Evasion of a Team of Dubins Vehicles from a Hidden Pursuer

Shih-Yuan Liu Zhengyuan Zhou Claire Tomlin J. Karl Hedrick

Abstract— **We consider a single-pursuer-multiple-evader pursuit-evasion game in which a team of evaders aims to delay the capture by a faster pursuer. We extend our previous open-loop formulation (and its solution) of the game to incorporate more realistic settings: a pursuer with uncertain position and evaders with limited turning rates. The formulation provides a guaranteed lower bound on the team survival time. The survival time performance of the proposed approach is evaluated through extensive simulations and compared to that of the existing approaches. It is shown to be highly effective even when the evaders can not detect the pursuer. A noticeable trend of potentially practical importance is that larger teams benefit more from an increase in turning rates than smaller teams.**

I. INTRODUCTION

The capabilities of autonomous vehicles such as unmanned aerial vehicles (UAVs) and unmanned ground vehicles (UGVs) have been drastically improved in the past decade. It is now possible for a team of autonomous vehicles to accomplish tasks such as cooperative information gathering [1], which require a high level of communication and collaboration from all members of the team. In some of these applications it might be necessary for the team of autonomous agents to be deployed in an adversarial environment where hostile agents can attack the members of the team. In these scenarios, the team must be able to respond promptly to the presence of the hostile agents to enhance its chance of survival.

When the team is facing a single hostile agent, the scenario can be formulated as a single-pursuer-multiple-evader pursuit-evasion game. Extensive literature exists with various formulations of this game with different objective functions, information patterns, and capabilities of the players. For example, in [2] the evaders aim to maximize the time-minimal distance to a pursuer with limited sensing range; in [3] the evaders use a behavior-based approach to evade the pursuer as a herd; in [4] each of the evaders aims to minimize its own chance of being the closest evader to a hidden pursuer. A good overview on the different objective functions used for single-pursuer-multiple-evader pursuit-evasion games is provided in [5].

In this work, we focus on a special case of the single-pursuer-multiple-evader pursuit-evasion game where the evaders are trying to evade a faster pursuer for as long as possible as a team. This problem is coined as the successive pursuit problem by Breakwell et al. in [6]. It has been shown that in the fixed-sequence variant of the game, where the pursuer must capture the evaders in a pre-determined sequence known to the evaders, the optimal trajectories of the evaders are straight lines. It has also been shown that if the pursuer is free to choose any capture sequence, the optimal trajectories for the evaders must in general be solved by dynamic programming: an approach that is hardly applicable for a team with more than 2 evaders due to computational intractability.

In [7], Chikrii et al. formulate the fixed-sequence problem as a directional optimization problem and are able to compute the optimal headings of a team of 3 evaders by solving the optimization problem numerically. Very recently in [8], Belousov et al. have proposed a way to convert the directional optimization problem into a root searching problem which can be solved very efficiently even for a team with a large number of evaders. The fixed-sequence problem is formulated to be conservative towards the pursuer; it provides the pursuer with a capture sequence that has a guaranteed upper bound on the capture time of the team. However, due to the unrealistic assumption that the pursuer will disclose the capture sequence to the evaders, the fixed-sequence formulation is not as useful to the evaders.

By adapting the framework of open-loop games proposed by Takei et al. in [9] and Zhou et al. in [10], we have proposed a sequence-free open-loop formulation of the successive pursuit game that is conservative towards the evaders in [11]. Although the resulting open-loop optimal trajectories of the evaders are not always the same as the actual optimal trajectories, when implemented iteratively with a small update time the resulting team survival time is often very close to the actual optimal for a 2-evader team.

In this work, we expand the formulation proposed in [11] to handle uncertainties in the pursuer's position and also evaders with limited turning rates, both of which are practical concerns which often arise in more realistic scenarios.

II. PRELIMINARIES

In this section, a concise introduction to the sequence-free open-loop formulation for the team evasion problem proposed in [11] is provided; the work in this paper is built on this previously proposed framework.

Consider a pursuit-evasion game with $N+1$ players: a single pursuer and N evaders taking place in $\mathbb{R}^2$. The goal of the pursuer is to capture all the evaders as soon as possible

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while the goal of the team of evaders is to delay the capture of the whole team, and hence the last evader, for as long as possible. The case of interest here is when the pursuer is faster than all evaders since if any of the evaders is faster than the pursuer, the team can delay the capture indefinitely.

Throughout this paper, the subscript p is used to indicate the pursuer and the subscript $1,\dots,N$ are used for the N evaders. The position of agent j at time t is denoted by $x_j(t) \in \mathbb{R}^2$. The joint position of the agents at time t is denoted by $X(t) = [x_p(t), X_e(t)]$ where $x_p(t)$ denotes the position of the pursuer and $X_e(t) = [x_1(t),\dots,x_N(t)]$ denotes the joint position of all the N evaders. The control of an agent as a function of time is denoted by $u(\cdot) : \mathbb{R} \to \mathbb{R}^2$. The direction of $u(t)$ indicates the direction of velocity of the agent at time t and the length of $u(t)$ indicates the ratio between the speed of the agent at time t to its maximum speed. Following this definition, the set of general admissible controls for an agent is defined as $\mathbb{U} = \{u(\cdot) | \|u(t)\| \leq 1, t \in [0,\infty)\}$. The set of admissible joint controls for the N evaders is defined by $\mathbb{U}^N = \{[u_1(\cdot),\dots,u_N(\cdot)] | u_i(\cdot) \in \mathbb{U} \text{ for } i = 1,\dots,N\}$. The maximum speed of agent j is denoted by v_j. We assume that $v_p = 1$ and $v_i < 1$, for $i = 1,\dots,N$, indicating that all the evaders are slower than the pursuer. The agents have simple motion dynamics that can be described by the following differential equations:

$$\dot{x}_j(t) = v_j u_j(t), \text{ for } j = p,1,\dots,N. \tag{1}$$

In the open-loop formulation proposed in [11], the evaders make the following assumptions when planning for their joint control $U_e(\cdot) = [u_1(\cdot),\dots,u_N(\cdot)] \in \mathbb{U}^N$. The evaders plan their joint control function at the beginning of the game and then commit to it. The joint control is revealed to the pursuer and the pursuer act optimally against the joint control.

The minimum time it takes for a pursuer starting at $x_p(0)$ to capture the N evaders starting at $X_e(0)$ with known joint control $U_e(\cdot)$ in a specific capture sequence $s = [s_1,\dots,s_N]$ is defined to be $f_s(x_p(0), X_e(0), U_e(\cdot))$. Since the actual capture sequence the pursuer will take is unknown to the evaders, the evaders aim to maximize the worst-case capture time of all the possible capture sequences. The optimal open-loop survival time of the team of evaders is therefore defined as:

$$\tau^{ol\star}(x_p, X_e) = \sup_{U_e(\cdot) \in \mathbb{U}^N} \inf_{s \in \mathbb{S}_N} f_s(x_p, X_e, U_e(\cdot)), \tag{2}$$

where $s = [s_1,\dots,s_N]$ is a capture sequence of N evaders and $\mathbb{S}_N = \{s | s_i \in \mathbb{N}_{>0}, s_i \neq s_j \text{ for } i \neq j, \max_i s_i = N\}$ is the set of all possible capture sequences of N evaders. The maximizer of the sup problem, $U_e^{ol\star}(\cdot)$, is the optimal open-loop joint control of the evaders. For a specific $U_e(\cdot)$, the minimizer of the inf problem, $s^\star$, can be solved by comparing the minimum capture time of all the possible capture sequences. It is worth noting that there can be multiple optimal capture sequences for a given $U_e(\cdot)$.

It has been shown in [11] that the optimal open-loop joint control for the evaders always belongs to the set $\mathbb{U}_{\Theta_e} = \{[u_1(\cdot),\dots,u_N(\cdot)] | u_i(\cdot) = [\cos\theta_i, \sin\theta_i], i = 1,\dots,N\}$ where $\Theta_e = [\theta_1,\dots,\theta_N] \in \mathbb{R}^N$ is the joint heading of all the evaders. In other words, the optimal open-loop controls for the evaders are always constant headings with maximum speeds. As a result the admissible joint control set can be parameterized simply by the joint heading of the evaders Θ_e. The optimal open-loop survival time can be written as:

$$\tau^{ol\star}(x_p, X_e) = \sup_{\Theta_e \in \mathbb{R}^N} \inf_{s \in \mathbb{S}_N} f_s(x_p, X_e, \Theta_e). \tag{3}$$

This is a finite-dimensional minimax problem that can be solved numerically with non-linear optimization methods such as sequential quadratic programming (SQP). The optimal open-loop joint control of the evaders with respect to the initial condition of the game provides a guaranteed open-loop survival time for the team.

III. PROBLEM FORMULATION

The goal of this work is to expand the previously proposed open-loop formulation to handle uncertainties in the pursuer's position and evaders with limited turning rates to accommodate more realistic scenarios. Firstly, the original formulation requires that the exact position of the pursuer be known to the evaders. In reality, the pursuer's position must be measured and estimated by the evaders and hence uncertainties arise. We deal with a specific kind of uncertainty where the pursuer can be anywhere within a circular disk with a specific center and radius. Secondly, while the original formulation assumes that the evaders have unlimited turning rates, autonomous agents in practice, such as fixed-wing aerial vehicles and wheeled ground vehicles, have limited turning rates due to physical constraints. These vehicles are often modeled as Dubins vehicles that travel in constant speeds and have limited turning rates. The formulation is modified so that the admissible control set of the evaders reflects the limitation on the turning rates of the evaders.

The modified open-loop survival time of the cooperative team evasion problem is defined as follows:

$$\tau^{ol\star}(\mathbb{D}(x_c, r), X_e, \Theta_e^0, \Omega) = \sup_{\Theta_e(\cdot) \in \mathbb{U}_{\Omega,\Theta_e^0}} \inf_{s \in \mathbb{S}_N} \inf_{x_p \in \mathbb{D}(x_c, r)} f_s(x_p, X_e, \Theta_e^0, \Theta_e(\cdot)). \tag{4}$$

The joint state of the evaders with finite turning rate is specified by the current joint position X_e and the current joint heading of N evaders $\Theta_e^0 = [\theta_1^0,\dots,\theta_N^0]$. Since Dubins evaders always travel with constant speeds, it is sufficient to use the heading of an evader as a function of time $\theta(\cdot) : \mathbb{R} \to \mathbb{R}$ to specify the control of an evader. Given a heading $\theta(\cdot)$, the control $u(\cdot)$ can be generated via $u(\cdot) = [\cos\theta(\cdot), \sin\theta(\cdot)]$. The joint heading of the evaders is denoted by $\Theta_e(\cdot) = [\theta_1(\cdot),\dots,\theta_N(\cdot)]$. Here, the evaders' headings come with an additional turning rate constraint despite sharing the same notation as in the previous section. The admissible heading control set of a Dubins evader with an initial heading θ^0 and a maximum turning rate ω is defined as $\mathbb{U}_{\omega,\theta^0} = \{\theta(\cdot) | \theta(0) = \theta^0, |\dot{\theta}(t)| \leq \omega \text{ for } t > 0\}$. The joint admissible Dubins control set for the N evaders is defined as $\mathbb{U}_{\Omega,\Theta_e^0} = \{[\theta_1(\cdot),\dots,\theta_N(\cdot)] | \theta_i(\cdot) \in \mathbb{U}_{\omega_i,\theta_i^0} \text{ for } i = 1,\dots,N\}$ with $\Omega = [\omega_1,\dots,\omega_N]$ being the joint maximum turning rate of the N evaders. The minimum time it takes for a pursuer starting at x_p to capture the N evaders with joint initial position X_e, joint initial heading Θ_e^0, and joint heading control $\Theta_e(\cdot)$ according to the capture sequence s is denoted by $f_s(x_p, X_e, \Theta_e^0, \Theta_e(\cdot))$. Note that (4) is a sup-inf-inf problem with the inner-most inf problem being the pursuer picking its position x_p from the circular disk $\mathbb{D}(x_c, r) = \{x_p | \|x_p - x_c\| \leq r\}$. In the inf problem over $s \in \mathbb{S}_N$, the pursuer picks a capture sequence that results in the minimum capture time given the current positions and joint control of the evaders.

The inf over $x_p \in \mathbb{D}(x_c, r)$ in (4) comes after the inf over $s \in \mathbb{S}_N$, implying that the evaders assume that the pursuer can be at different positions within the disk when different capture sequences are considered. Additionally, since the sup over $\Theta_e(\cdot)$ comes first, the pursuer's decisions on the capture sequence and the position are made after knowing the exact joint control of the evaders. Both of these factors contribute to the conservatism of this formulation towards the evaders. As a result the optimal open-loop survival time, $\tau^{ol\star}$, is a lower bound on the team survival time. By following the optimal open-loop joint control, $\Theta_e^{ol\star}(\cdot)$, the evaders are guaranteed to survive at least for $\tau^{ol\star}$ against an optimal pursuer that can be anywhere within the disk $\mathbb{D}(x_c, r)$.

IV. Solution Methods

In this section, the procedures used to solve the proposed open-loop problem are presented in detail.

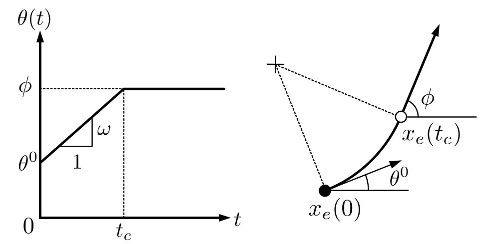
A. Single Parameter Dubins Path

The joint admissible Dubins control set $\mathbb{U}_{\Omega,\Theta_e^0}$ in the sup problem in (4) is an infinite dimensional set containing all the joint admissible Dubins heading controls. To solve (4) efficiently, the search of the optimizer has to be restricted to a finite-dimensional subset of $\mathbb{U}_{\Omega,\Theta_e^0}$.

In Dubins' seminal paper [12], it was pointed out that the minimum length Dubins trajectory between two position-heading pairs in $\mathbb{R}^2$ can be composed by two kinds of elementary paths: paths with curvatures equal to the maximum curvature everywhere, and straight lines. In other words, the admissible heading control set can be limited to $\{\theta(\cdot) | \theta(0) = \theta^0, |\dot{\theta}(t)| = \omega \text{ or } 0, \text{ for } t > 0\}$ without affecting the optimality. Since the orientation at which an evader is captured does not affect the team survival time, we choose to further limit the individual admissible heading control set to the set:

$$\hat{\mathbb{U}}_{\omega,\theta^0} = \{\theta(\cdot) | \theta(0) = \theta^0, \exists t_c \geq 0 \text{ such that}$$
$$|\dot{\theta}(t)| = \omega \text{ for } t \in [0, t_c) \text{ and } \dot{\theta}(t) = 0 \text{ for } t \in [t_c, \infty)\}. \quad (5)$$

An evader adopting a control in this set starts from the initial heading θ^0, turns with maximum turning rate ω in one direction for a finite amount of time t_c and then keeps in constant heading thereafter. For given θ^0 and ω, a heading control in $\hat{\mathbb{U}}_{\omega,\theta^0}$ can be uniquely specified by the direction of the turn and the goal heading $\phi = \theta(t_c)$. In this work, the direction of the turn is picked to be the direction that can reach the goal heading in the minimum amount of time. Figure 1a shows a heading control $\theta(\cdot)$ specified by an initial



(a) Heading control (b) Resulting trajectory

Fig. 1. Example of Dubins heading control and resulting trajectory

heading θ^0, a maximum turning rate ω, and a goal heading ϕ. The turning direction is picked to be counter-clockwise in this case. Figure 1b shows the resulting trajectory of an evader adopting this heading control. Since the joint initial heading Θ_e^0 and the joint maximum turning rate Ω are given as a part of the initial condition, the joint heading control within the set $\mathbb{U}_{\Omega,\Theta_e^0}$ can be parameterized by the joint goal heading $\Phi = [\phi_1, \ldots, \phi_N] \in \mathbb{R}^N$. With this parameterization of the search space, the optimization problem in (4) can be re-written as:

$$\tau^{ol\star}(\mathbb{D}(x_c, r), X_e, \Theta_e^0, \Omega) =$$
$$\sup_{\Phi \in \mathbb{R}^N} \inf_{s \in \mathbb{S}_N} \inf_{x_p \in \mathbb{D}(x_c, r)} f_s(x_p, X_e, \Theta_e^0, \Omega, \Phi), \quad (6)$$

where $f_s(x_p, X_e, \Theta_e^0, \Omega, \Phi)$ is the minimum time it takes a pursuer starting at x_p to capture the evaders under the prescribed parameters X_e, Θ_e^0, Ω, Φ and s. We drop the dependence on the maximum speeds of the players for notational simplicity. Note that the search space for the sup problem is now finite-dimensional. The optimal open-loop joint goal heading of the evaders is defined as the optimizer of (6) and denoted by $\Phi^{ol\star}(\mathbb{D}(x_c, r), X_e, \Theta_e^0, \Omega)$.

B. Solving for the Optimal Open-loop Solution

To solve the optimization problem in (6), $\inf_{x_p \in \mathbb{D}(x_c, r)} f_s(x_p, X_e, \Theta_e^0, \Omega, \Phi)$ must be computed. For the single evader case where the pursuer with maximum speed v_p can be anywhere within a circular disk $\mathbb{D}(x_c, r)$, and the evader with state and control specified by x_e, θ^0, ω, and ϕ, the minimum capture time $t^\star$ is the minimum positive real root of

$$\|x_e(t^\star) - x_c\| = v_p t^\star + r, \quad (7)$$

where $x_e(\cdot)$ denotes the resulting trajectory of the evader given the prescribed parameters and heading control. The computation of $t^\star$ is simple if the capture happens after the evader starts to travel in a straight line at time t_c. As shown in Fig. 2, at time t_c the evader is at $x_e(t_c)$ and will travel in the direction of $e_\phi = [\cos\phi, \sin\phi]$ for all $t \geq t_c$; the pursuer can be anywhere within the disk $\mathbb{D}(x_c, r + v_p t_c)$. A closed-form solution for $t^\star$ exists when $x_e(t_c) \notin \mathbb{D}(x_c, r + v_p t_c)$. For cases where $x_e(t_c) \in \mathbb{D}(x_c, r + v_p t_c)$, $t^\star$ is smaller than t_c and must be solved with a root searching routine on (7).

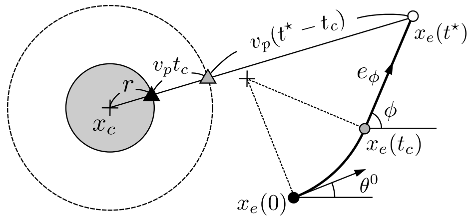


Fig. 2. Minimum capture time of a Dubins path given a pursuer in a circular disk

Algorithm 1 $\inf_{x_p \in \mathbb{D}(x_c, r^0)} f_s(x_p, X_e, \Theta_e^0, \Omega, \Phi)$

$\tau \leftarrow 0$, $x_p \leftarrow x_c$, $r \leftarrow r^0$
for $i = 1, \ldots, N$ **do**
  $x_{s_i}(\cdot) \leftarrow \texttt{traj}(x_{s_i}, \theta_{s_i}^0, \omega_{s_i}, \phi_{s_i})$
  $x_{traj}(t) \leftarrow x_{s_i}(t + \tau)$, for $t \in [0, \infty)$
  $\tau \leftarrow \tau + \texttt{mincap}(\mathbb{D}(x_p, r), x_{traj}(\cdot))$
  if $i = N$ **then return** τ
  else
    $r \leftarrow 0$
    $x_p \leftarrow x_{s_i}(\tau)$
  end if
end for

We introduce two routines: $\texttt{mincap}(\mathbb{D}(x_c, r), x_e(\cdot))$ returns the minimum capture time it takes a pursuer that can be anywhere within the disk $\mathbb{D}(x_c, r)$ to capture an evader traversing the trajectory $x_e(\cdot)$; $\texttt{traj}(x_e, \theta^0, \omega, \phi)$ returns the trajectory of an evader starting at x_e and follows the heading control specified by θ^0, ω, and ϕ. With those two routines, $\inf_{x_p \in \mathbb{D}(x_c, r^0)} f_s(x_p, X_e, \Theta_e^0, \Omega, \Phi)$ can be computed by Algorithm 1 for a given capture sequence $s = [s_1, \ldots, s_N]$. The radius of the pursuer disk is always set to zero once the first evader is captured since the pursuer must be at the capture position and must start pursuing the remaining evaders from that position. Also, the algorithm proposed in [11] is a special case of this algorithm with $r^0 = 0$ and $\Omega = \infty$.

With the ability to efficiently compute the minimum team survival time, the sup-inf-inf problem in (6) is now a sup-inf problem in standard max-i-min form where the max player is trying to maximize the point-wise minimum over a finite number of continuous functions. There are $N!$ functions corresponding to the $N!$ possible capture sequences for a team of N evaders. The minimax problem is then transformed into a constrained non-linear optimization problem and solved by sequential quadratic programming (SQP) routines. The MATLAB function `fminimax` is used to transform and solve the max-i-min problem.

C. Iterative Open-loop Approach against a Hidden Pursuer

In the open-loop (OL) approach, (6) is solved for the open-loop joint optimal goal heading $\Phi^{ol\star}(\mathbb{D}(x_c, r), X_e, \Theta_e^0)$ where $\mathbb{D}(x_c, r)$, X_e, and Θ_e^0 are the pursuer disk, joint evader position, and joint evader heading taken from the initial conditions of the game respectively. The evaders then keep this joint heading until they are all captured. While this does provide a guaranteed lower bound on the team survival time, the survival time can be further improved by employing the iterative open-loop (iOL) approach.

In the iOL approach, the evaders still solve (6) for the optimizer $\Phi^{ol\star}(\mathbb{D}(x_c, r), X_e, \Theta_e^0)$. However, the initial conditions, $\mathbb{D}(x_c, r), X_e$, and Θ_e^0, are taken from the most recent state of the game instead of the initial state of the game. The evaders only keep their joint goal heading for a predetermined amount of time before resolving (6) with the most current state of the game and adjust their joint heading. This iteration continues until all the evaders are captured. An evader at x_e is considered captured when $x_e \in \mathbb{D}(x_c, r)$. At the end of an iteration if no evader is inside the pursuer disk, the radius of the disk is increased by $v_p \Delta t$, which is the distance the pursuer can travel within time Δt; the center of the disk x_c is kept at the same position. If one or more of the evaders are within the pursuer disk, the pursuer can choose to capture one of the evaders within the disk. The center of the disk, x_c, is then moved to the capture point and the radius of the disk is set to zero.

The resulting survival time of the team against a hidden pursuer given initial states $\mathbb{D}(x_c, r), X_e, \Theta_e^0$, and Ω is denoted by $\tau^{\mathrm{iOL}\star}(\mathbb{D}(x_c, r), X_e, \Theta_e^0, \Omega)$. Unlike the visible pursuer case, this survival time can be obtained without specifying a heading control of the pursuer. Since the pursuer is hidden, the actual position of the pursuer within the disk does not affect the actions of the evaders. The value of $\tau^{\mathrm{iOL}\star}(\mathbb{D}(x_c, r), X_e, \Theta_e^0, \Omega)$ only depends on the actual capture sequence. The uncertainties in the pursuer's position introduce additional conservatism towards the evaders in the open-loop formulation, the effect of which will be further discussed in the next section.

V. RESULTS & DISCUSSIONS

We set the following parameters. The maximum speed of the pursuer and the evaders are 1 and 0.25 units per second respectively. The maximum turning rates of the evaders are $\pi/2$ radians per second. The update time of the iterative approaches, Δt, is 0.01 second. It's worth noting that our formulation is able to handle a team of evaders with different maximum speeds and maximum turning rates as long as the pursuer is faster than all evaders. We choose the above-mentioned to simulate a team of fixed-wing unmanned aerial vehicles (UAVs) evading from a faster and more agile UAV.

A. Resulting Behavior

In this section, the resulting trajectories of the agents using the iOL approach against a hidden pursuer are illustrated. Figure 3 shows four snapshots taken from a simulation of a team of evaders using the iOL approach against a hidden pursuer. It's important to note that the actual position and the intended capture sequence of the pursuer are not revealed to the evaders; they are shown in the figures to help visualizing the intention of the players only.

The game starts at $t = 0$ (Fig. 3a); the first evader is captured by the pursuer at the origin. Before this, the team of evaders is not aware of the presence of the pursuer since

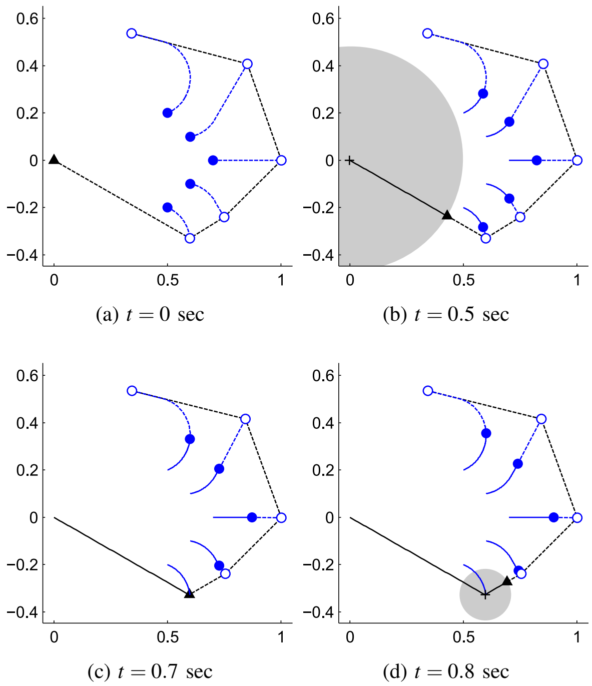


Fig. 3. Snapshots of a simulation. The dark triangle marks the position of the pursuer and the solid blue circles mark the positions of the evaders. The gray disk represents the pursuer disk $\mathbb{D}(x_c, r)$ and the center of the disk is marked by a cross. The solid lines are the trajectories of the players starting from $t = 0$ while the dashed lines represent the planned future trajectories of the players. The hollow circles are the predicted capture points of the evaders. For clarity, only the predicted trajectories that are associated with one of the optimal capture sequence are shown in the figures.

they can not detect it. After the first evader is captured, the 5 remaining evaders then know the pursuer and its position. The evaders start to evade the pursuer using the iOL approach with the pursuer disk centered at the origin with $r = 0$. Figure 3b shows the state of the game at $t = 0.5$ sec. The uncertainty in the pursuer's position has grown with time and the pursuer can now be anywhere within a circular disk centered at the origin with $r = 0.5$. Figure 3c shows the state of the game at $t = 0.7$ sec. An evader is captured at this time, revealing the position of the pursuer to the team; the center of the disk is moved to the capture point with its radius set to 0. Figure 3d shows that at time $t = 0.8$ sec the center of the disk is at the last known position of the pursuer and the radius of the disk has been increased to 0.1. The resulting trajectories of the rest of the game coincide with the predicted trajectories shown in Fig. 3d.

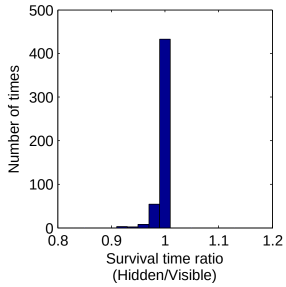
B. Survival Time Performance Against a Hidden Pursuer

In this section, the survival time performance of the team against a hidden pursuer is compared to that against a visible pursuer. The initial positions of the evaders are sampled uniformly from a unit square centered at the origin with the initial headings set to be away from the origin. To simulate a capture that happens at the origin at time $t = 0$, the initial position of the pursuer is set to the origin and is known to the evaders at the beginning of the game. The difference between evading a visible pursuer and a hidden pursuer lies in how the center and radius of the pursuer disk are updated. Against a visible pursuer, the radius of the disk is always zero and the center of the disk is moved to the actual pursuer position at every update time. Against a hidden pursuer, when no capture happens, the center of the disk is kept at the last known position of the pursuer and the radius of the disk increases with time according to the speed of the pursuer; when a capture happens the center of the disk is moved to the capture position with the radius set to zero.

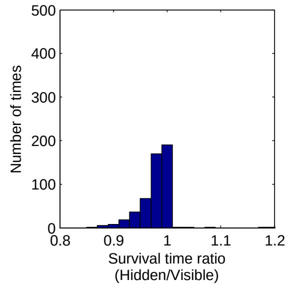
In the simulations the pursuer uses a near optimal strategy against the iOL evaders as proposed in [11]: At every update time, the pursuer computes the optimal trajectory against the current optimal open-loop trajectories of the evaders assuming that the evaders will stick to their current plan. The pursuer follows this open-loop optimal trajectory until the next update time when the evaders update their joint control. This pursuer strategy has been shown to be near optimal against iOL evaders in [11]. The pursuer can capture an evader when the evader is within or on the boundary of the pursuer disk. When there are multiple such evaders the pursuer can only choose to capture one of them. There are two important implications of this capture mechanism. Firstly, the actual position of the hidden pursuer does not affect the team survival time since the actions of the evaders only depend on the center and the radius of the pursuer disk. Secondly, the resulting team survival time is a lower bound of the minimum team survival time against a hidden pursuer. This will be discussed in detail later in this section.

Figure 4 is the histogram of the ratio of the team survival time against a hidden pursuer to the survival time against a visible pursuer. The results are gathered from 500 different initial conditions for a 3-evader team and a 5-evader team. To measure the survival time ratio consistently, each initial condition is simulated twice: once with a visible pursuer and once with a hidden pursuer. The capture sequence taken by the hidden pursuer is set to be the same as that of the visible pursuer. The evaders do not know which sequence the pursuer will take during the simulation. Note that in Fig. 4a most of the initial conditions with 3 evaders have a survival time ratio of 1, indicating that the team can often survive roughly the same time against both a hidden pursuer and a visible pursuer. As shown in Fig. 4b, the 5-evader team achieves a survival time ratio of 1 in 200 out of the 500 initial conditions. (It is consistently above 0.9 for the rest.) This high ratio demonstrates the effectiveness of the iOL approach. Knowing the pursuer's maximum speed, the inability to detect the pursuer does not degrade the survival time performance of the team against a near optimal pursuer by much. This result also shows that the near optimal strategy for the pursuer against iOL evaders proposed in [11] is very close to the optimal because it achieves the lower bound of the minimum team survival time most of the time.

Figure 4 also shows that against a hidden pursuer, the survival time of the 5-evader team degrades more than that of the 3-evader team. This is due to the additional conservatism towards the evaders introduced by the uncertain pursuer



(a) 3 evaders



(b) 5 evaders

Fig. 4. Distribution of the ratio of team survival time against a hidden pursuer to the survival time against a visible pursuer.

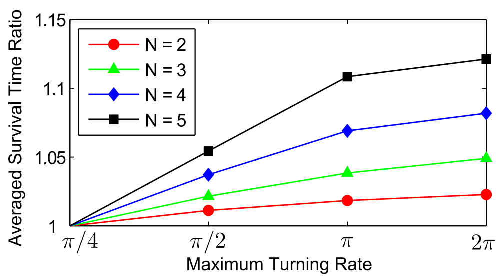


Fig. 5. Averaged ratio of team survival time of different maximum turning rates. N is the number of evaders in the team.

position in the open-loop formulation in (6). In the open-loop formulation, the evaders plan their joint control with the conservative assumption that the pursuer can be at different positions within the pursuer disk when different capture sequences are considered. A team with more evaders has more capture sequences to consider and hence will act more conservatively in response to a hidden pursuer compared to a smaller team.

C. Effects of the Turning Rates on the Team Survival Time

To exam the effect of the maximum turning rates of the evaders on the team survival time, 500 random initial conditions are generated for teams with 2 to 5 evaders. Each initial condition is simulated with the maximum turning rates of the evaders being $\pi/4$, $\pi/2$, π, and 2π rad/sec. For each team size, the average team survival time of different turning rates are compared to that of the lowest turning rate, $\pi/4$ rad/sec. As shown in Fig. 5, the team survival time increases with the turning rates of the evaders. However, this gain in survival time gradually diminishes as the turning rate gets higher. This is to be expected since even a team of evaders with unlimited turning rates will be captured in finite time by a faster pursuer. Another trend to be noticed from the result is that a bigger team benefits more in terms of the team survival time from the increase in turning rates.

VI. CONCLUSION & FUTURE WORK

In this paper we have proposed an open-loop solution to the single-pursuer-multiple-evader pursuit-evasion game where the team of evaders with limited turning rates cooperate to delay the capture of the whole team by a hidden pursuer that is faster than all evaders. Due to the additional conservatism introduced by the uncertainties in the position of the pursuer, the team survival time does degrade gradually as the size of the team increases. However, the approach is shown to be highly effective in that the team survival time against a hidden pursuer is similar to that against a visible pursuer. Also, the results demonstrate that a bigger team can benefit more in terms of the team survival time from an increase in the turning rate of the evaders.

In the future, the authors would like to improve the computation time and scalability of the proposed approach by approximating the nonlinear optimization problem with a linear one and making use of the column generation technique in [13]. Another interesting direction is to investigate evaders' strategies under different objective functions such as the sum or expected value of the survival time of all the evaders.

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