Integrating Traffic Estimation and Routing Optimization for Multi-Radio Multi-Channel Wireless Mesh Networks

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***Abstract*—Traffic routing plays a critical role in determining the performance of a wireless mesh network. To investigate the best solution, existing work proposes to formulate the mesh network routing problem as an optimization problem. In this problem formulation, traffic demand is usually implicitly assumed as static and known a priori. Contradictorily, recent studies of wireless network traces show that the traffic demand, even being aggregated at access points, is highly dynamic and hard to estimate. Thus, in order to apply the optimization-based routing solution into practice, one must take into account the dynamic and unpredictable nature of wireless traffic demand.**

This paper presents an integrated framework for network routing in multi-radio multi-channel wireless mesh networks under dynamic traffic demand. This framework consists of two important components: traffic estimation and routing optimization. By analyzing the traces collected at wireless access points, the traffic estimation component predicts future traffic demand based on its historical value using time-series analysis, and represents the prediction result in two forms – mean value and statistical distribution. The optimal mesh network routing strategies then take these two forms of traffic demand estimations as inputs. In particular, two routing algorithms are proposed based on linear programming which consider the mean value and the statistical distribution of the predicted traffic demands, respectively. The trace-driven simulation study demonstrates that our integrated traffic estimation and routing optimization framework can effectively incorporate traffic dynamics in mesh network routing, where both algorithms outperform the shortest path algorithm in about 80% of the test cases.

I. Introduction

Wireless mesh networks have attracted increasing attention and deployment as a high-performance and low-cost solution to last-mile broadband Internet access. In a wireless mesh network, local access points and stationary wireless mesh routers communicate with each other and form a backbone structure which forwards the traffic between mobile clients and the Internet. To alleviate the problem of location-dependent interference in wireless communication, mesh routers are usually equipped with multiple radios which enable them to transmit and receive simultaneously or transmit on multiple channels simultaneously.

Traffic routing and channel assignment jointly play a critical role in determining the performance of a wireless mesh network. Thus it attracts extensive research recently. The proposed approaches usually fall into two ends of the spectrum. On one end of the spectrum are the heuristic algorithms (*e.g.*, [1]–[4]). Although many of them are adaptive to the dynamic environments of wireless networks, these algorithms lack the theoretical foundation to analyze how well the network performs globally (*e.g.*, whether the traffic shares the network in a fair fashion).

On the other end of the spectrum, there are theoretical studies based on optimization methods (*e.g.*, [5], [6]). The algorithms derived from these optimization formulations can usually claim analytical properties such as resource utilization optimality and throughput fairness. In these optimization frameworks, traffic demand is usually implicitly assumed as static and known a priori. Contradictorily, recent studies of wireless network traces [7] show that the traffic demand, even being aggregated at access points, is highly dynamic and hard to estimate. Such observations have significantly challenged the practicability of the existing optimization-based routing solutions in wireless mesh networks.

To address this challenge, this paper investigates the optimal mesh network routing framework which takes into account the dynamic nature of wireless traffic demand. This routing framework could work as a part of the joint routing and channel assignment solution in [5]. To incorporate the traffic dynamics, the following two components must be seamlessly integrated into this framework.

- *Traffic demand estimation* which derives the traffic model of a wireless mesh network. The model should be dependable at predicting the mean demand at long term, yet agile at containing often uncertain dynamics at short term.
- *Routing optimization* which works with channel assignment and distributes the traffic along different routes, channels and radio interfaces so that minimum congestion will be incurred even under dynamic traffic. The routing strategy should effectively take into account the traffic demand estimation results.

By studying the traces collected at Dartmouth College campus wireless network [8], the traffic prediction method derives future traffic demand based on its historical value using time-series analysis. The mean value of the predicted demand, together with its prediction error distribution, are used in establishing a statistical model for the traffic demand at a local access point.

This paper further identifies an optimization framework which integrates the demand prediction into traffic routing. In particular, two forms of traffic demands are considered as the inputs for routing optimization, namely the *mean value* of the demand prediction and its *statistical distribution*. We present two routing algorithms for each form of the traffic demand estimation respectively. For the first case, based on the classical maximum concurrent flow problem, we formulate multi-radio multi-channel mesh (M^3) network routing as a linear programming problem, which maximizes the minimum scaling factor (λ) of throughput to fixed-value demand among all flows. We then present a fast $(1-\epsilon)$-approximation algorithm (fixed-demand multi-radio multi-channel mesh network routing (FM^3R) algorithm) which could accept the mean value of the demand prediction as the input. For the second case, in order to incorporate the statistical distribution of the demand estimation into the problem formulation, we characterize the traffic demand using a random variable. Now the scaling factor λ under a given routing solution is also a random variable. The throughput optimization problem is then extended to a stochastic optimization problem where the expected value of the scaling factor λ is maximized. Finally, based on the design of the FM^3R algorithm, a $(1-\epsilon)$-approximation algorithm (uncertain-demand multi-radio multi-channel mesh network routing (UM^3R) algorithm) is presented for mesh network routing under uncertain demand.

To evaluate the performance of our algorithms under realistic wireless networking environment, we conduct a trace-driven simulation study. In particular, we derive the traffic demand for the local access points of our simulated wireless mesh network based on the traffic traces collected at Dartmouth College campus wireless networks. Our simulation results demonstrate that our integrated traffic estimation and optimal routing framework could effectively incorporate the traffic dynamics into the routing optimization of wireless mesh networks, where both algorithms outperform the shortest path algorithm in about 80% of the test cases.

The original contributions of this paper are two-fold. Practically, the integration of traffic estimation and routing optimization effectively improves the network performance of multi-radio multi-channel wireless mesh networks under dynamic and uncertain traffic. The full-fledged simulation study based on real wireless network traffic traces provides convincing validation of the practicability of our solution. Theoretically, upon the classical linear optimization algorithm which only considers wireline network connection and accepts only singlar-value demands as inputs, we extend it into a stochastic optimization solution which is capable of handling wireless mesh network with multiple radio interfaces operating on multiple channels and serving uncertain demands that are modelled by their statistical distributions.

The remainder of this paper is organized as follows. Sec. II presents the system model and solution overview. Sec. III formulates the mesh network routing problem under fixed-value traffic demand and uncertain traffic demand and two fast approximation algorithms (FM^3R and UM^3R). Sec. IV describes the traffic prediction method. We show simulation results in Sec. V, present related work in Sec. VI and finally conclude the paper in Sec. VII.

II. System Model and Solution Overview

A. Network and Interference Model

In a multi-hop wireless mesh network, local access points aggregate and forward the traffic from the mobile clients that are associated with them. They communicate with each other, also with the stationary wireless routers to form a multi-hop wireless backbone network. This wireless mesh *backbone* network forwards the user traffic to the gateways which are connected to the Internet. We use $w \in W$ to denote the set of gateways in the network. In the following discussion, local access point, gateway and mesh router are collectively called mesh nodes and denoted by set V (Note that $W \subset V$). Further, we assume that node v is equipped with $\kappa(v)$ radios. The network could use a set of orthogonal wireless channels denoted by C. For example, in the IEEE 802.11b standard, $|C| = 3$.

In a wireless network, packet transmissions in the same channel are subject to location-dependent interference. We assume that all mesh nodes have the uniform transmission range denoted by R_T. Usually the interference range is larger than its transmission range. We denote the interference range of a mesh node as $R_I = (1+\Delta)R_T$, where $\Delta \geq 0$ is a constant. In this paper, we consider the *protocol model* presented in [9]. Let $r(u,v)$ be the distance between u and v $(u, v \in V)$. In the protocol model, packet transmission from node u to v on channel $c \in C$ is successful, if and only if (1) the distance between these two nodes $r(u,v)$ satisfies $r(u,v) \leq R_T$; (2) any other node $w \in V$ within the interference range of the receiving node v, *i.e.*, $r(w,v) \leq R_I$, is not transmitting on the same channel. If node u can transmit to v directly on channel c, they form an edge $e(c)$. We denote the capacity of this edge as $\phi_e(c)$ which is the maximum data rate that can be transmitted. Let E_c be the set of all edges $e(c)$. We say two edges $e(c), e'(c)$ interfere with each other, if they can not transmit simultaneously based on the protocol model. Further we define *interference set* $I_e(c)$ which contains the edges that interfere with edge $e(c)$ and $e(c)$ itself.

Finally, we introduce a virtual node w^* to represent the Internet. w^* is connected to each gateway with a virtual edge $e' = (w^*, w), w \in W$. For simplicity, we assume that the link capacity in Internet is much larger than the wireless channel capacity, and thus the bottleneck always appears in the wireless mesh network. Under this assumption, the virtual edges could be regarded as having unlimited capacity. Note that none of the virtual links interferes with any of the wireless transmissions.

B. Solution Overview

The performance of a multi-radio multi-channel wireless mesh network critically depends on the design of three interdependent components: scheduling, channel assignment, and routing. Their joint design has been studied in several existing works [5], [6]. In this paper, we adopt the same approach as in [5] which formulates this problem as an integer linear programming problem. To solve this problem, [5] first solves its LP (linear programming) relaxation and derives the routing solution based on the necessary conditions of channel assignment and schedulability. Then the channel assignment and post processing algorithms are designed to adjust the flows to yield a feasible solution.

We assume that the system operates synchronously in a time-slotted mode. The result we obtain will provide an upper bound for systems using IEEE 802.11 MAC. We further assume that the traffic between a local access point and the Internet could be infinitesimally divided and routed over multiple paths to multiple gateways achieving the optimal load balancing and the least congestion.

Formally, let $y_e(c)$ be the flow rate on edge $e(c) \in E_c$, $\boldsymbol{y}$ be the link flow vector, $\rho_e(c) = \frac{y_e(c)}{\phi_e(c)}$ be the utilization of channel c over link e, and $E(v)$ be the set of links that is adjacent to node v. Based on the results presented in [5], the necessary conditions of channel assignment and scheduling are summarized in the following claim:

Claim 1 (Necessary Condition of Channel Assignment and Schedulability). For the multi-channel, multi-radio wireless mesh network, if a given link flow vector $\boldsymbol{y}$ does not satisfy the following inequalities:

$$\sum_{e'(c)\in I_e(c)} \rho_{e'}(c) \leq \gamma(\Delta); \forall e(c) \in E_c, \forall c \in C \tag{1}$$

$$\sum_{c\in C} \sum_{e(c)\in E(v)} \rho_e(c) \leq \kappa(v); \forall v \in V \tag{2}$$

then $\boldsymbol{y}$ is not schedulable.

In particular, Inequality (1) is the *congestion constraint* over an individual channel. $\gamma(\Delta)$ is a constant that only depends on the interference model. Inequality (2) gives the *node radio constraint*. Recall that a mesh node $v \in V$ has $\kappa(v)$ radios, and thus can only support $\kappa(v)$ simultaneous communications.

The focus of this paper is to investigate the optimal routing scheme under dynamic traffic based on the above necessary conditions of channel assignment and schedulability. Once the flow routes are derived, we simply apply the same method presented in [5] to adjust the flow routes and scale the flow rates to yield a feasible routing and channel assignment.

III. Optimal Routing

This paper investigates the optimal routing strategy for wireless mesh *backbone* network. Thus it only considers the aggregated traffic among the mesh nodes. In particular, we regard the virtual node w^* that connects to gateways as the source of all incoming traffic and the destination of all outgoing traffic of a mesh network. Similarly, the local access points, which aggregate the client traffic, serve as the sources of all outgoing traffic and the destinations of incoming traffic. It is worth noting that although we consider only the aggregated traffic between gateway access points and local access points in this paper, our problem formulations and algorithms could be easily extended to handle inter-mesh-router traffic.

For simplicity, we call the aggregated traffic from a local access point to the Internet a *flow* and denote it as $f \in F$, where F is the set of all aggregated flows. We also denote the rate of an aggregated flow $f \in F$ as x_f, and use $\boldsymbol{x} = (x_f, f \in F)$ to represent the aggregated flow rate vector.

A. Fixed Demand Multi-Radio Multi-Channel Mesh Network Routing (FM^3R)

We first study the formulation of throughput optimization routing problem in a wireless mesh backbone network under the fixed traffic demand. We use d_f to denote the demand of flow f and $\boldsymbol{d} = (d_f, f \in F)$ to denote the demand vector consisting of all flow demands. Consider the fairness constraint that, for each flow f, its throughput being routed is in proportion to its demand d_f. Our goal is to maximize λ (so called *scaling factor*) where at least $\lambda \cdot d_f$ amount of throughput can be routed for flow f.

We assume an infinitesimally divisible flow model where the aggregated traffic flow could be routed over multiple paths and use $\mathcal{P}_f$ to denote the set of unicast paths that connect the source of f and w^*. Let $x_f(P)$ be the rate of flow f over path $P \in \mathcal{P}_f$. Obviously the link flow rate $y_e(c)$ is given by $y_e(c) = \sum_{f:P\in\mathcal{P}_f \& e(c)\in P} x_f(P)$, which is the sum of the flow rates that are routed through paths P passing edge $e(c) \in E_c$. Based on the necessary conditions of scheduling and channel assignment in Claim 1 (Eq.(1) and Eq.(2)), we have that

$$\sum_{e'(c)\in I_e(c)} \frac{1}{\phi_{e'}(c)} \sum_{f:P\in\mathcal{P}_f \& e'(c)\in P} x_f(P) \leq \gamma(\Delta); \forall e(c) \in E_c \tag{3}$$

$$\sum_{c\in C} \sum_{e(c)\in E(v)} \frac{1}{\phi_e(c)} \sum_{f:P\in\mathcal{P}_f \& e(c)\in P} x_f(P) \leq \kappa(v); \forall v \in V \tag{4}$$

To simplify the above equations, we define $A_{e(c)P} = \sum_{e'(c)\in I_{e(c)}, e'(c)\in P} \frac{1}{\phi_{e'}(c)}$ and $B_{vP} = \sum_{c\in C} \sum_{e(c)\in E(v), e(c)\in P} \frac{1}{\phi_e(c)}$. The throughput optimization routing with fairness constraint is then formulated as the following linear programming (LP) problem:

$$\begin{aligned}
\mathbf{P_T}: \quad & \text{maximize} && \lambda && (5)\\
& \text{subject to} && \sum_{P\in\mathcal{P}_f} x_f(P) \geq \lambda \cdot d_f, \forall f \in F && (6)\\
& && \sum_{f\in F} \sum_{P\in\mathcal{P}_f} x_f(P) A_{e(c)P} \leq \gamma(\Delta), && \\
& && \forall e(c) \in E_c, \forall c \in C && (7)\\
& && \sum_{f\in F} \sum_{P\in\mathcal{P}_f} x_f(P) B_{vP} \leq \kappa(v), \forall v \in V && (8)\\
& && \lambda \geq 0, x_f(P) \geq 0, \forall f \in F, \forall P \in \mathcal{P}_f && (9)
\end{aligned}$$

In this problem, the optimization objective is to maximize λ, such that at least $\lambda \cdot d_f$ units of data can be routed for each aggregated flow f with demand d_f. Inequality (6) enforces fairness by requiring that the comparative ratio of traffic routed for different flows satisfies the comparative ratio of their demands. Inequality (7) and (8) come from the necessary conditions of channel assignment and scheduling. This problem formulation follows the same form as the maximum concurrent flow problem.

Problem $\mathbf{P_T}$ could be solved by a LP-solver such as [10]. To reduce the complexity for practical use, we present a fully polynomial time approximation algorithm for problem $\mathbf{P_T}$, which finds an ϵ-approximate solution. This algorithm also enlightens the design of the uncertain-demand routing algorithm. The key to a fast approximation algorithm lies on the dual of this problem, which is formulated as follows.

$$\mathbf{D_T}: \quad \text{minimize} \quad \sum_{c \in C} \sum_{e(c) \in E_c} \gamma(\Delta) \cdot \mu_{e(c)} + \sum_{v \in V} \kappa(v) \mu_v \tag{10}$$

$$\text{subject to} \quad \sum_{c \in C} \sum_{e(c) \in E_c} A_{e(c)P} \mu_{e(c)} + \sum_{v \in V} B_{vP} \mu_v \geq \mu_f, \quad \forall f \in F, \forall P \in \mathcal{P}_f \tag{11}$$

$$\sum_{f \in F} \mu_f d_f \geq 1 \tag{12}$$

We assign a price $\mu_{e(c)}$ to each set $I_e(c)$ for $e(c) \in E_c$ and a price μ_v to each node $v \in V$. The objective is to minimize the aggregated price for all interference sets and all nodes. As the constraint, Inequality (11) requires that the price $\sum_{e(c) \in E_c} A_{e(c)P} \mu_{e(c)} + \sum_{v \in V} B_{vP} \mu_v$ of any path $P \in \mathcal{P}_f$ for flow f must be at least μ_f, the price of flow f. Further, Inequality (12) requires that the weighted flow price μ_f over its demand d_f must be at least 1.

Based on the above dual problem $\mathbf{D_T}$, our fast approximation algorithm is presented in Table I. The algorithm design follows the idea of [11] and extends the work of [12] with multi-radio multi-channel consideration. In particular, Line 1 and Line 2 initialize the algorithm. Then for each flow f, we route d_f units of data. We do so by finding the lowest priced path in the path set $\mathcal{P}_f$ (Line 7), then filling traffic to this path by its bottleneck capacity (Lines 8 to 10). Then we update the prices for the interference sets and the nodes appeared in this path based on the function defined in Line 11 and Line 12. We keep filling traffic to flow f in the above fashion until all d_f units are routed. This procedure is repeated until the weighted aggregated price of the interference sets and the nodes exceeds 1 (Line 3).

We formally analyze the properties of our algorithm in the following theorem. The proofs of the theorems in this paper are available in the technical report [13].

Theorem 1: If $\beta = ((|E_c| + |V|)/(1-\epsilon))^{-1/\epsilon}$, then the final flow generated by the FM^3R algorithm is at least $(1-3\epsilon)$ times the optimal value of $\mathbf{P_T}$. The running time is $O(\frac{1}{\epsilon^2}[\log(|E_c|+|V|)(2|F|\log|F|+|E_c|+|V|)+\log U)]) \cdot T_{mp}$, where U is the length of the longest path in G, and T_{mp} is the running time to find the shortest path.

B. Uncertain Demand Mesh Network Routing

Now we proceed to investigate the throughput optimization routing problem for wireless mesh backbone network when the aggregated traffic demand is uncertain. We model such uncertain traffic demand of an aggregated flow $f \in F$ using a random variable D_f. We assume that D_f follows the following discrete probability distribution $Pr(D_f = d_f^i) = q_f^i$, where $\mathcal{D}_f = \{d_f^1, d_f^2, ..., d_f^m\}$ is the set of of values for D_f with non-zero probabilities. Let $\boldsymbol{d} = (d_f, d_f \in \mathcal{D}_f, f \in F)$ be a sample traffic demand vector, $\boldsymbol{D}$ be the corresponding random variable, and $\mathcal{D}$ be the sample space. Thus the distribution of $\boldsymbol{D}$ is given by the joint distribution of these random variables: $Pr(\boldsymbol{D} = \boldsymbol{d}) = Pr(D_f = d_f^i, f \in F)$. We abbreviate $Pr(\boldsymbol{D} = \boldsymbol{d})$ as $p(\boldsymbol{d})$. It is obvious that $\sum_{\boldsymbol{d} \in \mathcal{D}} p(\boldsymbol{d}) = 1$.

FM^3R: Fixed-Demand Multi-Radio Multi-Channel Mesh Network Routing

1 $\forall c \in C, \forall e(c) \in E_c, \gamma \leftarrow \gamma(\Delta), \mu_{e(c)} \leftarrow \beta/\gamma,$ $\mu_v \leftarrow \beta/\kappa(v)$
2 $x_f(P) \leftarrow 0, \forall P \in \mathcal{P}_f, \forall f \in F$
3 **while** $\sum_{c \in C} \sum_{e(c) \in E(c)} \gamma \cdot \mu_{e(c)} + \sum_{v \in V} \kappa(v) \mu_v < 1$
4   **for** $\forall f \in F$ **do**
5     $d'_f \leftarrow d_f$
6   **while** $\sum_{c \in C} \sum_{e(c) \in E(c)} \gamma \cdot \mu_{e(c)} + \sum_{v \in V} \kappa(v) \mu_v < 1$ **and** $d'_f > 0$ **do**
7       $P \leftarrow$ lowest priced path in $\mathcal{P}_f$ using $\mu_{e(c)}$ and μ_v
8       $\delta \leftarrow \min\{d'_f, \min_{e(c) \in P} \frac{\gamma}{A_{e(c)P}}, \min_{v \in V} \frac{\kappa(v)}{B_{vP}}\}$
9       $d'_f \leftarrow d'_f - \delta$
10       $x_f(P) \leftarrow x_f(P) + \delta$
11       $\forall c \in C, \forall e(c) \in E_c$ s.t. $A_{e(c)P} \neq 0$, $\mu_{e(c)} \leftarrow \mu_{e(c)}(1 + \epsilon\delta A_{e(c)P}/\gamma)$
12       $\forall v \in V$ s.t. $B_{vP} \neq 0$, $\mu_v \leftarrow \mu_v(1 + \epsilon\delta B_{vP}/\kappa(v))$
13   **end while**
14   **end for**
15 **end for**

TABLE I
ROUTING ALGORITHM UNDER FIXED DEMAND

Let us consider a traffic routing solution $(x_f(P), P \in \mathcal{P}_f, f \in F)$ that satisfies the capacity and node-radio constraints (Inequality (7) and (8)). It is obvious that λ is a function of $\boldsymbol{d}$: $\lambda(\boldsymbol{d}) = \min_{f \in F}\{\frac{x_f}{d_f}\}$, where $x_f = \sum_{P \in \mathcal{P}_f} x_f(P)$. Further let us consider the optimal routing solution under demand vector $\boldsymbol{d}$. A ϵ-optimal solution could be easily derived based on Algorithm I shown in Table I. We denote the optimal value of λ as $\lambda^*(\boldsymbol{d})$.

We further define the *performance ratio* ω of routing solution $(x_f(P), P \in \mathcal{P}_f, f \in F)$ as $\omega(\boldsymbol{d}) = \frac{\lambda(\boldsymbol{d})}{\lambda^*(\boldsymbol{d})}$ Obviously, the performance ratio is also a random variable under uncertain demand. We denote it as Ω which is a function of random variable $\boldsymbol{D}$. Now we extend the wireless mesh network routing problem to handle such uncertain demand. Our goal is to maximize the expected value of Ω, which is given by $E(\Omega) = \sum_{\boldsymbol{d} \in \mathcal{D}} p(\boldsymbol{d}) \times \frac{\lambda(\boldsymbol{d})}{\lambda^*(\boldsymbol{d})}$

Formally, we formulate the throughput optimization routing problem for wireless mesh backbone network under uncertain traffic demand as follows.

$\mathbf{P_U}$:

$$
\begin{aligned}
\text{maximize} \quad & \sum_{\boldsymbol{d}\in\mathcal{D}} p(\boldsymbol{d})\frac{\lambda(\boldsymbol{d})}{\lambda^*(\boldsymbol{d})} && (13)\\
\text{subject to} \quad & \forall \boldsymbol{d}\in\mathcal{D}, \text{where } \boldsymbol{d}=(d_f, f\in F) \\
& \sum_{P\in\mathcal{P}_f} x_f(P) \geq \lambda(\boldsymbol{d})\cdot d_f, \forall f\in F && (14)\\
& \sum_{f\in F}\sum_{P\in\mathcal{P}_f} x_f(P)A_{e(c)P} \leq \gamma(\Delta), \\
& \forall e(c)\in E_c, \forall c\in C && (15)\\
& \sum_{f\in F}\sum_{P\in\mathcal{P}_f} x_f(P)B_{vP} \leq \kappa(v), \forall v\in V && (16)\\
& \lambda\geq 0, x_f(P)\geq 0, \forall f\in F, \forall P\in\mathcal{P}_f && (17)
\end{aligned}
$$

Similar to problem $\mathbf{P_T}$, the constraints of $\mathbf{P_U}$ come from the fairness requirement and the wireless mesh network capacity and node radio interfaces. In particular, Inequality (14) enforces fairness for all demand $\boldsymbol{d}\in\mathcal{D}$, Inequality (15) enforces capacity constraint as Inequality (7) in problem $\mathbf{P_T}$, and Inequality (16) enforces the node-radio constraint as Inequality (8) in problem $\mathbf{P_T}$.

Now we consider the dual problem $\mathbf{D_U}$ of $\mathbf{P_U}$. Similar to $\mathbf{D_T}$, the objective of $\mathbf{D_U}$ is to minimize the aggregated price for all adjusted interference sets. However, in Inequality (20), for each sample demand vector $\boldsymbol{d}$, the aggregated price of all flows weighted by their demand needs to be larger than its probability.

UM^3R: Uncertain-Demand Multi-Radio Multi-Channel Mesh Network Routing

1 $\forall c\in C$, $\forall e(c)\in E_c$, $\gamma\leftarrow\gamma(\Delta)$, $\mu_{e(c)}\leftarrow\beta/\gamma$, $\mu_v\leftarrow\beta/\kappa(v)$
2 $x_f(P)\leftarrow 0$, $\forall P\in\mathcal{P}_f$, $\forall f\in F$
3 **loop**
4 **for** $\forall f\in F$ **do**
5 $\bar{P}\leftarrow$ lowest priced path in $\mathcal{P}_f$ using $\mu_{e(c)}$, μ_v
6 $\mu_f\leftarrow\sum_{c\in C}\sum_{e(c)\in E_c} A_{e(c)\bar{P}}\mu_{e(c)} + B_{v\bar{P}}\mu_v$
7 **end for**
8 **for** $\forall \boldsymbol{d}\in\mathcal{D}$ **do**
9 $\mu_{\boldsymbol{d}}\leftarrow\sum_{f\in F}\mu_f d_f\frac{\lambda^*(\boldsymbol{d})}{p(\boldsymbol{d})}$
10 **end for**
11 $\mu^{\min}\leftarrow\min_{\boldsymbol{d}\in\mathcal{D}}\mu_{\boldsymbol{d}}$
12 $\boldsymbol{d}^{\min}\leftarrow\arg\min_{\boldsymbol{d}\in\mathcal{D}}\mu^{\min}$
13 **if** $\mu^{\min}\geq 1$
14 **return**
15 **for** $\forall f\in F$ **do**
16 $d'_f\leftarrow d_f^{\min}$
17 **while** $d'_f>0$ **do**
18 $P\leftarrow$ lowest priced path in $\mathcal{P}_f$ using $\mu_{e(c)}$, μ_v
19 $\delta\leftarrow\min\{d'_f, \min_{e(c)\in P}\frac{\gamma}{A_{e(c)P}}, \min_{v\in V}\frac{\kappa(v)}{B_{vP}}\}$
20 $d'_f\leftarrow d'_f-\delta$
21 $x_f(P)\leftarrow x_f(P)+\delta$
22 $\forall c\in C$, $\forall e(c)\in E_c$ s.t. $A_{e(c)P}\neq 0$, $\mu_{e(c)}\leftarrow\mu_{e(c)}(1+\epsilon\delta A_{e(c)P}/\gamma)$
23 $\forall v\in V$ s.t. $B_{vP}\neq 0$, $\mu_v\leftarrow\mu_v(1+\epsilon\delta B_{vP}/\kappa(v))$
24 **end while**
25 **end for**
26 **end loop**

TABLE II
ROUTING ALGORITHM UNDER UNCERTAIN DEMAND

$$
\begin{aligned}
\mathbf{D_U}: \quad \text{minimize} \quad & \sum_{c\in C}\sum_{e(c)\in E_c}\gamma(\Delta)\cdot\mu_{e(c)} + \sum_{v\in V}\kappa(v)\mu_v && (18)\\
\text{subject to} \quad & \sum_{c\in C}\sum_{e(c)\in E_c} A_{e(c)P}\mu_{e(c)} + \sum_{v\in V} B_{vP}\mu_v \geq \mu_f, \\
& \forall f\in F, \forall P\in\mathcal{P}_f && (19)\\
& \sum_{f\in F}\mu_f d_f \geq \frac{p(\boldsymbol{d})}{\lambda^*(\boldsymbol{d})}, \forall \boldsymbol{d}\in\mathcal{D} && (20)\\
& \text{where } \boldsymbol{d}=(d_f, f\in F)
\end{aligned}
$$

Now we present an approximation algorithm for $\mathbf{P_U}$ in Table II. This algorithm (UM^3R) has the same initialization as the algorithm for problem $\mathbf{P_T}$ (FM^3R). Then we march into the iteration, in which we find $\boldsymbol{d}^{\min}$, the demand whose price $\mu^{\min}$ is the minimum among others (Lines 4 to 12). If $\mu^{\min}\geq 1$, then the algorithm stops (Lines 13 and 14), since Inequality (19) and (20) would be satisfied for all demand. Otherwise, we will increase the price of $\boldsymbol{d}^{\min}$ by routing more traffic. This procedure (Lines 16 to 23) is the same as what has been described in Lines 4 to 12 of FM^3R algorithm. Following the same proving sequence for FM^3R, we are able to prove the similar properties with UM^3R.

Theorem 2: If $\beta=((|E_c|+|V|)/(1-\epsilon))^{-1/\epsilon}$, then the final flow generated by the UM^3R algorithm is at least $(1-3\epsilon)$ times the optimal value of $\mathbf{P_U}$. The running time is $O(\frac{1}{\epsilon^2}[\log(|E_c|+|V|)(2|\mathcal{D}||F|\log|F|+|E_c|+|V|)+\log U)])\cdot T_{mp}$, where U is the length of the longest path in G, T_{mp} is the running time to find the shortest path.

IV. TRAFFIC ESTIMATION

The goal of this section is to (1) develop a reliable estimation method that is able to predict the aggregated traffic demand of an access point based on its historical data, and (2) develop a statistical model to characterize the prediction results. The estimated traffic demands serve as the inputs for the FM^3R and UM^3R algorithms respectively.

In order to develop such a traffic demand model, first let's examine the traces collected at the campus wireless LAN network of Dartmouth College in Spring 2002 [8]. By analyzing the *snmp* log from each access point, we can derive its aggregated traffic behavior. Given the present availability of wireless network traces, we argue that the above trace best resembles the traffic condition of a wireless mesh network, since the access points of a wireless LAN serve a similar role and thus exhibit similar behavior as the local access points of a wireless mesh network.

To illustrate our analysis procedure, we choose one of the access points (ResBldg97AP3) as an example. The time series of its incoming traffic is plotted in Fig. 1. The first step of our analysis is to identify and remove the daily and

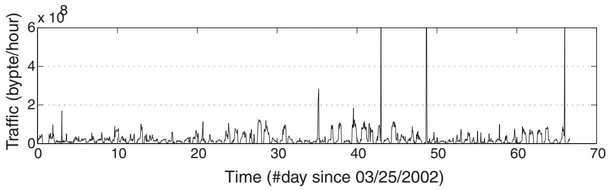


Fig. 1. Incoming Traffic Time Series of ResBldg97AP3 (March 25, 12am, 2002 - June 9, 11pm, 2002 EST).

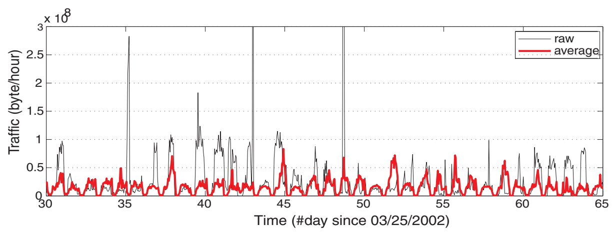


Fig. 2. Raw Traffic vs. Moving Average Series

weekly cyclic patterns in the time series. This requires us to calculate the weekly/daily cyclic average. Formally, let us denote $x(t)$ as the *raw traffic series*. We estimate the moving average of this series based on the same time of the day: $\bar{x}(t) = \sum_{i=1}^{W} x(t - 24 \times i)/W$, where W is the size of the moving window. To eliminate the effect of bursty traffic, we also filter out the spike traffic during the above averaging procedure. Fig. 2 plots the raw traffic as well as its moving average with $W = 5$ for the interval $[30, 65]$ day.

By removing the cyclic effect from the raw data, we derive the *adjusted traffic series* $z(t)$ as $z(t) = x(t) - \bar{x}(t)$. This adjusted traffic exhibits short-term (a few hours) traffic correlations. We model the adjusted traffic series with an autoregressive process as follows.[1]

$$z(t) = \beta_1 z(t-1) + \beta_2 z(t-2) + ... + \beta_K z(t-K) + \epsilon \quad (21)$$

where K is the process order. To apply this model for prediction, we estimate the parameters of this process. Given N observations $z_1, z_2, ..., z_N$, the parameters β_1, ..., β_K are estimated via least squares by minimizing:

$$\sum_{t=K+1}^{N} \left[z(t) - \beta_1 z(t-1) ... - \beta_K z(t-K)\right]^2 \quad (22)$$

Based on these parameters, we further derive the adjusted traffic prediction $\hat{z}(t)$ as $\hat{z}(t) = \beta_1 z(t-1) + \beta_2 z(t-2) + ... + \beta_K z(t-K)$. Fig. 3 illustrates the estimation results for the adjusted traffic series for the interval $[780, 840]$ hour, where $K = 2$, $\beta_1 = 0.531$, $\beta_2 = 0.469$. The figure plots the predicted series for the adjusted traffic as well as its raw data. In this figure, the number of observations used for parameter estimation is $N = 60$. The fitted traffic series is also plotted for the interval $[720, 779]$ hour for the purpose of comparison.

[1]Ideally, $z(t)$ should have zero mean. In some cases, $z(t)$ has a small mean value which needs to be removed before fitting an autoregressive process.

We now consider the errors involved in this prediction process. In particular, we define the adjusted traffic prediction error as $\epsilon_z(t) = z(t) - \hat{z}(t)$. Through normality test, we find the error distribution fits the normal distribution with a mean close to zero.

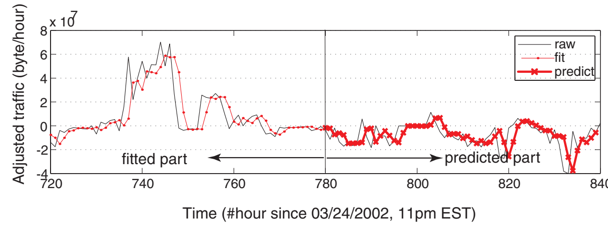


Fig. 3. Adjusted Traffic and Its Prediction

Finally, we define traffic prediction $\hat{x}$ as follows:

$$\hat{x}(t) = [\bar{x}(t) + \hat{z}(t)]^+ \quad (23)$$

where $[x]^+ = \max\{0, x\}$. Fig. 4 plots the predicted traffic series $\hat{x}(t)$ in comparison with the raw traffic. We can see the predicted traffic closely matches the real(raw) traffic. Again the prediction error $\epsilon_x(t) = x(t) - \hat{x}(t)$ also fits the normal distribution with a near-zero mean. To this end, we could consider the estimated traffic demand at time t as a random variable $X(t)$ which follows the normal distribution with mean $\hat{x}(t)$ and the same variance as $\epsilon_x(t)$.

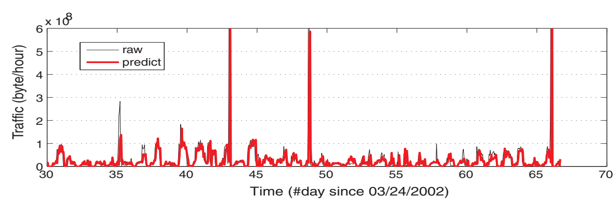


Fig. 4. Raw Traffic vs. Predicted Traffic

To summarize, the presented estimation method provides two prediction models: mean value and statistical distribution, which serve as the inputs for the FM^3R and the UM^3R algorithms respectively.

V. Simulation Study

A. Simulation Setup and Performance Metrics

We evaluate the performance of our algorithms via simulation study. We develop a simulator which provides flow-level wireless mesh network simulation with multi-radio and multi-channel capability. In the simulated wireless mesh network, 60 mesh nodes are randomly deployed over a $1000 \times 2000m^2$ region. In the default simulation setup, 10 nodes at the edge of this network are selected as the local access points (LAP) that forward traffic for clients. 4 nodes in the center of the deploy region are selected as the gateway access points. Each mesh node has a transmission range of $250m$ and an interference range of $500m$, which means $\Delta = 1$. The channel capacity $\phi_c(e)$ is the same for all links e and channels c, which is set as 54 Mbps. In the basic setting, each mesh node is equipped with 3 radio interfaces. And there are 3 orthogonal channels in the network. Aside from this basic setting, we have also evaluated the performance of our algorithms with different configurations of radio and channel numbers, which we will show in the later part of this section.

To realistically simulate the traffic demand at each LAP, we employ the traces collected in the campus wireless LAN network. The network traces used in this work are collected in Spring 2002 at Dartmouth College and provided by CRAWDAD [8]. By analyzing the *snmp* log trace at each access point, we are able to derive its 1108-hour incoming and outgoing traffic volume since 12:00AM, March 25, 2002 EST. We select the access points from the Dartmouth campus wireless LAN and assign their traffic traces to the LAPs in our simulation.[2]

We evaluate and compare different traffic prediction and routing strategies for this simulated network. In particular, we consider the following strategies.

- *Oracle Routing (OR)*. In this strategy, the traffic demand is known a priori. It runs the FM^3R algorithm (presented in Tab. I) based on this demand. This solution runs every hour based on the up-to-date traffic demand from the trace and returns the optimal set of routes. This ideal strategy is designed to return the benchmark result, which the rest of the practical strategies compare to.
- *Mean-Value Prediction Routing (MVPR)*. This strategy does not know the traffic demand a priori. Instead, it only predicts the traffic demand based on its historical data. In particular, it employs the mean value prediction model and runs the FM^3R algorithm based on this predicted demand. This solution also runs every hour to provide the set of routes for the next hour.
- *Statistical-Distribution Prediction Routing (SDPR)*. Similar to MVPR, this strategy also relies on traffic prediction. It predicts not only the mean-value of the traffic demand in the next hour, but also its distribution. It runs the UM^3R algorithm (presented in Tab. II) with the predicted traffic demand distribution as its input. Since UM^3R only accepts discrete probability distribution, we need to discretize the demand distribution by sampling the following values, the mean value μ, and values $\mu - \sigma$, $\mu + \sigma$, $\mu - 2\sigma$, and $\mu - 2\sigma$. Since about 95% of all traffic demand values fall within the range $[\mu - 2\sigma, \mu + 2\sigma]$, we ignore the values which has a probability smaller than 5%.
- *Shortest-Path Routing (SPR)*. This strategy is agnostic of traffic demand, and returns fixed routing solution purely based on the shortest distance (number of hops) from each mesh node to the gateway. The purpose to evaluate this strategy is to quantitatively contrast the advantage of our traffic-predictive routing strategies.

Note that the flows derived from the above routing strategies will be adjusted by the channel assignment, post processing and flow scaling algorithms in [5]. We denote the final rate of flow f along path P as $x_f^A = \sum_{P \in \mathcal{P}_f} x_f^A(P)$. This is the maximum flow throughput under the fairness constraint weighted by the traffic demand, which maximizes the scaling factor λ. However, for performance study, λ is not a suitable performance metric. First, we are more interested in the network performance (*i.e.*, congestion) incurred by the given traffic demand, instead of the achievable throughput. Second, the absolute value of λ could be misleading, especially when the actual demand is not the same as the predicted demand which is being used for routing.

Now we proceed to define the performance metric we use in the simulation study. First, we derive $x'_f(P)$, the actual traffic load that is imposed on path P under our routing and channel assignment scheme, by scaling the achievable flow rate x_f^A by its actual traffic demand d_f:

$$x'_f(P) = x_f^A(P) \cdot \frac{d_f}{x_f^A} \tag{24}$$

Thus the traffic being routed within the interference set $I_e(c)$ over channel c is given by $\sum_{f \in F} \sum_{P \in \mathcal{P}_f} x'_f(P) A_{e(c)P}$. We define the *congestion of an interference set* $I_e(c)$ using its utilization and denote it as $\theta_e^{ch}(c) = \frac{\sum_{f \in F} \sum_{P \in \mathcal{P}_f} x'_f(P) A_{e(c)P}}{\gamma(\Delta)}$. Then $\theta^{ch} = \max_{e(c) \in E_c} \theta_e^{ch}(c)$ is the maximum congestion among all the interference sets. We further consider the congestion at a single mesh node incurred by the traffic from all channels. The *congestion of a node* v is defined as $\theta_v^{rd} = \frac{\sum_{f \in F} \sum_{P \in \mathcal{P}_f} x'_f(P) B_{vP}}{\kappa(v)}$. And $\theta^{rd} = \max_{v \in V} \theta_v^{rd}$. Finally, the *network congestion* θ is defined as $\theta = \max\{\theta^{rd}, \theta^{ch}\}$. Note that θ could be larger than 1, since it reflects the network congestion caused by real traffic demands. In this case, the capacity constraints and node-radio constraints can be violated, causing packet delay and loss.

B. Simulation Results

We experiment with the above routing strategies along the time range $[108, 1108]$, a 1000-hour period excerpted from the trace.[3] Note that all the simulation results presented in this section are using 108 as the zero point.

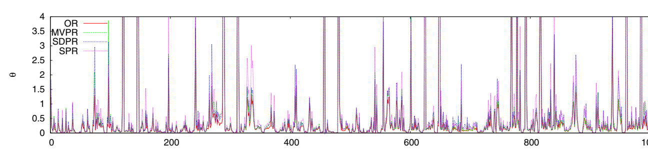


Fig. 5. Overview of All Strategies

We start by presenting the congestion achieved by all strategies (*OR*, *MVPR*, *SDPR*, and *SPR*) during the entire 1000-hour simulation period. As seen in Fig. 5, *OR* constantly achieves the minimum worst-case congestion among others, due to its unrealistic capability to know the actual traffic demand. We

[2] For the details of the simulated network topology, gateway selection and traffic assignment, please refer to our technical report [13].

[3] Note that the beginning part of the trace $[0, 107]$ is used as training data, thus is not included in the simulation result.

note that the burstiness of θ applies to all strategies including *OR*. Such observation comes from the burstiness of the traffic load in the *snmp* log trace, which is caused by the insufficient level of traffic multiplexing at wireless local access points.

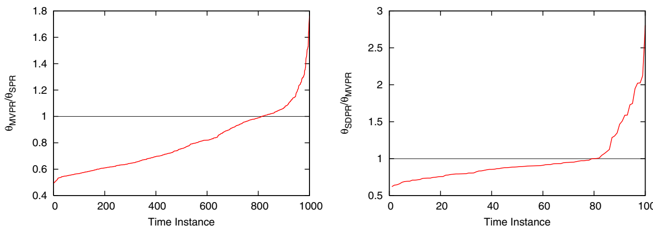


Fig. 6. (a) $\frac{\theta_{MVPR}}{\theta_{SPR}}$ Sorted View (b) $\frac{\theta_{SDPR}}{\theta_{MVPR}}$ Sorted View

To have a clearer comparison among the *MVPR*, *SDPR*, and *SPR* strategies, in Fig. 6 we show the sorted congestion ratio between *MVPR* and *SPR* ($\frac{\theta_{MVPR}}{\theta_{SPR}}$) and the sorted congestion ratio between *SDPR* and *SPR* ($\frac{\theta_{SDPR}}{\theta_{MVPR}}$). From the figure, *MVPR* outperforms *SPR* in 81.4% of all time instances (test cases) with an average congestion ratio of 0.803. While *SDPR* outperforms *MVPR* in 79% of all time instances.

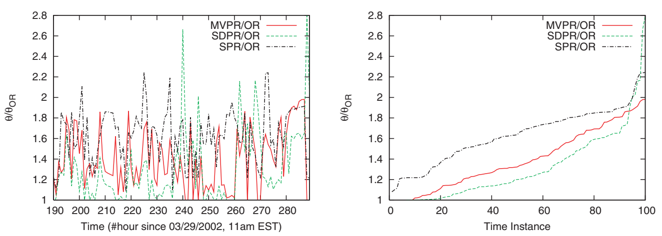
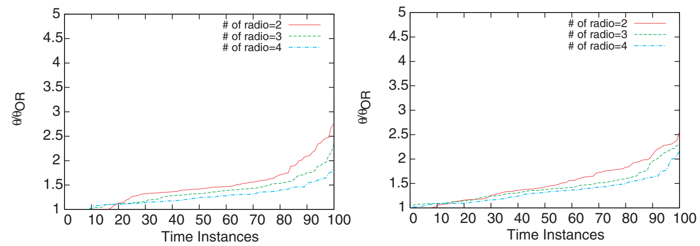


Fig. 7. (a) Congestion Ratio ($\frac{\theta}{\theta_{OR}}$) (b) Sorted View

Also in Fig. 7(a), we take a close-up look during the hour range $[190, 290]$. Here, the *MVPR* and *SDPR* strategies achieve less than 2 times of the optimal congestion in most cases, while the *SPR* strategy performs worse than the previous two in most cases. The above observations get clearer when we sort out the normalized congestion ratio for the three strategies in Fig. 7(b). It is clear that our *MVPR* and *SDPR* strategies which integrate the traffic prediction with the optimal routing outperform the *SPR* strategy which is agnostic about the traffic demand. Further, *SDPR* achieves lower congestion than *MVPR* in most of the time due to more comprehensive representation of the traffic demand estimation. However, in a few cases (less than 10% of the time), the worst-case congestion of *SDPR* is higher than *MVPR*. This problem can be mostly attributed to the inaccuracy of traffic prediction.

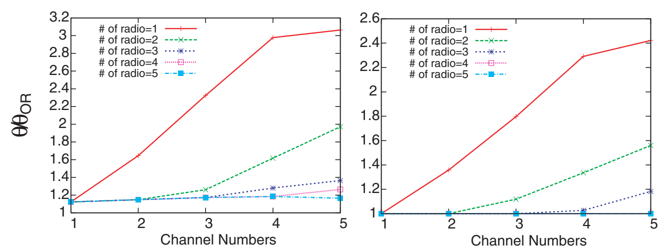
In what follows, we alter our simulation configurations to examine the abilities of different strategies at adapting various network settings, such as radio interface numbers and channel numbers. Here, we focus on the traffic prediction strategies, namely, *MVPR* and *SDPR*. Also we plot their performances by the congestion ratio θ/θ_{OR} normalized by the *OR* routing results. We first vary the number of radio interfaces from 2 to 4 and study the congestion θ during the time interval



(a) MVPR (c) SDPR

Fig. 8. Impact of Number of Radio Interfaces

$[190, 290]$. Fig. 8 plots the sorted normalized congestion $\frac{\theta}{\theta_{OR}}$ of the two strategies. Comparing these two figures, we could see that the *SDPR* strategy performs slightly better than the *MVPR* strategy. The improvement of both strategies over the *OR* strategy increases (*i.e.*, normalized congestion decreases) with the radio number.



(a) MVPR (b) SDPR

Fig. 9. Impact of Channel/Radio

Finally, Fig. 9 plots the normalized congestion under different radio and channel numbers at a single time instance 271 for these two strategies. The results show that the improvement of both strategies over the *OR* strategy decreases with the channel number. This is because when the network has more channels, the algorithms are likely to find more paths and the prediction error is more likely to be magnified.

If our algorithms are implemented in real systems, the processing overhead includes the computation overhead at the central node and the message exchange overhead between the central node and mesh nodes. The computation overhead of each mesh node is relatively small. Our approximate algorithms have shown that the computation overhead at the centralized node is polynomial. For the message exchange overhead, each mesh node report the traffic information to central node and central node sends the routing information back to mesh nodes. The complexity of message overhead is $O(n)$ where n is the number of mesh nodes in the network.

VI. Related Work

We evaluate and highlight our original contributions in light of previous related work.

The problem of wireless mesh network routing, channel assignment, and the joint solution of these two have been extensively studied in the existing literature. For example, routing algorithms are proposed to improve the throughput for wireless mesh networks via integrating MAC layer information [2], such as expected packet transmission time [1], channel cost metric (CCM) which is the sum of expected transmission time weighted by the channel utilization [4]. Joint solutions for channel allocation and routing are explored in [14] using a centralized algorithm and in [3] in a distributed fashion. These heuristic solutions are designed to adapt to the dynamic network condition. However, they lack the theoretical foundation to analyze how well the network performs globally (*e.g.,* whether the network resource is fully utilized, whether the flows share the network in a fair fashion) under their routing schemes.

There are also theoretical studies that formulate these network planning decisions into optimization problems. For example, the works of [5], [6] study the optimal solution of joint channel assignment and routing for maximum throughput under a multi-commodity flow problem formulation and solve it via linear programming. The work of [15] presents bandwidth allocation schemes to achieve maximum throughput and lexicographical max-min fairness respectively. Further, the work of [16] presents a rate limiting scheme to enforce the fairness among different local access points. These results provide valuable analytical insights to the mesh network design under ideal assumptions such as known static traffic input. However, they may be unsuitable for practical use under highly dynamic traffic situation. Different from these existing works, our work explicitly incorporates traffic behavior analysis and prediction into the routing optimization, thus better fits the routing need in the dynamic wireless mesh networks. Distributed algorithms are presented for joint scheduling and routing in [17], and for joint channel assignment, scheduling and routing in [18]. These distributed algorithms only use local information for traffic routing, thus have the potential to accommodate dynamic traffic. However, their crucial properties, such as convergence speed and messaging overhead, are yet to be evaluated under realistic traffic conditions.

Trace analysis has been used to study the behavior of wireless networks in many recent works. For example, [7] statistically characterizes both static flows and roaming flows in a large campus wireless network. Different from these existing works, which focus on either user behavior, network flow or link performances, we provide a framework that integrates traffic uncertainty model with its performance optimization.

Our work is also related to dynamic traffic engineering [19] in Internet, which also considers the impact of demand uncertainty in make routing decisions. The major difference between our work and these existing works lies in the different network and traffic models of wireless mesh network and Internet. This work also extends our early work [12] from single-radio single-channel network to multi-radio multi-channel wireless mesh network.

VII. Conclusion

This paper studies the optimal routing strategies for wireless mesh networks. Different from existing works which implicitly assume traffic demand as static and known a priori, this work considers the traffic demand uncertainty. It studies the dynamic behavior of wireless network traffic, establishes two prediction models based on time series analysis, and extends the classical maximum concurrent flow problem with statistical demand input. Simulation study is conducted based on the traffic demand from the real wireless network traces. The results show that our problem formulation and algorithm could effectively incorporate the traffic demand dynamics.

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