Correlated Failures of Power Systems: Analysis of the Nordic Grid

Martin Andreasson[*], Saurabh Amin[†], Galina Schwartz[†],
Karl H. Johansson[*], Henrik Sandberg[*], Shankar S. Sastry[†]

[*] ACCESS Linnaeus Centre, KTH
[†] TRUST Center, UC Berkeley

April 11, 2011 - FDSCPS Workshop

Correlated failures in power systems

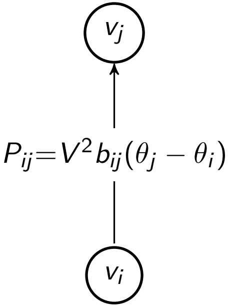
- New technology and software advancements pose new threats to power systems (e.g. PMUs, FACTS, WAMS)
- Failures due to software or hardware faults are more likely to be more correlated than other failures
- Current reliability criteria, such as the $N-1$ criterion assume independent failures and do not work well for correlated failures
- How does increased correlation between faults affect losses, and how can these be measured?

DC power flow model

- Power injections P are given by:

$$P = V^2 A^T B A \theta =: L_B \theta$$

  - $P \in \mathbb{R}^n$ is a vector of power injections
  - θ is a vector of phase angles
  - V is the bus voltage
  - $A \in \mathbb{R}^{m \times n}$ is the node-edge incidence matrix of the graph corresponding to the power system
  - $B = \text{diag}(b_1, \ldots, b_n) \in \mathbb{R}^{m \times m}$ is a matrix of edge admittances
  - $L_B \in \mathbb{R}^{n \times n}$ is the weighted Laplacian of the power system, with weight matrix $V^2 B$



Load shedding

- Load shed defined as:

$$S := P_{load} - P_{load}^{demand}$$

  where P_{load} is the total power load of the load nodes, and P_{load}^{demand} is the total power demand

- The load shed represents the gap between power supply and demand, and is used as a last resort to maintain stability in the power system
- In normal operation, the load shed $S = 0$

Optimal load shedding

- By using the DC-model, the problem of minimizing the load shed problem can be cast as a linear program[1]:

$$\begin{aligned} \min \quad & \text{Load shed} \\ \text{s.t.} \quad & \text{Physical constraints} \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} \min_{\theta} \quad & c^T\theta \\ \text{s.t.} \quad & C\theta \preceq d \end{aligned}$$

$$\Leftrightarrow$$

$$\min_{\theta} \begin{bmatrix} \mathbf{0} & \mathbf{1} \end{bmatrix} L_B\theta \quad \text{s.t.} \begin{bmatrix} V^2BA \\ -V^2BA \\ L_B \\ -L_B \\ A \\ -A \end{bmatrix} \theta \leq \begin{bmatrix} P^{line}_{max} \\ P^{line}_{max} \\ P^{g}_{max} \\ \mathbf{0} \\ \mathbf{0} \\ P^{demand}_{load} \\ \Delta\theta_{max} \cdot \mathbf{1} \\ \Delta\theta_{max} \cdot \mathbf{1} \end{bmatrix}$$

[1] Abur, Gomez. *Power System State Estimation: Theory and Implementation*. Dekker, Abingdon, 2004

Connection to the $N - 1$ criterion

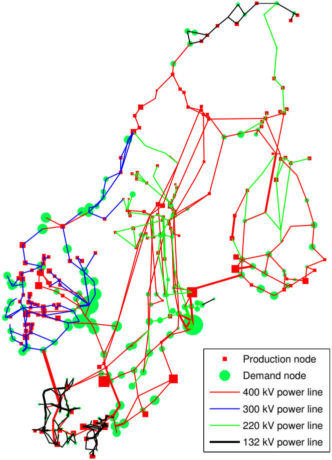
Definition

A power system is $N - 1$ reliable if for all single disconnections of power lines, it holds that $S = 0$

- The $N - 1$ criterion is a widely used deterministic reliability criterion
- The $N - 1$ criterion does not allow measuring the size of the losses
- There is an implicit assumption that 2 or more failures are very unlikely

Model of the Nordic power grid

- We derive a static model of the Nordic HV power grid
  - 470 buses, 717 power lines
  - Topology available from public sources
  - Line admittances estimated by line length
  - Generation capacities and line capacities are collected from public sources
  - Demand data estimated from census data



Problem formulation

- Failures are modeled as a binary random variable X:
  - $X_i = 0 \Leftrightarrow$ line i is connected
  - $X_i = 1 \Leftrightarrow$ line i is disconnected $(B_{ii} = 0)$
- Note that the line admittances $B_X = B(I - \mathrm{diag}(X))$ are random variables, hence the load shed $S(X) = \min_\theta\{c(X)^T\theta | C(X)\theta \preceq d\}$ is also a random variable
- Keep $\bar{X} = \mathrm{E}[X]$ constant, and vary $\sigma_X = \mathrm{E}[(X - \bar{X})^T(X - \bar{X})]$

Problem

How does the distribution of $S(X)$ depend on the correlation of the failures X?

Monte Carlo sampling algorithm

for different covariances **do**
  for i=1:number of samples **do**
    Draw a sample $\bar{X}$ from the Bernoulli RV X with given mean and covariance, then compute:

$$S(\bar{X}) = \min_{\theta}\{c^T\theta | C(\bar{X})\theta \leq d\}$$

  end for
end for

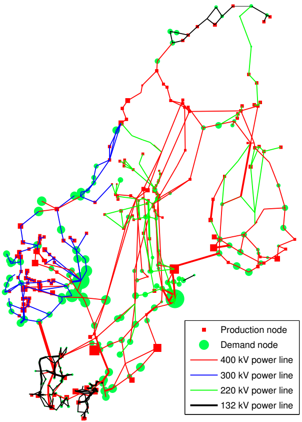
- The sampled mean converges asymptotically to the actual mean, in particular given $\epsilon > 0, \delta > 0$, there exists a number of samples $N \sim \frac{1}{\delta\epsilon^2}$ s.t.:

$$\Pr[|\hat{S}_N - \bar{S}| \geq \epsilon] \leq \delta$$

where $\hat{S}_N, \bar{S}$ are the sampled and the actual mean of S

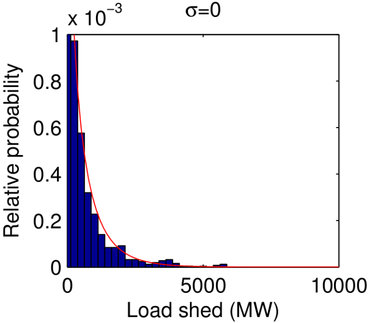
Simulation results

- Consider a model where neighboring lines are more likely to fail simultaneously
- Let the covariance between power lines be nonzero iff they are incident
- Run the Monte Carlo simulation for $N = 1000$ samples

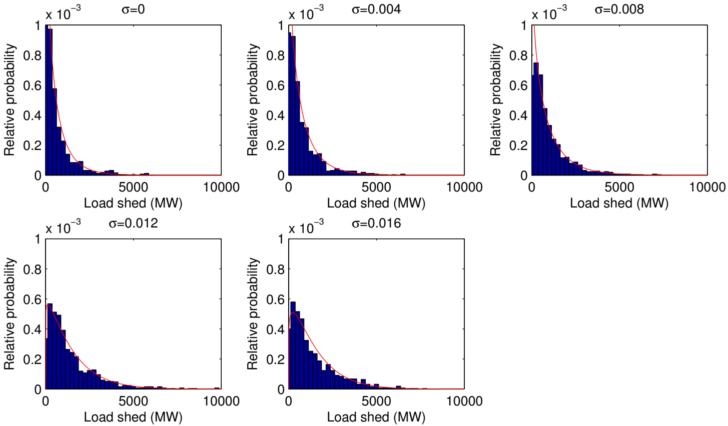


Sampled load shed distributions

- The load shed distribution for the uncorrelated case, $\sigma_X = \mathbf{0}$
- It is observed that the load shed distribution can be well approximated with a Weibull distribution

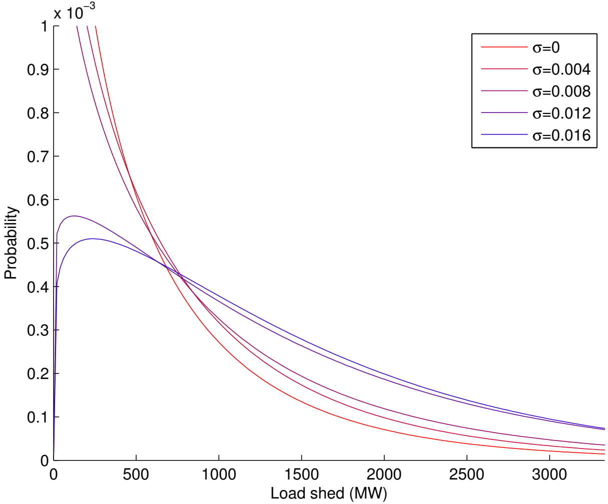


Sampled load shed distributions



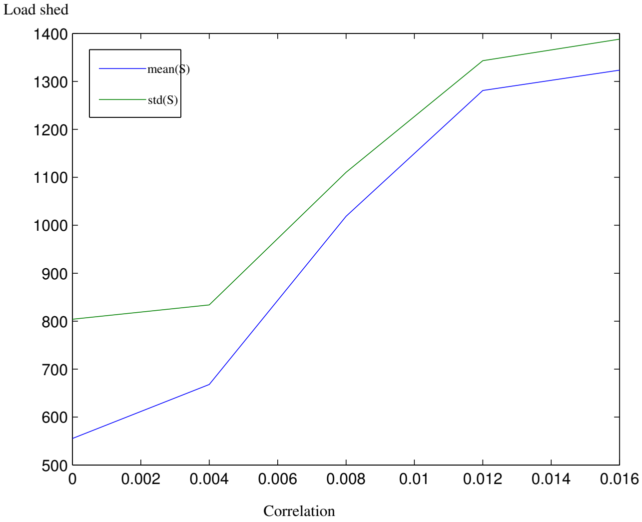
- As correlations increase, $\Pr(X = 0)$ decreases
- The tails of the load shed distribution get fatter as correlations increase

Fitted Weibull distributions for different correlations



- When comparing the distributions side-by-side the differences are evident

Mean and standard deviation as a function of correlation



- Both the mean and the vaiance of S increase with correlations

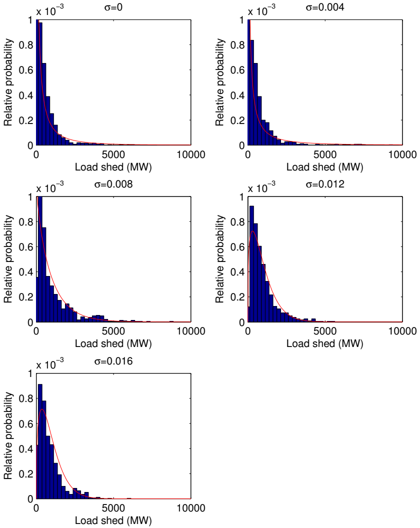
Conclusions

- Traditional reliability criteria such as the $N-1$ criterion are not sufficient when the failure distributions are correlated
- $N-2$ may not be computationally or economically feasible. There is need for new measures of reliability
- Monte Carlo study is used for analyzing power system reliability, in particular under correlated failures
- Increased correlation between power line failures can increase the expected cost of system operation, as well as the variance, leading to higher risks
- Analytical studies are needed to provide further insight in the consequences of correlated failures

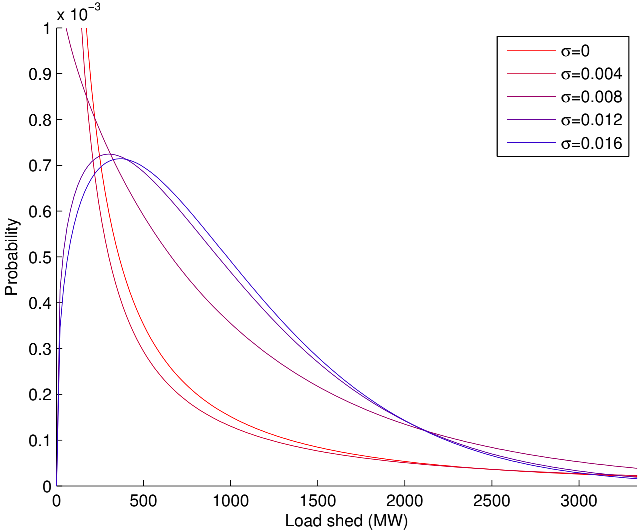
Thank you!

Correlation between power lines connected to PMU nodes

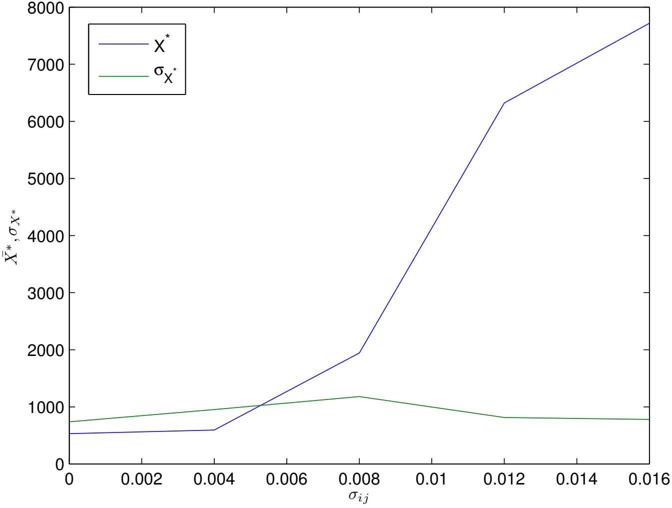
- Let:the failures between power lines connected to PMUs be correlated



- As correlations increase, the expected load shed increases.
- The tails of the load shed get fatter with correlations



- When comparing the distributions side-by-side the differences are evident



- Both the mean and the vaiance of S increase with correlations