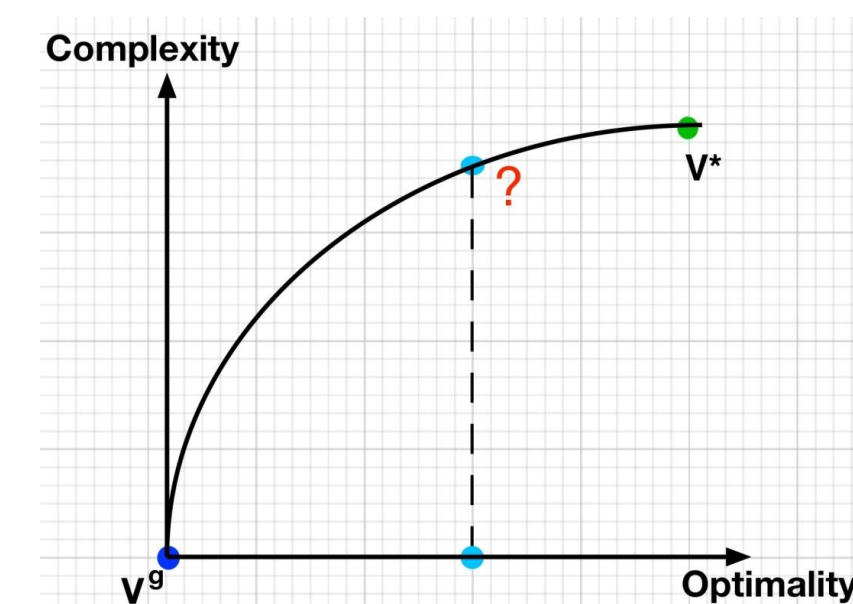
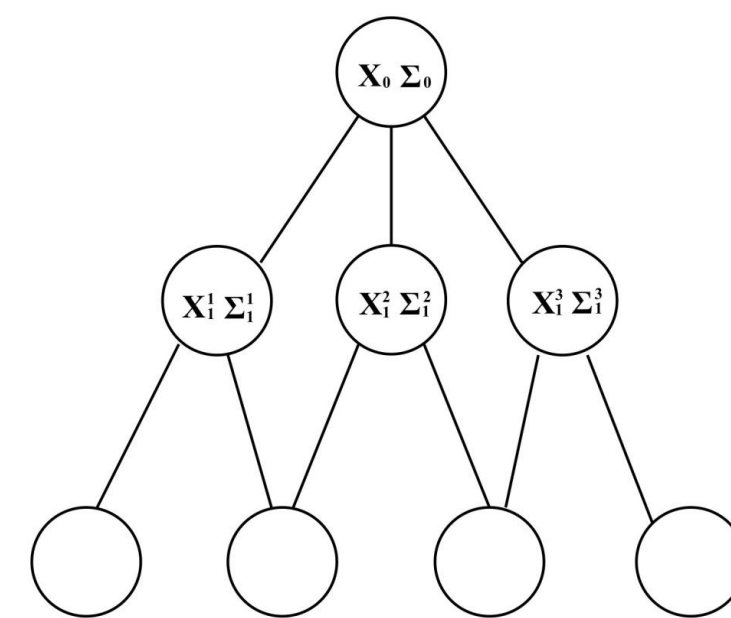


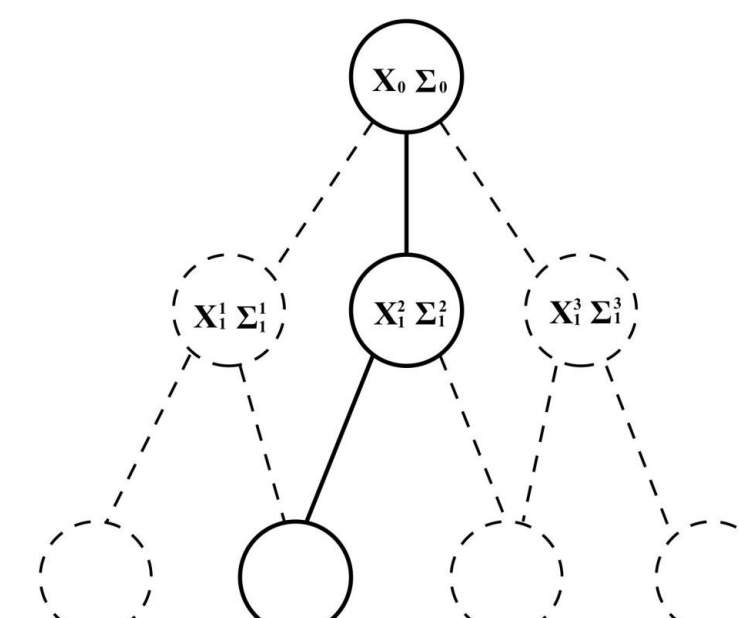
### Forward Value Iteration:

- keeps all reachable nodes
- obtains optimal solution



### Greedy policy:

- keeps only the best node per stage
- no sub-optimality guarantees



### Our Contribution

- Non-myopic control strategy** for mobile sensors with linear Gaussian observation models to track targets with linear Gaussian dynamics
  - Sub-optimality guarantees on the performance
  - Considers **sensor dynamics** & **continuous space** of configurations
  - Trade-off between computation and optimality via the parameters
  - Provably better than the widely used greedy control policy
- Linearization & Model Predictive Control to obtain **closed-loop control policy** for sensors with non-linear observation models
- Target state inference is still done with a **non-linear estimator**

### The Objective (Task D.3.3)

Design a scalable control strategy for a swarm of mobile sensors to observe and evolving phenomenon of interest (target) efficiently

### Sensor & Target Characteristics

- Sensor Motion Model (SMM):  $x_{t+1} = f(x_t, u_t)$ ,  $x_t \in \mathcal{X}, u_t \in \mathcal{U}$
- Target Motion Model (TMM):  $y_{t+1} = a(y_t, w_t)$ ,  $y_t \in \mathcal{Y}, w_t = \text{noise}$
- Sensor Observation Model (SOM):  $z_t = h(x_t, y_t, v_t)$ ,  $v_t = \text{noise}$

### Active Information Acquisition Problem:

Given a finite horizon  $T$ , choose a control policy to maximize the **mutual information** between the measurements and the final target state:

$$\max \mathbb{I}(y_T; z_{1:T} | x_{1:T})$$

### Linear Gaussian Assumptions

- TMM:  $y_{t+1} = Ay_t + w_t$ ,  $y_t \in \mathcal{Y}, w_t \sim \mathcal{N}(0, W)$
- SOM:  $z_t = H(x_t)y_t + v_t(x_t)$ ,  $v_t \sim \mathcal{N}(0, V(x_t))$

### Separation Principle:

Under **linear Gaussian assumptions**, **open-loop** policies are optimal and the problem reduces to a **deterministic control** problem:

$$\begin{aligned} & \max_{u_0, \dots, u_{T-1}} \log \det(\Sigma_T) \\ & \text{s.t.} \quad x_{t+1} = f(x_t, u_t), \quad t = 0, \dots, T-1, \\ & \quad \quad \Sigma_{t+1} = \rho(\Sigma_t, x_{t+1}), \quad t = 0, \dots, T-1, \end{aligned}$$

$\Sigma$  is the covariance of the target distribution,  $\rho$  is the Riccati map

### Partial Order on Sensor Trajectories

#### Algebraic Redundancy:

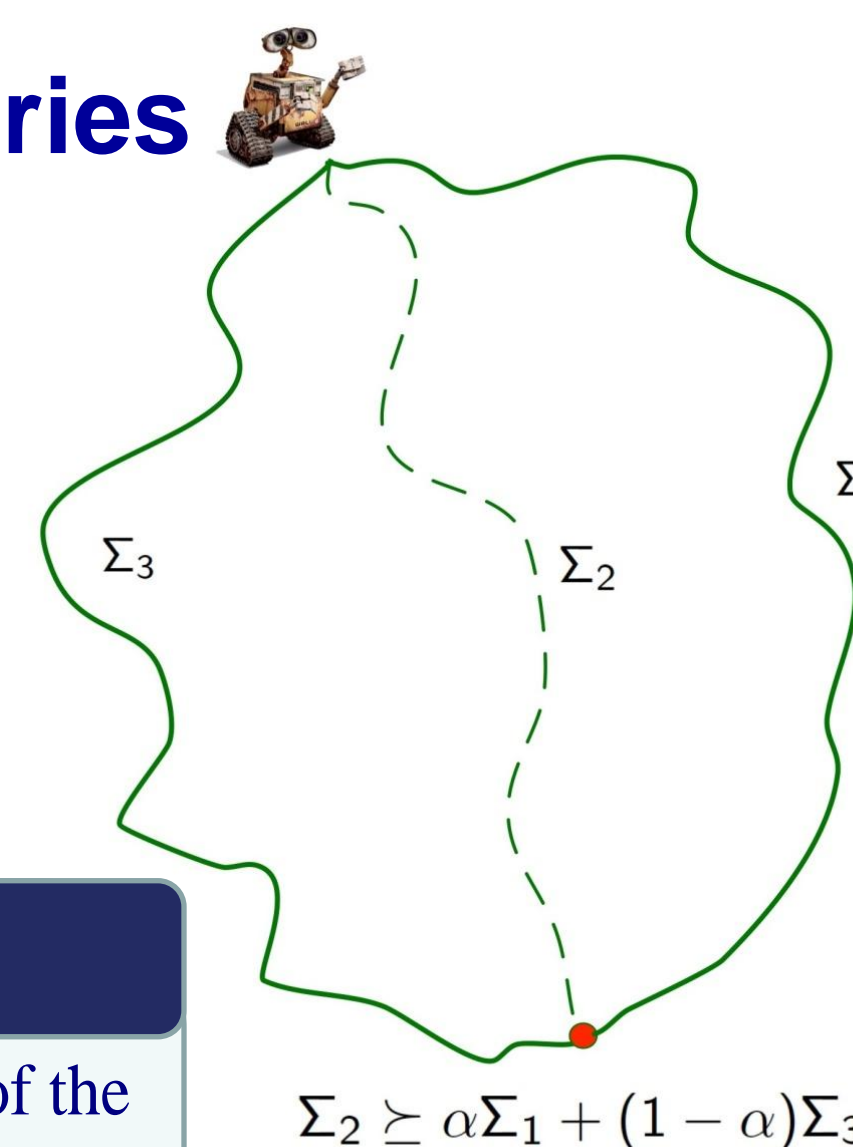
$\Sigma \in S_+^n$  is algebraically redundant wrt  $\{\Sigma_k\}_{k=1}^K \subset S_+^n$  when  $\Sigma \succeq \sum_{k=1}^K \alpha_k \Sigma_k$  for some non-negative constants  $\{\alpha_k\}$ , which sum to 1.

#### Theorem:

For some  $t \in [1, T]$ , let  $(x, \Sigma) \in S_t$  be a node in the  $t$ -th level of the search tree. If there exists a set  $\{x^i, \Sigma^i\} \subseteq S_t \setminus \{(x, \Sigma)\}$  such that:

$$x = x^i, \quad \forall i$$

$\Sigma$  is algebraically-redundant with respect to  $\{\Sigma^i\}$ , then  $(x, \Sigma)$  can be removed from  $S_t$  **without eliminating the optimal sensor path**.

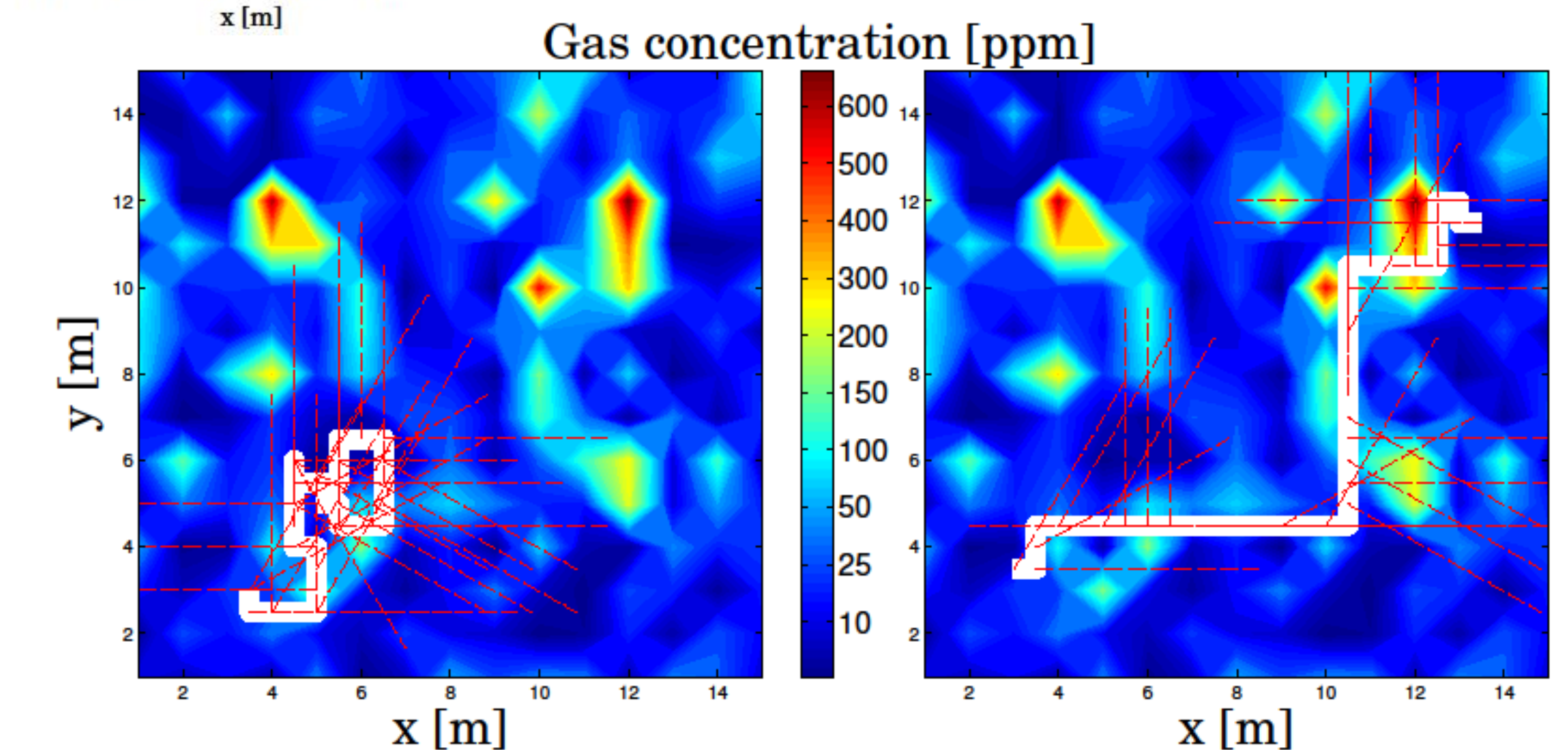
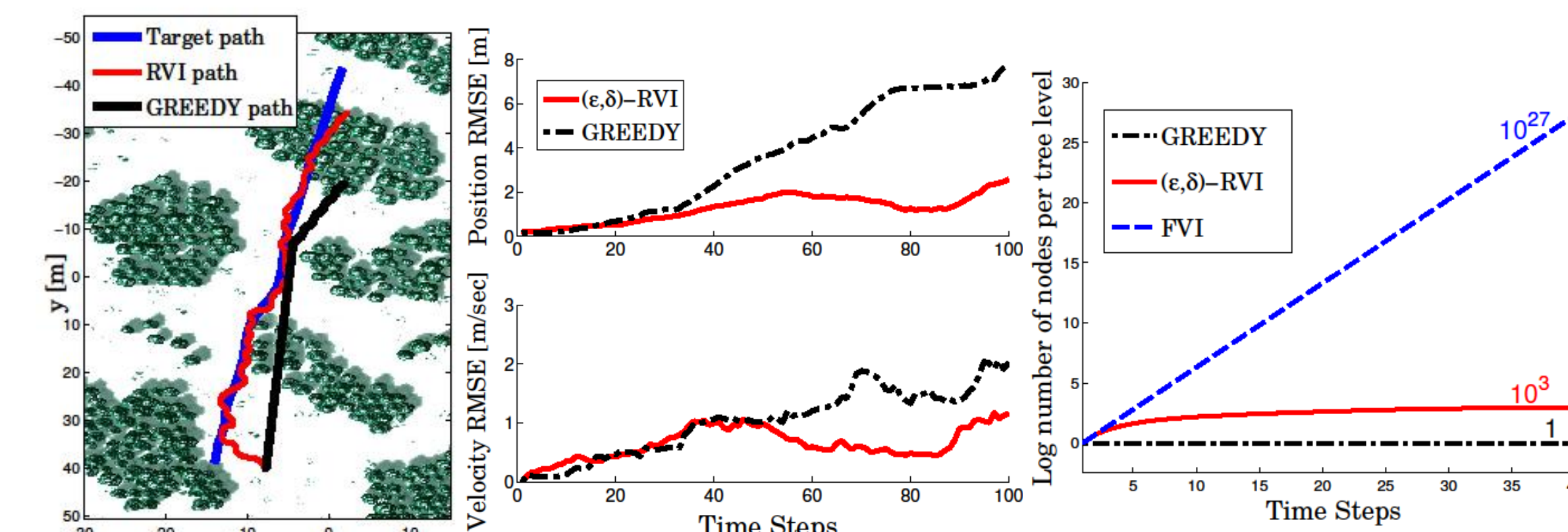
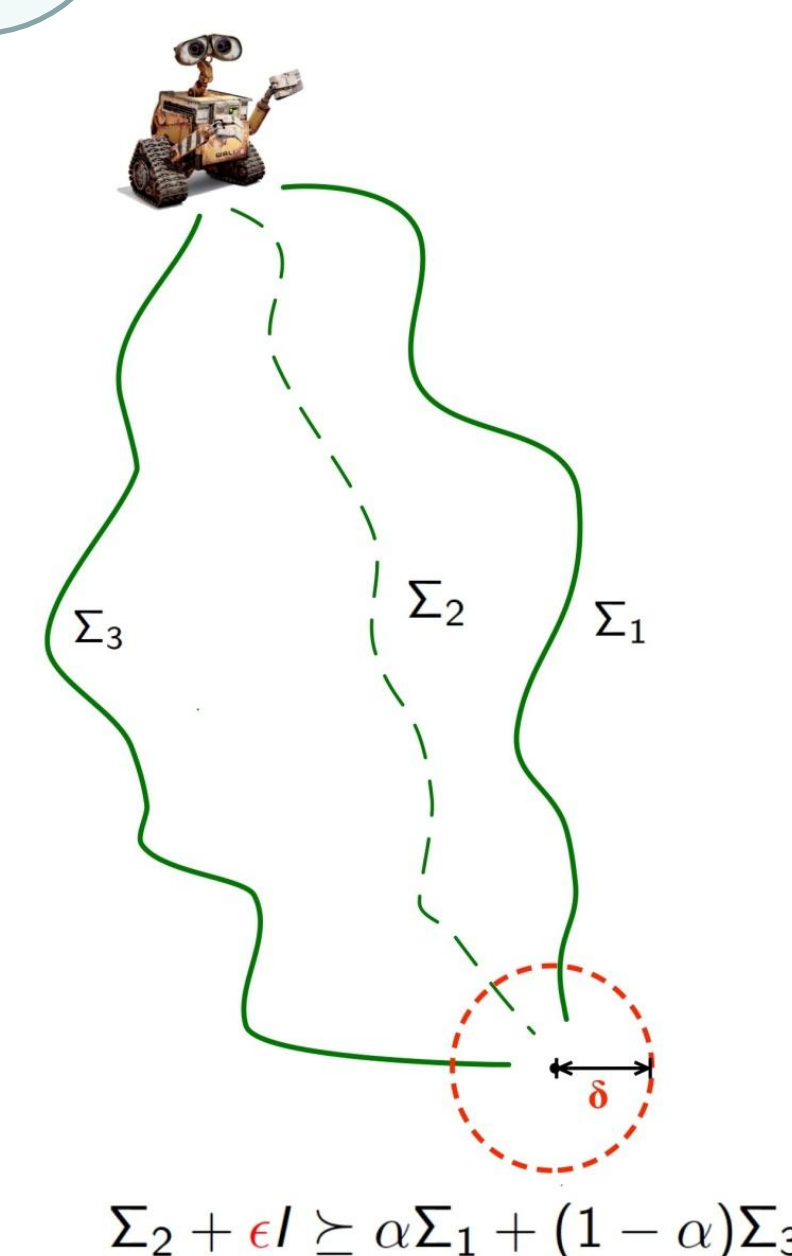


### Relaxing the Conditions

- Remove paths which are:
  - almost crossing at time  $t$
  - almost algebraically-redundant paths
- Regularity assumptions are necessary

#### Theorem:

The sub-optimality gap between the cost  $V_T^*$  of the optimal sensor path and the cost  $V_T^{c, \delta}$  of the path obtained after the reductions satisfies:

$$0 \leq V_T^{c, \delta} - V_T^* \leq (\Gamma_T^\delta - 1)(V_T^* - \log \det(W)) + \epsilon \Delta_T^\delta$$


### Future Direction

- Distributed control and estimation:**
  - Estimation from local observations and interaction with neighbors
  - Choose controls locally, while maintaining performance guarantees
- Applications & experimental validation in the Smart City testbed
- Applications with more general sensing models such as **active semantic localization**
- Distributed self-localization** method with consistency guarantees
  - Collaborate with others working on localization (Task D.3.2)