly defined, which results in ambiguities. More precisely, given any combination (H_d^0, H_h^d) that satisfies (7), the combination $(H_d^0 \tilde{H}, \tilde{H}^{-1} H_h^d)$ also satisfies (7), for any $\tilde{H}$ of the form (1). This ambiguity can be resolved by choosing a convenient initial position and orientation for Ψ_d, e.g. such that $p_h^d(0) = 0$ and $R_d^0(0) = I$.

B. Filtering periodic components

Given a general quasi-periodic signal $x(t)$, composed of two periodic components $x_1(t)$ and $x_2(t)$. Suppose we have sufficient measurements of $x(t)$ and of the two phases $\phi_1(t)$ and $\phi_2(t)$ of the composing signals, then we can estimate the periodic components $x_1(\phi_1)$ and $x_2(\phi_2)$ themselves as functions of their respective phases. The approach we take is illustrated in Fig. 2. We parameterize each periodic signal
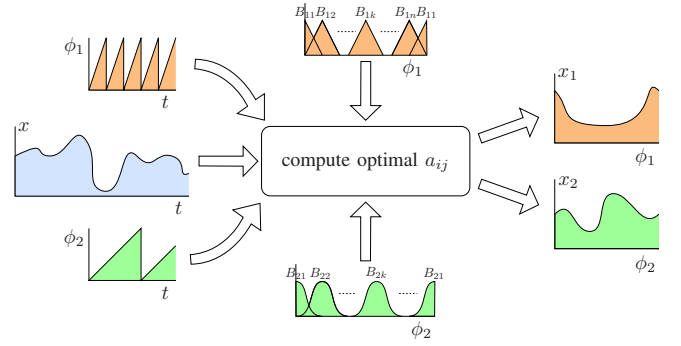

Fig. 2. Schematic representation of the algorithm. Based on measurements of a quasi-periodic signal x and the phase trajectories of its two periodic components, and given a choice of basis functions B_{ij}, the optimal parameters a_{ij} are computed. The resulting representation of the components $x_i(\phi_i)$ can be used to predict future $x(t)$.

as a sum of basis elements $B_{ij}(\phi_i)$ and write the total signal $x(t)$ at time t as

$$x(t) = x_1(\phi_1(t)) \oplus x_2(\phi_2(t)) \oplus \epsilon(t) \quad (8)$$

$$= \left(\sum_k a_{1k} B_{1k}(\phi_1(t)) \right) \oplus \left(\sum_m a_{2m} B_{2m}(\phi_2(t)) \right) \oplus \epsilon(t)$$

where $\oplus$ denotes the composition of the two periodic signals, $\epsilon(t)$ is a measurement error, and a_{ij} are the weights for the basis elements. Given sufficient measurements, we can estimate the parameters a_{ij} such that $\epsilon(t)$ is minimized, e.g. in the least-squares sense. Depending on the choice of basis functions B_{ij}, the coefficients may not be uniquely determined. For example, the two periodic components can only be determined up to a constant: if $\oplus$ denotes the regular sum operator, then adding a constant to one of the components and subtracting the same constant from the other component does not change their composition, and similarly when $\oplus$ is multiplication. Other ambiguities will occur if the basis functions are chosen in an unfortunate way that makes them 'fit' in both periodic components.

We now apply this general approach separately to the rotation and translation components of the quasi-periodic motion $H_h^0(t)$, as given by (7). We start with rotation and choose to use a representation in unit quaternion coordinates, as described in Section II. Unit quaternions provide a good balance between minimizing the dimension of the signal space (four for quaternions) and simplifying the expression for 'combining' two rotations (defined by the Grassman product for quaternions). Although choosing exponential coordinates [13] would further reduce the dimension of the signal space to three, writing the product of two rotations in terms of their exponential coordinates produces a quite involved expression, which is much harder to optimize than the simple bi-linear expression for quaternions:

$$Q(t) = \epsilon(t) \circ Q_h(\phi_h(t)) \circ Q_d(\phi_d(t)) \quad (9)$$

in which $Q(t)$ is the quaternion expression for the measured total rotation at time t, Q_d and Q_h are the quaternion expressions for the rotations R_d^0 and R_h^d, respectively, $\epsilon(t)$ describes

the measurement error (close to the $(1,0)$ quaternion), and $\circ$ is the Grassman product defined in (3).

We parameterize each component in (9) as in (8) by choosing a set of basis functions B_{ij} for the quaternions $Q_d(\phi_d)$ and $Q_h(\phi_h)$. With this choice and sufficient measurements $Q(t)$, the next step is to estimate the coefficients a_{ij}. An extra complication arises here because of the constraint that Q_d and Q_h should have unit length at all times, and hence their four components cannot be chosen arbitrarily. This can be solved by including constraints in the optimization process, or, more pragmatically though not mathematically sound, to just freely optimize over all quaternions and normalize the result afterwards. A second aspect is the choice of metric for the optimization, *i.e.* how to judge what ϵ in (9) is minimal. The simplest solution is to minimize $(Q^{-1} \circ Q_d \circ Q_h) - (1,0)$ in the least squares sense, or even just $Q - Q_d \circ Q_h$. Both are not proper metrics on $SO(3)$ but are easy to optimize and give good results in simulations.

Once the rotation components, in particular R_d^0, have been determined, we can write the estimation problem (8) for the translational components as follows.

$$p_h^0(t_0) = R_d^0(t_0) p_h^d(\phi_h(t_0)) + p_d^0(\phi_d(t_0)) + \epsilon(t_0) \quad (10)$$

with p_h^0 and R_d^0 known and p_h^d and p_d^0 to be estimated. This relation is unconstrained and linear in the unknowns and can be solved *e.g.* by a standard linear least squares computation.

For both rotation and translation components, several design choices have to be made. First, the number of basis functions $B_{ij}(\phi_i)$ and their shapes need to be determined. Secondly, a metric on the data space needs to be chosen, *i.e.* a mapping that weighs the relative influence of older and newer samples in the estimation of the coefficients a_{ij} and hence in the prediction of the future signal. If the signal is highly periodic, then older samples should be weighed as well as new samples, such that any random noise is averaged and thus reduced. If the shape of the signal changes significantly over several periods, then only the newer samples should be taken into account. Studying experimental data from various patients should help make these choices.

Once the coefficients a_{ij} for rotation and translation have been determined and hence the shapes of the two periodic components of H_h^0 are known, future values of H_h^0 can be predicted by combining the values of the two components $H_d^0(\phi_d)$ and $H_h^d(\phi_h)$, evaluated at their predicted phases. The prediction of the phases ϕ_d and ϕ_h can be a simple extrapolation at the current respiration and heart beat rate, respectively. These rates are stabilized by a mechanical ventilator and by medication and generally only vary quite slowly.

Fig. 3 shows a simulation of the extraction of the periodic diaphragm motion (with period 4 s) and the periodic cardiac motion (with period 1.1 s) from an artificial combined motion signal, contaminated with zero-mean Gaussian noise with a standard deviation of 0.1. We used 20 equally spaced triangular windows as the basis functions for each component. From the measurements (noisy signals in the top graphs) between 0 and 6 seconds, the coefficients a_{ij} were optimized

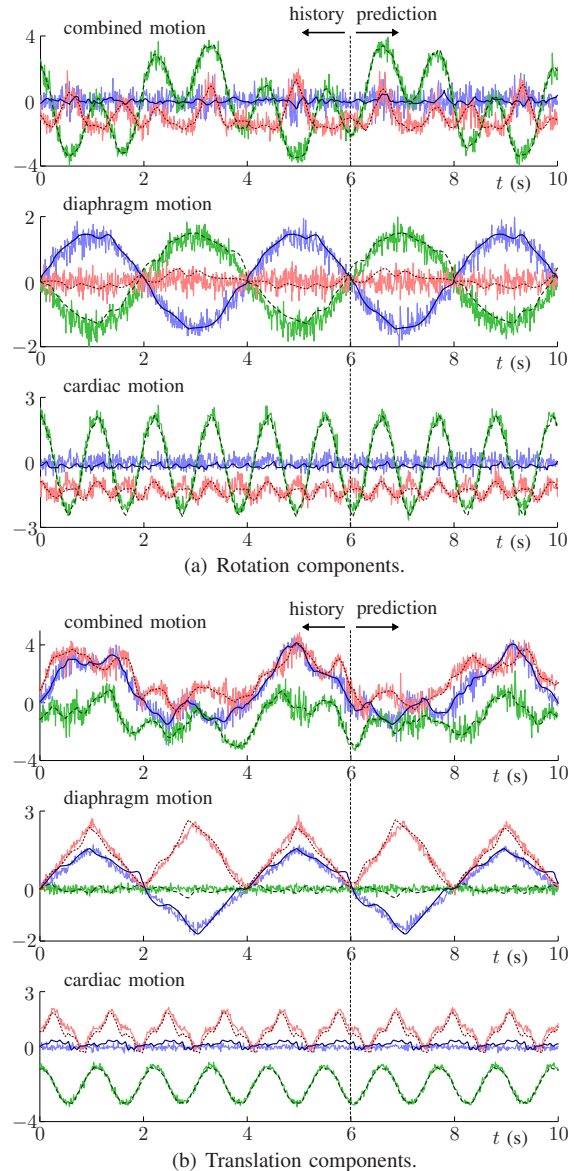(a) Rotation components.

(b) Translation components.

Fig. 3. Extraction of diaphragm (period 4 s) and cardiac (period 1.1 s) motions from noisy measurements of the combined motion. The figures show the three degrees of freedom for rotation and translation, with x (solid), y (dashed), and z (dotted) components. Reconstructed signals are shown on top of measured noisy signals.

and the periodic signals reconstructed (clean signals in the bottom graphs, overlaid on the noisy real periodic source components, which are unknown to the algorithm). Using the obtained parameters and the basis functions, the combined signal can be reconstructed ($t < 6$ s) and its future values ($t > 6$ s) can be accurately predicted. Note that Fig. 3(a) shows three components for each rotation (exponential coordinates) although the optimization was performed using four components (quaternions).

IV. EXPLICIT ASYMPTOTIC CONTROL FOR RELATIVE MOTION SYNCHRONIZATION

After computing a prediction of the motion of the heart surface, the next step is to synchronize the motion of the

surgical tool to the motion of the area of interest. This problem is usually cast as a three-dimensional position tracking problem and often solved using a form of (adaptive) model predictive control [1], [2]. In this section, we generalize the problem to include not only position, but also orientation information, and we present an alternative explicit model-based controller.

A. Choice of objective function

The objective of the controller is to stabilize the relative configuration of the robot with respect to a certain area on the heart surface. This relative configuration is described by a homogeneous matrix of the form

$$H_r^h = \begin{bmatrix} R_r^h & p_r^h \\ 0 & 1 \end{bmatrix} =: \begin{bmatrix} r_x & r_y & r_z & p \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

with index r for 'robot'. To quantify the control objective, we choose an error function $J(H_r^h)$ as follows

$$J(H_r^h) := J_{\text{tran}} + J_{\text{rot}} := \frac{1}{2}k_p(p - \Delta r_z)^T(p - \Delta r_z)$$
$$+ k_x(1 - e_x^T r_x) + k_y(1 - e_y^T r_y) + k_z(1 - e_z^T r_z) \quad (12)$$

with Δ the desired relative distance between the tool and the heart, $E := \begin{bmatrix} e_x & e_y & e_z \end{bmatrix}$ a rotation matrix describing the desired orientation of the tool frame (with $e_z = (0,0,1)$), and $k_p, k_x, k_y, k_z > 0$ constant parameters. The function J has a minimum equal to zero only if $p = \Delta r_z$ and $R_r^h = E$, i.e. the origin of Ψ_r is Δ along the surface normal away from the origin of Ψ_h, and the axes of the tool frame are properly aligned. The error function increases for distances different from Δ and for deviations from the desired orientation specified by E. Note that when $k_x = k_y = k_z$, J_{rot} is proportional to $3 - \text{trace}(E^T R_r^h) = 1 - \cos(\theta)$, with θ the angle of rotation away from E.

One of the advantages of the cost function (12) is that its time-derivative along the trajectories of the system takes a particularly simple form, as shown below.

$$\dot{J} = k_p(p - \Delta r_z)^T(v_r^{h,h} - \hat{p}\omega_r^{h,h} + \Delta \hat{r}_z \omega_r^{h,h})$$
$$+ k_x e_x^T \hat{r}_x \omega_r^{h,h} + k_y e_y^T \hat{r}_y \omega_r^{h,h} + k_z e_z^T \hat{r}_z \omega_r^{h,h}$$
$$= \begin{bmatrix} k_x \hat{e}_x r_x + k_y \hat{e}_y r_y + k_z \hat{e}_z r_z \\ k_p(p - \Delta r_z) \end{bmatrix}^T \begin{bmatrix} \omega_r^{h,h} \\ v_r^{h,h} \end{bmatrix}$$
$$=: (dJ)^T T_r^{h,h} \quad (13)$$

Where we used the fact that, from (5) and (11), we have

$$\dot{H}_r^h = \begin{bmatrix} \hat{\omega}_r^{h,h} r_x & \hat{\omega}_r^{h,h} r_y & \hat{\omega}_r^{h,h} r_z & \hat{\omega}_r^{h,h}p + v_r^{h,h} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (14)$$

and that $(\hat{p} - \Delta\hat{r}_z)(p - \Delta r_z) = 0$. Equation (13) is used in the following sections, as is the expression for the time-derivative of dJ given below.

$$(\dot{dJ}) = \begin{bmatrix} -k_x \hat{e}_x \hat{r}_x \omega_r^{h,h} - k_y \hat{e}_y \hat{r}_y \omega_r^{h,h} - k_z \hat{e}_z \hat{r}_z \omega_r^{h,h} \\ k_p(v_r^{h,h} - \hat{p}\omega_r^{h,h} + \Delta\hat{r}_z\omega_r^{h,h}) \end{bmatrix}$$
$$= \begin{bmatrix} -k_x \hat{e}_x \hat{r}_x - k_y \hat{e}_y \hat{r}_y - k_z \hat{e}_z \hat{r}_z & 0 \\ -k_p(\hat{p} - \Delta\hat{r}_z) & k_p I \end{bmatrix} T_r^{h,h} \quad (15)$$

B. Asymptotically stabilizing control law

Using the error function (12), we propose the following explicit control law for the robot that moves the surgical tool. We do not consider specific robot dynamics at this point and only specify the desired inertial acceleration $\dot{T}_r^{0,0}$ of its end effector frame Ψ_r. For a particular robot, this desired acceleration should be achieved by a suitable lower level controller. The proposed desired acceleration is the following.

$$\left(\dot{T}_r^{0,0}\right)_{\text{des}} = \dot{T}_h^{0,0} - \text{Ad}_{H_h^0} K_1(\dot{dJ}) + \text{ad}_{T_r^{0,0}} T_r^{0,h}$$
$$- \text{Ad}_{H_h^0} K_2\left(T_r^{h,h} + K_1 dJ\right) \quad (16)$$

with $K_1, K_2 > 0$ two symmetric positive-definite matrices, and $H_h^0, T_h^{0,0}$, and $\dot{T}_h^{0,0}$ the estimated configuration, velocity, and acceleration of the frame Ψ_h at the area of interest on the heart surface. Intuitively, this controller drives $T_r^{h,h}$ to $-K_1 dJ$ (by a gain K_2), which makes the configuration move along the direction of steepest descent of J (defined using K_1 as a metric). To prove asymptotic stability of the equilibrium $J = 0$ for the closed loop system, we consider the following candidate Lyapunov function.

$$V := \kappa_1 J + \frac{1}{2}\left(T_r^{h,h} + K_1 dJ\right)^T K_2^{-1}\left(T_r^{h,h} + K_1 dJ\right)$$

with $0 < \kappa_1 < 4\sigma_{\min}(K_1)$, i.e. κ_1 is strictly less than four times the smallest singular value of K_1 (which is strictly positive since $K_1 > 0$). Assuming the robot achieves perfect tracking of $(\dot{T}_r^{0,0})_{\text{des}}$, we can compute the time-derivative of V along system trajectories as follows.

$$\dot{V} = \kappa_1(dJ)^T T_r^{h,h} + (T_r^{h,h} + K_1 dJ)^T K_2^{-1}(\dot{T}_r^{h,h} + K_1 \dot{dJ})$$
$$= \kappa_1(dJ)^T T_r^{h,h} - (T_r^{h,h} + K_1 dJ)^T\left(T_r^{h,h} + K_1 dJ\right)$$
$$= -\left(T_r^{h,h} + (K_1 - \frac{\kappa_1}{2}I)dJ\right)^T\left(T_r^{h,h} + (K_1 - \frac{\kappa_1}{2}I)dJ\right)$$
$$- \kappa_1(dJ)^T\left(K_1 - \frac{\kappa_1}{4}I\right)dJ \quad (17)$$

This expression is non-positive since we chose κ_1 such that $K_1 - \frac{\kappa_1}{4}$ is strictly positive definite. Thus, $\dot{V} = 0$ only when $T_r^{h,h} = 0$ and $dJ = 0$. From (13), we see that $dJ = 0$ when the following two equations are satisfied.

$$0 = p - \Delta r_z \quad (18)$$
$$0 = k_x \hat{e}_x r_x + k_y \hat{e}_y r_y + k_z \hat{e}_z r_z \quad (19)$$

The first equation has only one solution, $p = \Delta r_z$. The second equation, together with the constraint that (r_x, r_y, r_z) defines an orthonormal frame, gives rise to several possible solutions. Only the solution $R_r^h = E$ is stable, all other solutions are unstable and oriented at least 90 degrees away from E, hence they can be safely ignored for practical purposes. By La Salle's Principle [14], this proves that the proposed control law (16) semi-globally asymptotically stabilizes the system to the desired state.

Fig. 4 shows a simulation of the proposed controller applied to the ideal robot system

$$\dot{T}_r^{0,0} = \left(\dot{T}_r^{0,0}\right)_{\text{des}} \quad (20)$$
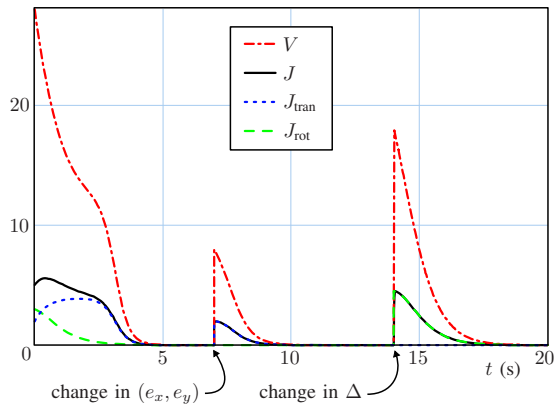
(a) Time trajectories of the Lyapunov and error functions.
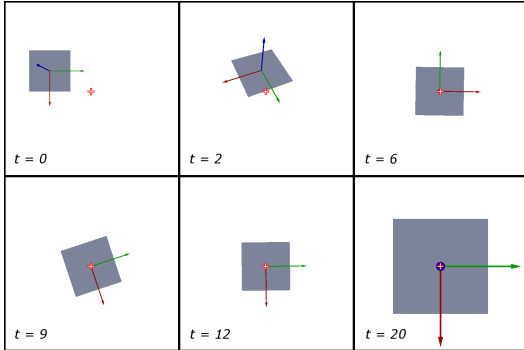
(b) Snapshots as viewed from the tool frame Ψ_h along the negative z direction (plus sign denotes the center of the picture).

Fig. 4. Simulation of the control law applied to an ideal robot.

with all control gains set to unity. Starting from some initial configuration and velocity, and with $H_h^0(t)$ an arbitrary motion similar to Fig. 3, the position and orientation of the robot tool converge to the desired relative distance and orientation. At $t = 7$, the desired orientation of the tool, specified by E, is changed by 90 degrees around the vertical axis (the viewing direction). At $t = 14$, the desired distance Δ is reduced. In both cases, the system recovers from the step change in the reference signals and converges asymptotically to the new equilibrium.

V. CONCLUSIONS AND FUTURE WORK

This paper presents a novel method to extract periodic 3-D motion signals from noisy measurements of a quasi-periodic motion and knowledge of the phase of the composing signals. The total quasi-periodic motion of a region on the surface of a beating heart is a nonlinear combination of respiratory and cardiac motions, if rotation of the diaphragm due to respiration is taken into account. Using a quaternion representation, the nonlinear signal estimation problem can be formulated as a reasonably simple optimization problem, from which the periodic respiratory and cardiac motions can be extracted and future heart surface motions can be predicted.

The predicted motion signal can then be used in the design of an explicit model based controller that asymptotically synchronizes the motion of a surgical tool to the motion of a region on the heart surface. This controller provides an alternative to the (implicit) model-based control algorithms known from literature.

In future work, we plan to extend the motion separation algorithm to a recursive version that can quickly recompute the optimal motion parameters when a few new samples arrive. We will also further investigate what sensors and sensing techniques can most reliably and robustly extract the heart motion, which is the input for the presented algorithm. When measurements of the full rigid motion of the heart surface are available, we can verify the signal estimation and control approach presented in this paper on real experimental data, and tune the various parameters in the algorithms.

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