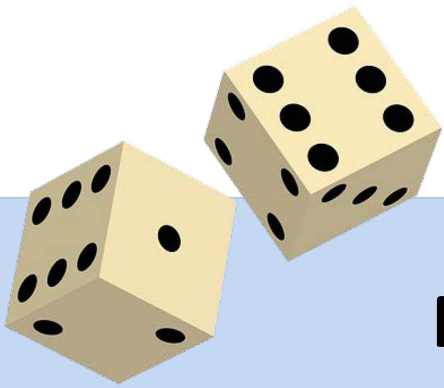
iCyPhy

Software Design for Cyber-Physical Systems

Edward A. Lee



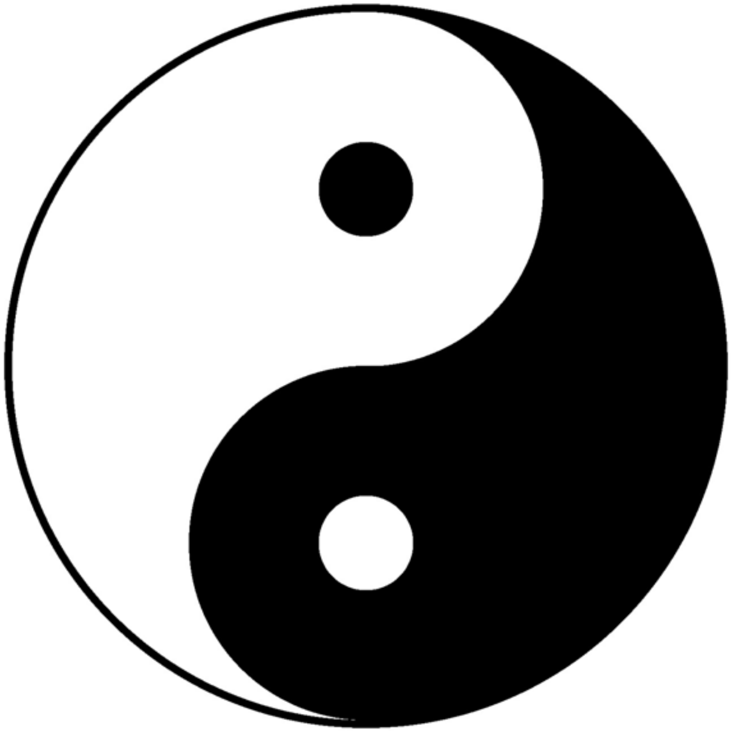
Module 5: Limits of Determinism

Technical University of Vienna
Vienna, Austria, May 2022

University of California, Berkeley

Despite the advantages…

Determinism has its limits.



- Complexity
- Uncertainty
- Chaos
- Incompleteness

Complexity

- Some systems are too complex for deterministic models.
- Nondeterministic abstractions become useful.

"Iron wing" model of an Airbus A350.

Complexity

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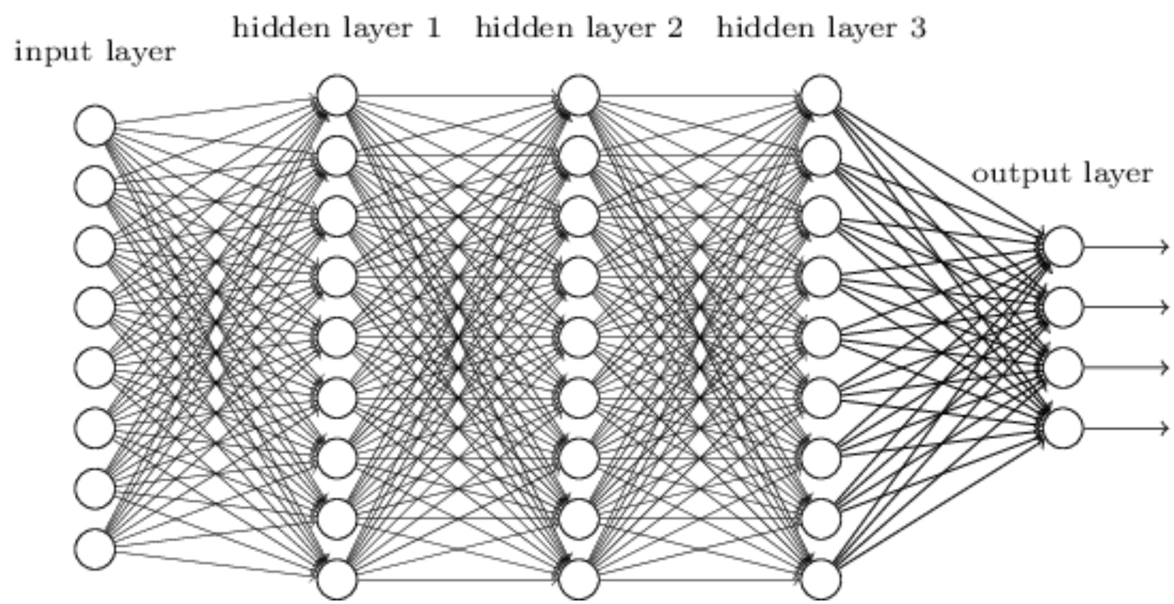
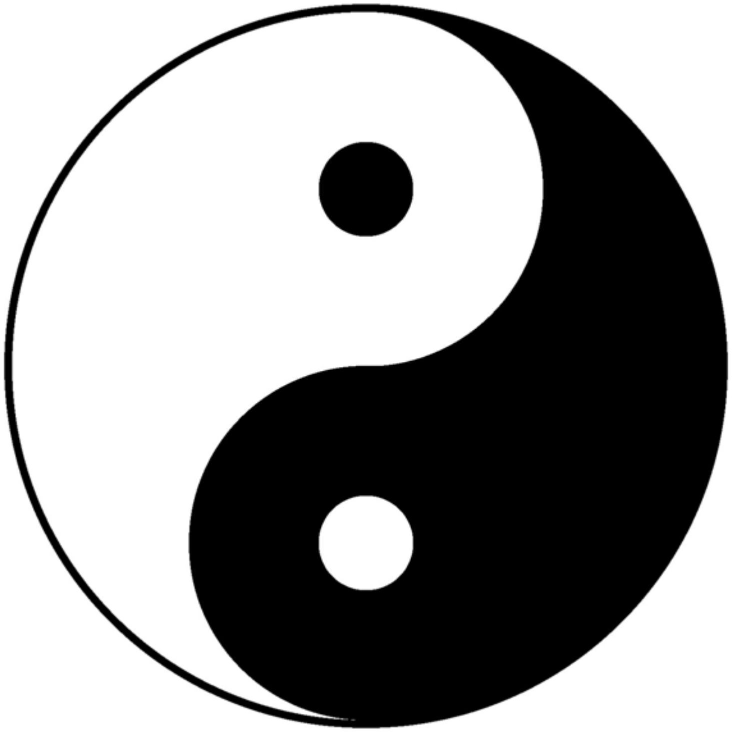


Deep Learning, draft book in preparation, by Yoshua Bengio, Ian Goodfellow, and Aaron Courville. http://www.deeplearningbook.org/

But…

Determinism has its limits.



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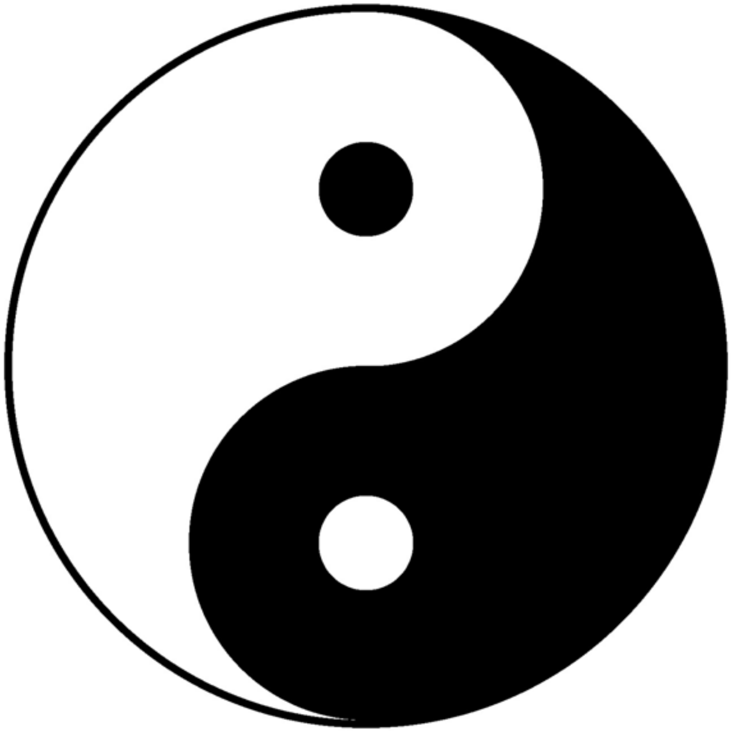
Uncertainty

- We can't construct deterministic models of what we don't know.
- For this, nondeterminism is useful.
- Bayesian probability (which is mostly due to Laplace) quantifies uncertainty.

Portrait of Reverend Thomas Bayes (1701 - 1761) that is probably not actually him.

But…

Determinism has its limits.



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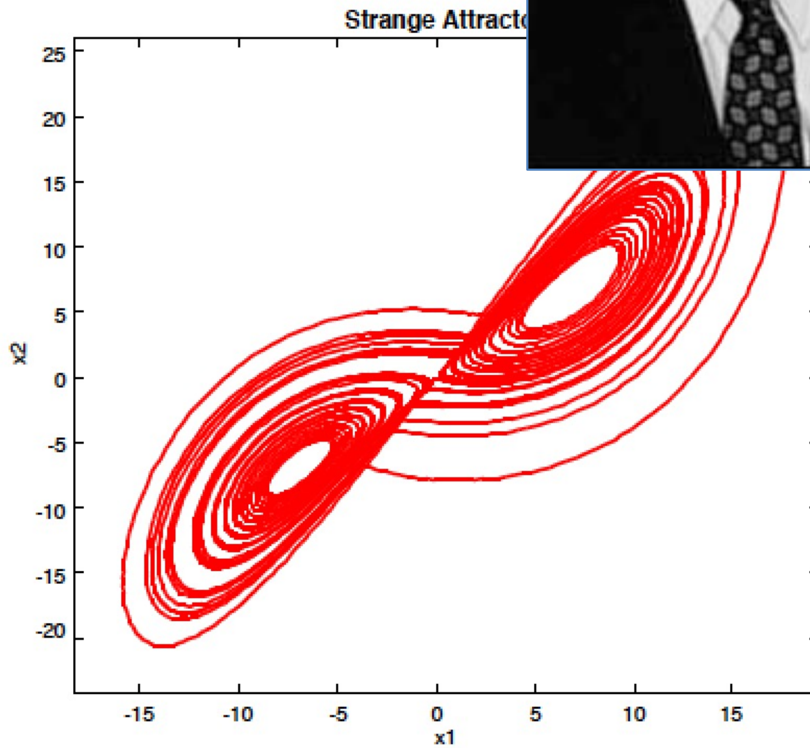
Chaos

Edward Lorenz

Lorenz attractor:

$$\dot{x}_1(t) = \sigma(x_2(t) - x_1(t))$$
$$\dot{x}_2(t) = (\lambda - x_3(t))x_1(t) - x_2(t)$$
$$\dot{x}_3(t) = x_1(t)x_2(t) - bx_3(t)$$

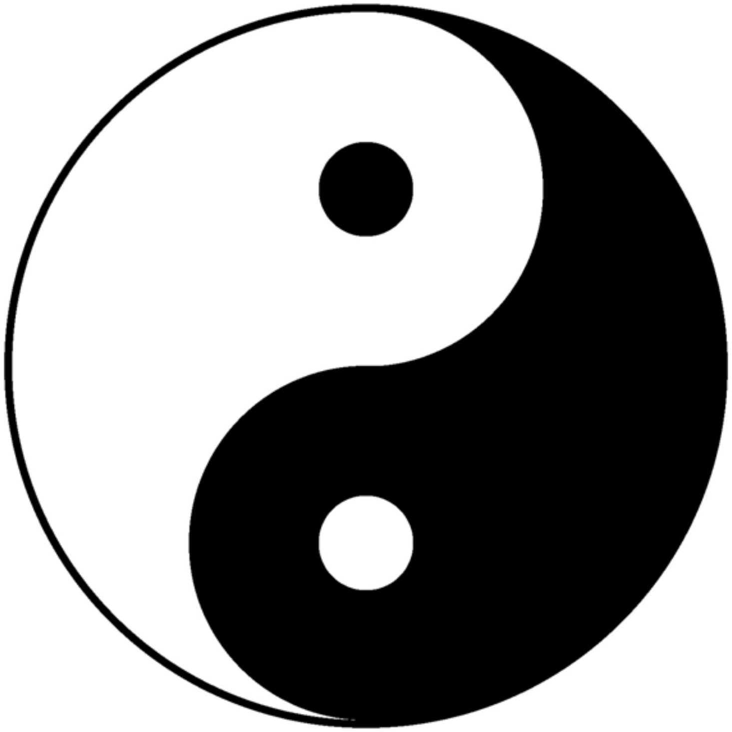
Plot of x_1 vs. x_2:



The error in x_1 and x_2 due to numerical approximation is limited only by the stability of the system.

But…

Determinism has its limits.



- Complexity
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Incompleteness of Determinism

Any set of deterministic models rich enough to encompass Newton's laws plus discrete transitions is incomplete.

Lee, *Fundamental Limits of Cyber-Physical Systems Modeling*, ACM Tr. on CPS, Vol. 1, No. 1, November 2016

Fundamental Limits of Cyber-Physical Systems Modeling

EDWARD A. LEE, EECS Department, UC Berkeley

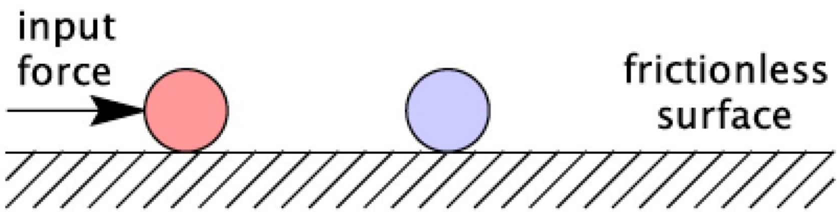
This article examines the role of modeling in the engineering of cyber-physical systems. It argues that the role that models play in engineering is different from the role they play in science, and that this difference should direct us to use a different class of models, where simplicity and clarity of semantics dominate over accuracy and detail. I argue that determinism in models used for engineering is a valuable property and should be preserved whenever possible, regardless of whether the system being modeled is deterministic. I then identify three classes of fundamental limits on modeling, specifically chaotic behavior, the inability of computers to numerically handle a continuum, and the incompleteness of determinism. The last of these has profound consequences.

CCS Concepts: • **Theory of computation** → **Timed and hybrid models**; • **Computing methodologies** → *Modeling methodologies*; • **Software and its engineering** → Domain specific languages

Additional Key Words and Phrases: Chaos, continuums, completeness

ACM Reference Format:
Edward A. Lee. 2016. Fundamental limits of cyber-physical systems modeling. ACM Trans. Cyber-Phys. Syst. 1, 1, Article 3 (November 2016), 26 pages.
DOI: http://dx.doi.org/10.1145/2912149

Illustration of the Incompleteness of Determinism



Conservation of momentum:

$$m_1 v_1' + m_2 v_2' = m_1 v_1 + m_2 v_2.$$

Conservation of kinetic energy:

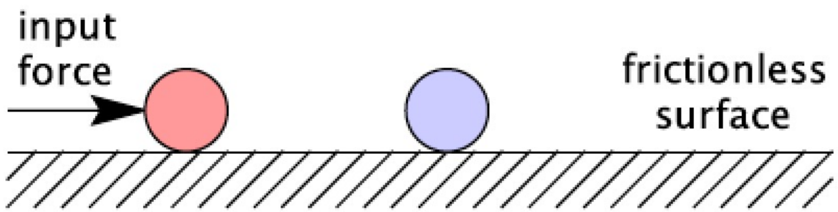
$$\frac{m_1 (v_1')^2}{2} + \frac{m_2 (v_2')^2}{2} = \frac{m_1 (v_1)^2}{2} + \frac{m_2 (v_2)^2}{2}.$$

We have two equations and two unknowns, v_1' and v_2'.

Illustration of the Incompleteness of Determinism

Quadratic problem has two solutions.

Solution 1: $v_1' = v_1$, $v_2' = v_2$ (ignore collision).



Solution 2:

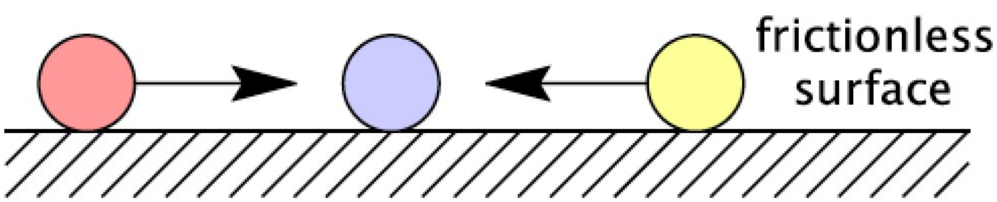
$$v_1' = \frac{v_1(m_1 - m_2) + 2m_2 v_2}{m_1 + m_2}$$

$$v_2' = \frac{v_2(m_2 - m_1) + 2m_1 v_1}{m_1 + m_2}.$$

Note that if $m_1 = m_2$, then the two masses simply exchange velocities (Newton's cradle).

Illustration of the Incompleteness of Determinism

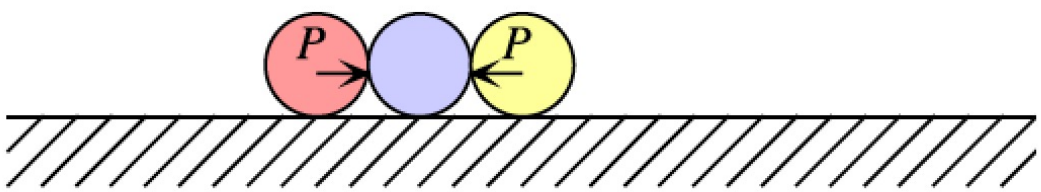
Consider this scenario:



Simultaneous collisions where one collision does not cause the other.

Illustration of the Incompleteness of Determinism

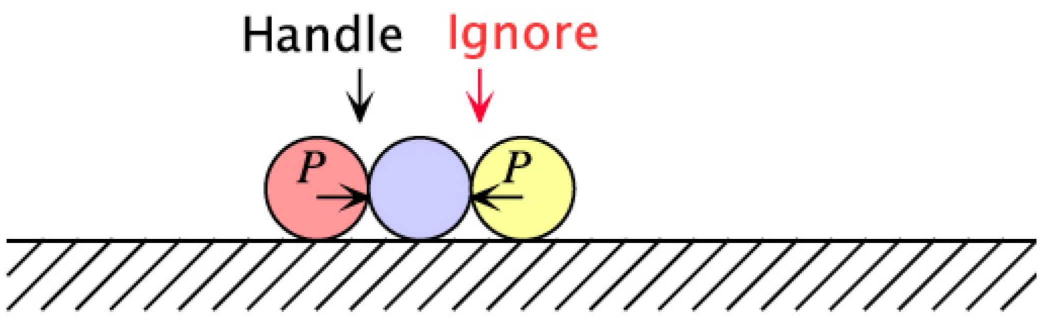
One solution: nondeterministic interleaving of the collisions:



At superdense time $(\tau, 0)$, we have two simultaneous collisions.

Illustration of the Incompleteness of Determinism

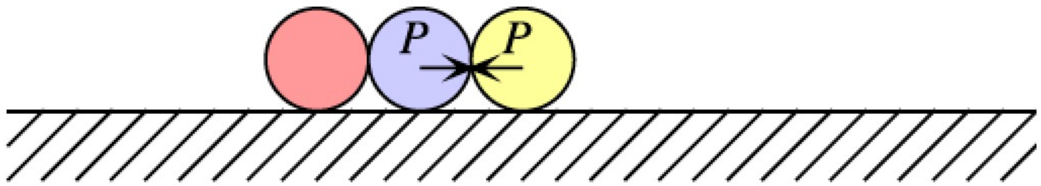
One solution: nondeterministic interleaving of the collisions:



At superdense time $(\tau, 1)$, choose arbitrarily to handle the left collision.

Illustration of the Incompleteness of Determinism

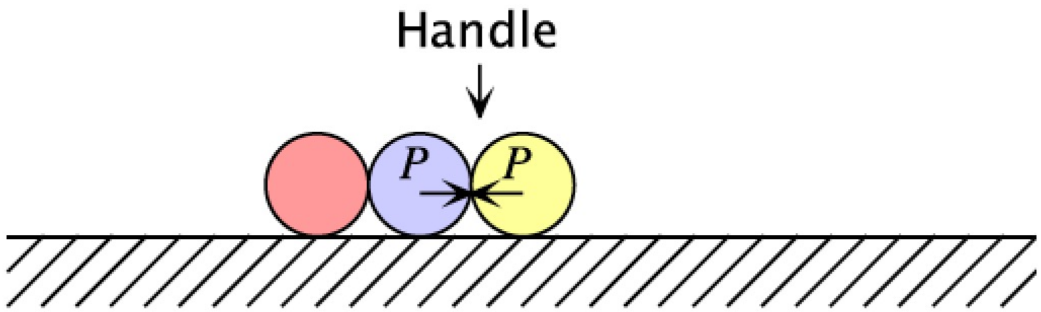
One solution: nondeterministic interleaving of the collisions:



After superdense time $(\tau, 1)$, the momentums are as shown.

Illustration of the Incompleteness of Determinism

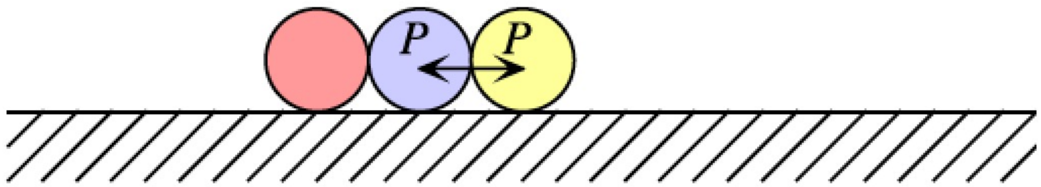
One solution: nondeterministic interleaving of the collisions:



At superdense time $(\tau, 2)$, handle the new collision.

Illustration of the Incompleteness of Determinism

One solution: nondeterministic interleaving of the collisions:



After superdense time $(\tau, 2)$, the momentums are as shown.

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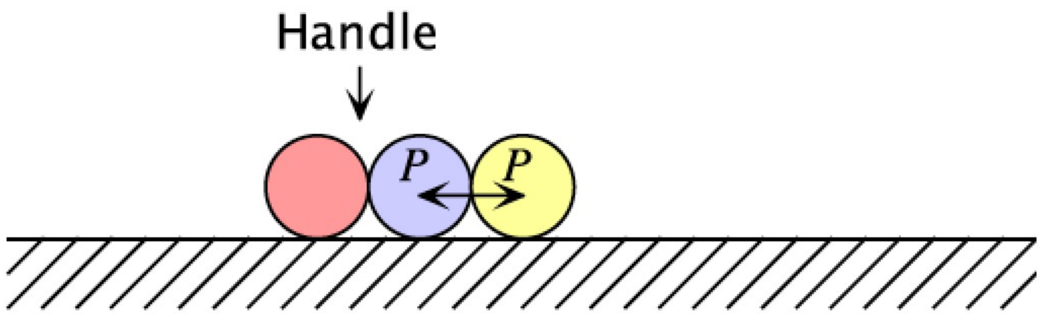
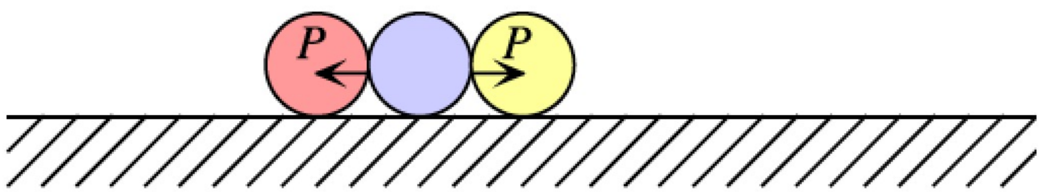
One solution: nondeterministic interleaving of the collisions:



At superdense time $(\tau, 3)$, handle the new collision.

Illustration of the Incompleteness of Determinism

One solution: nondeterministic interleaving of the collisions:



After superdense time $(\tau, 3)$, the momentums are as shown.

Illustration of the Incompleteness of Determinism

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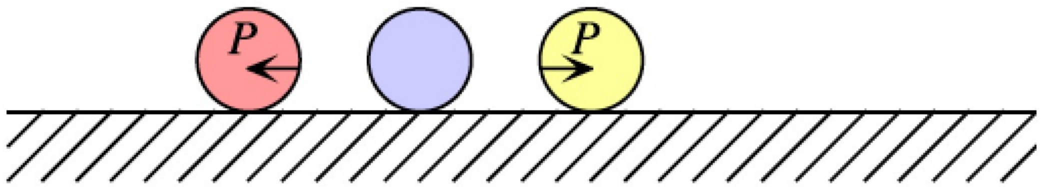
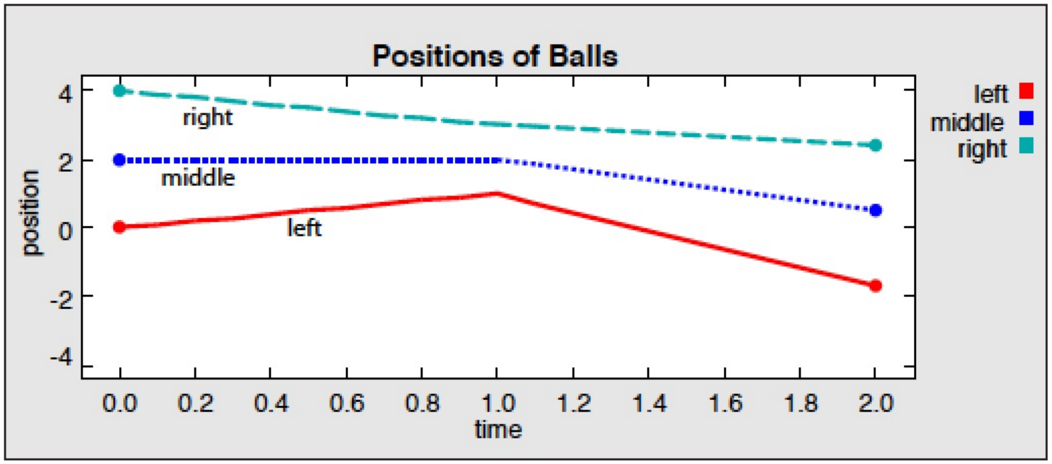


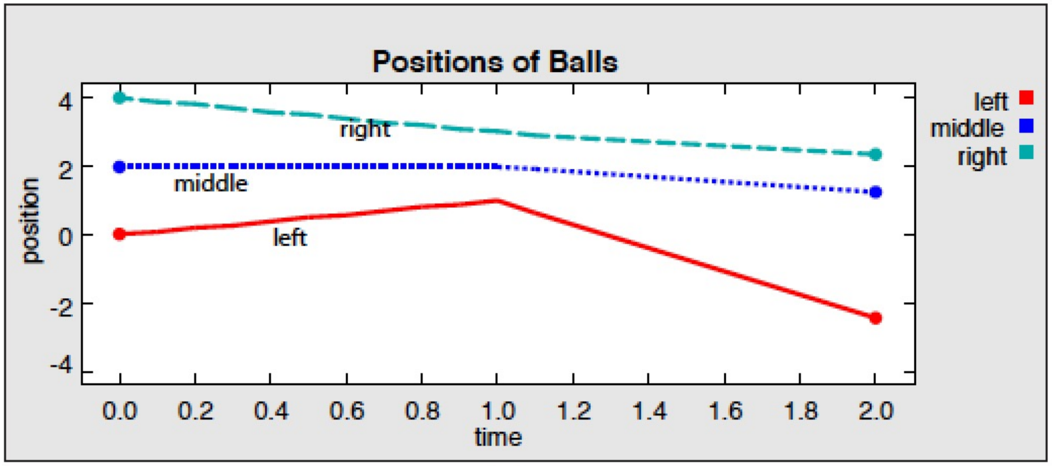
The balls move away at equal speed (if their masses are the same!)

Arbitrary Interleaving Yields Nondeterminism

If the masses are different, the behavior depends on which collision is handled first!



(a)

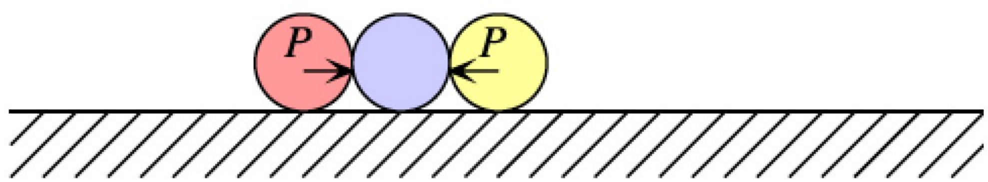


(b)

Recall the Heisenberg Uncertainty Principle

We cannot simultaneously know the position and momentum of an object to arbitrary precision.

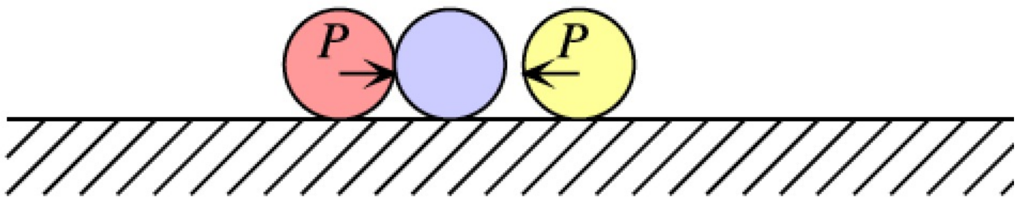
But the reaction to these collisions depends on knowing position and momentum precisely.



Is Determinism Incomplete?

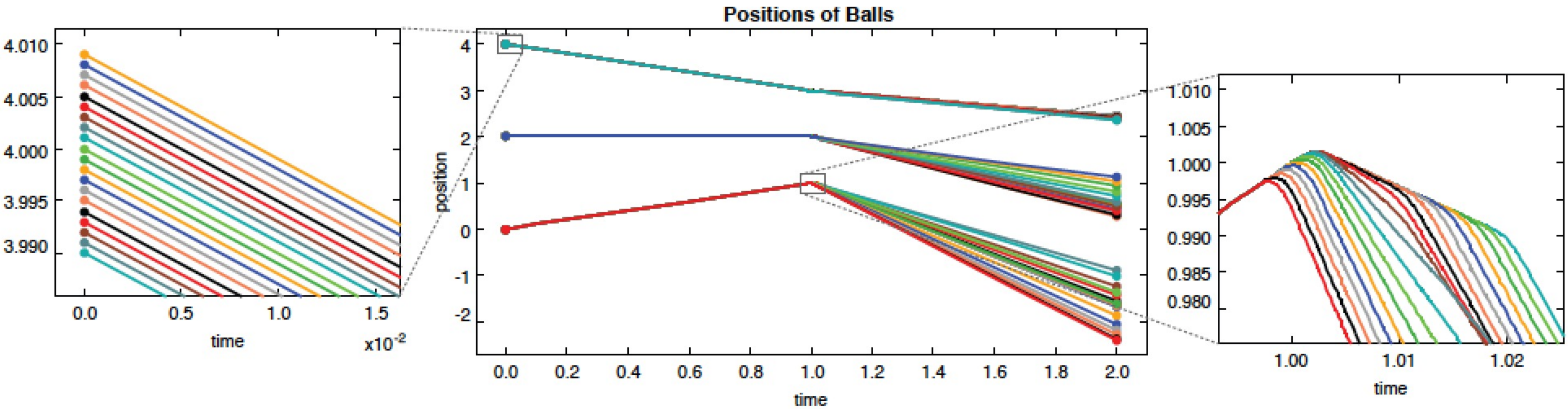
Let τ be the time between collisions. Consider a sequence of models for $\tau > 0$ where $\tau \to 0$.

Every model in the sequence is deterministic, but the limit model is not.



- In Lee (2016), I show that this sequence of models is Cauchy, so the space of deterministic models is incomplete (it does not contain its own limit points).
- In Lee (2014), I show that a direct description of this scenario results in a non-constructive model. The nondeterminism arises in making this model constructive.

Rejecting discreteness leads to deterministic chaos

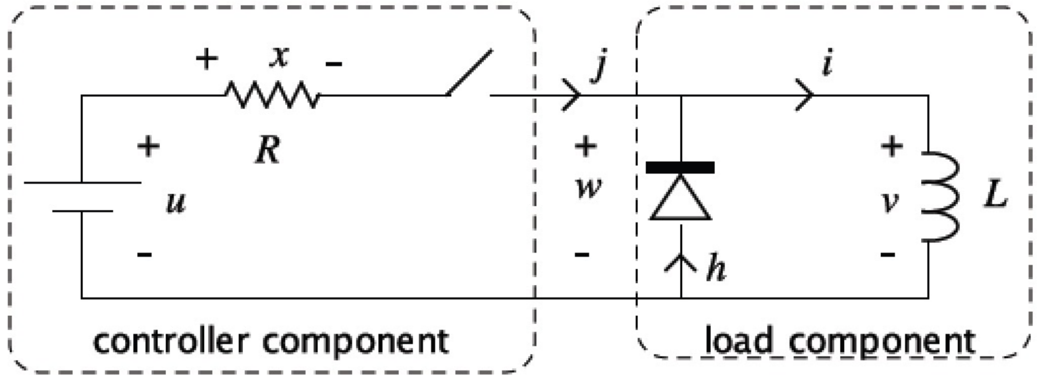


[Lee, ACM Tr. on CPS, 2016]

A continuous deterministic model that models collisions as elastic springs is chaotic.

Rejecting discreteness requires rejecting causality

Example from Lee, "Constructive Models of Discrete and Continuous Physical Phenomena," IEEE Access, 2014



A flyback diode is a commonly used circuit that prevents arcing when disconnecting an inductive load (like a motor) from a power source.

When the switch goes from closed to open, the causality and direct feedthrough properties of the two components reverse.

There is no logic that can transition from *A* causes *B* to *B* causes *A* smoothly without passing through non-constructive models.

If nondeterminism is unavoidable, shouldn’t we embrace it?

Death is unavoidable, but you may want to try to avoid it anyway…

For CPS, we need to change the question

The question is *not* whether deterministic models can describe the behavior of cyber-physical systems (with high fidelity).

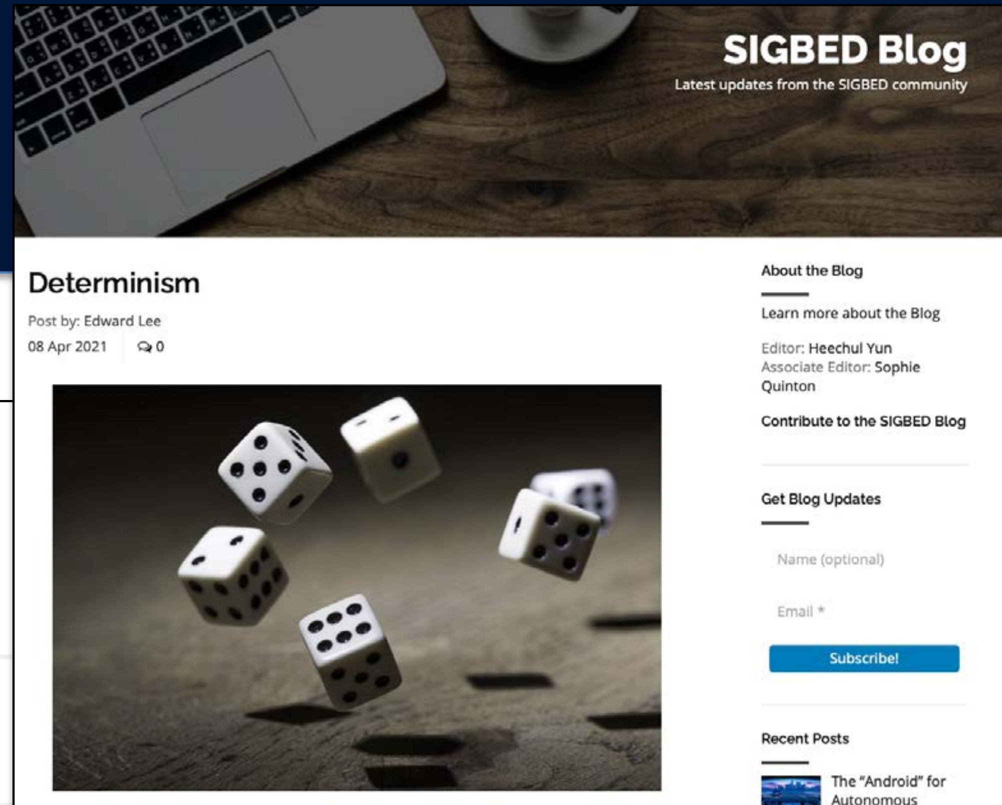
The question is whether we can build cyber-physical systems whose behavior matches that of a deterministic model (with high probability).

Determinism?
What about resilience? Adaptability?

Deterministic models do not eliminate the need for robust, fault-tolerant designs.

In fact, they *enable* such designs, because they make it much clearer what it means to have a fault!

References



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Determinism

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ACM Transactions on Embedded Computing Systems, Volume 20, Issue 5 • July 2021 • Article No.: 38, pp 1–34 • https://doi.org/10.1145/3453652

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