

Introducing Signals and Systems The Berkeley Approach

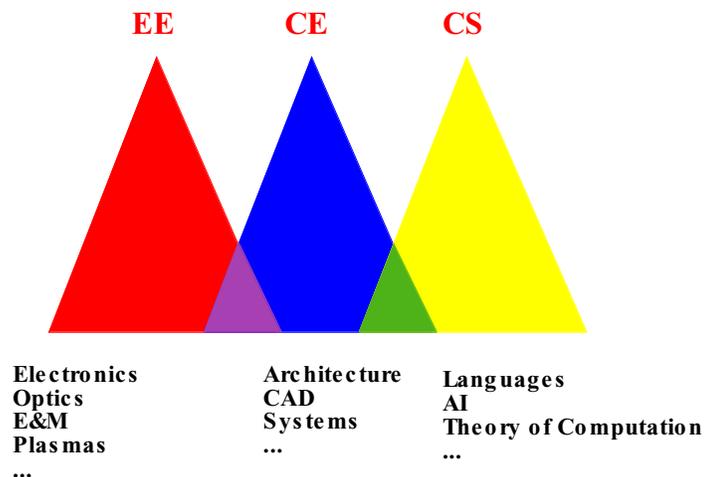


Edward A. Lee
Pravin Varaiya
UC Berkeley

A computer without
networking, audio,
video, or real-time
services.

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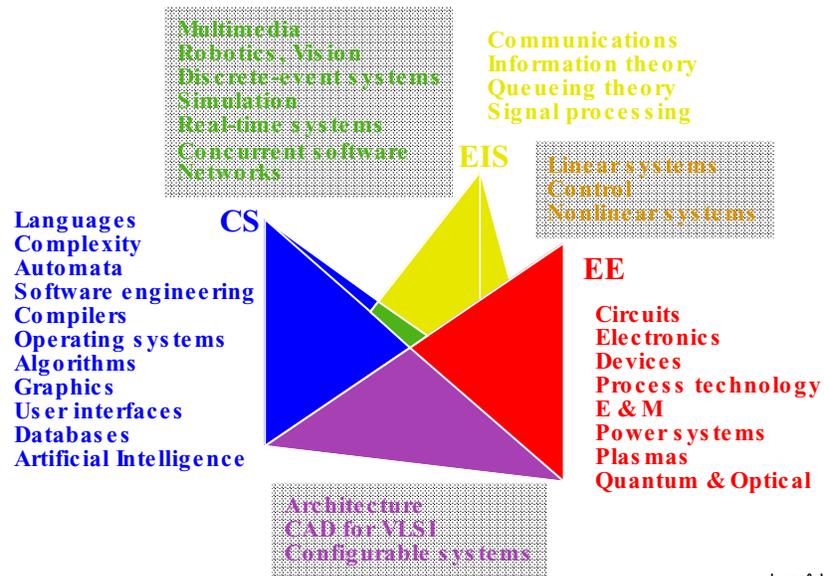
Starting Point



But the juncture of EE and CS is not just hardware.
It is also mathematical modeling and system design.

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Intellectual Grouping of EE, CE, CS



Six Intellectual Groupings

- **Blue:** Computer Science
- **Green:** Computer Information Systems
- **Yellow:** Electronic Information Systems
- **Orange:** Electronic Systems
- **Red:** Electronics
- **Purple:** Computer hardware

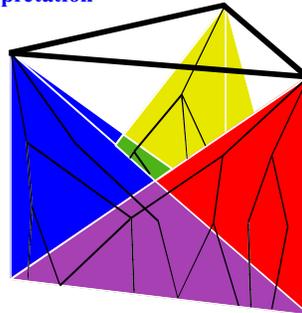
New Introductory Course Needed

CS
(CS 61A)

The Structure and Interpretation
of Computer Programs

EIS
(EECS 20)

Structure & Interpretation of
Signals & Systems



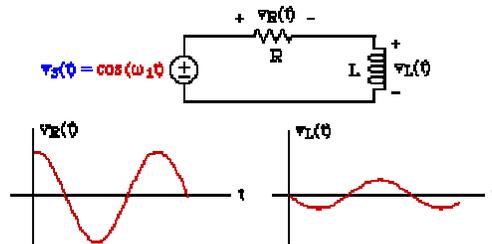
EE
(EE 40)

Introduction to
Electronic Circuits

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The Roots of Signals and Systems

- Circuit theory
- Continuous-time
- Calculus-based



Major models

- Frequency domain
- Linear time-invariant systems
- Feedback

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Changes in Content

- **Signal**
 - **used to be:** voltage over time
 - **now may be:** discrete messages
- **State**
 - **used to be:** the variables of a differential equation
 - **now may be:** a process continuation in a transition system
- **System**
 - **used to be:** linear time invariant transfer function
 - **now may be:** Turing-complete computation engine



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Changes in Intellectual Scaffolding

- **Fundamental limits**
 - **used to be:** thermal noise, the speed of light
 - **now may be:** chaos, computability, complexity
- **Mathematics**
 - **used to be:** calculus, differential equations
 - **now may be:** mathematical logic, topology, set theory, partial orders
- **Building blocks**
 - **used to be:** capacitors, resistors, transistors, gates, op amps
 - **now may be:** microcontrollers, DSP cores, algorithms, software components

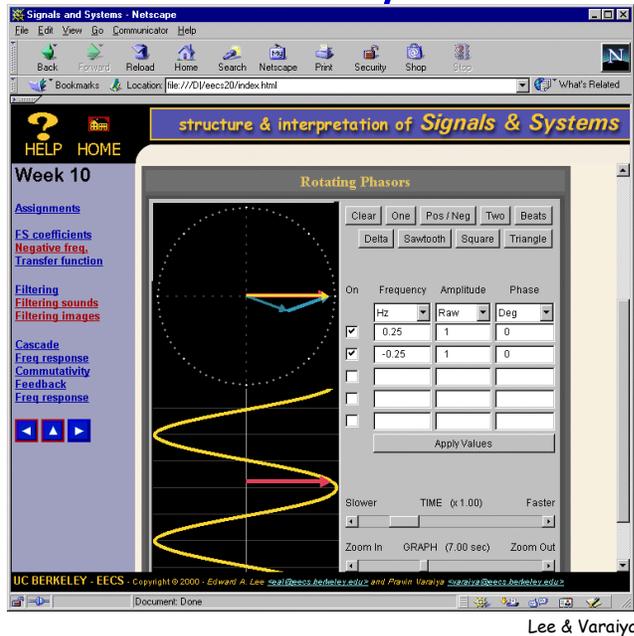


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Action at Berkeley

Berkeley has instituted a new sophomore course that addresses mathematical modeling of signals and systems from a very broad, high-level perspective.

The web page at the right contains an applet that illustrates complex exponentials used in the Fourier series.



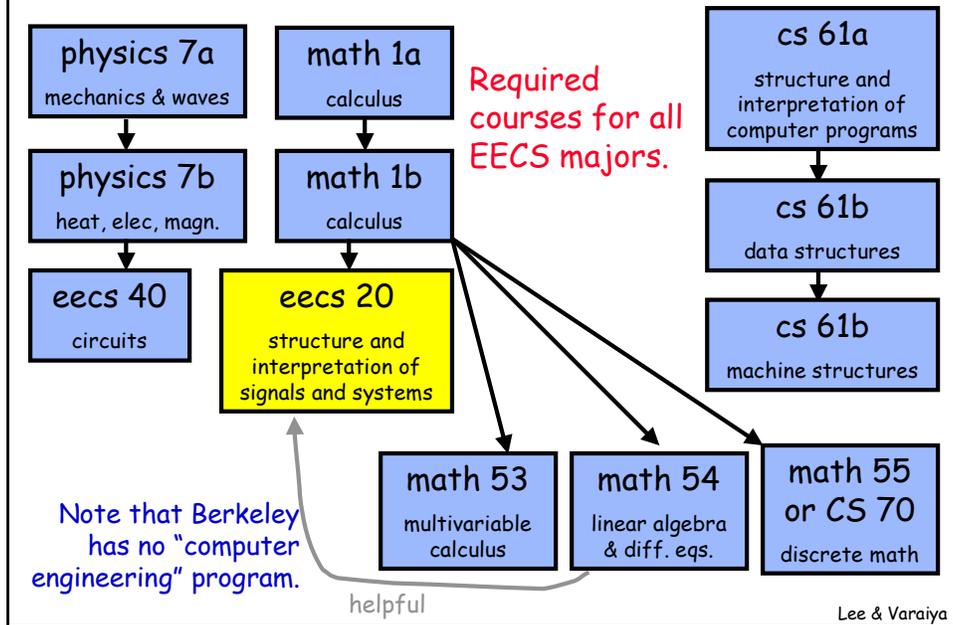
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Themes of the Course

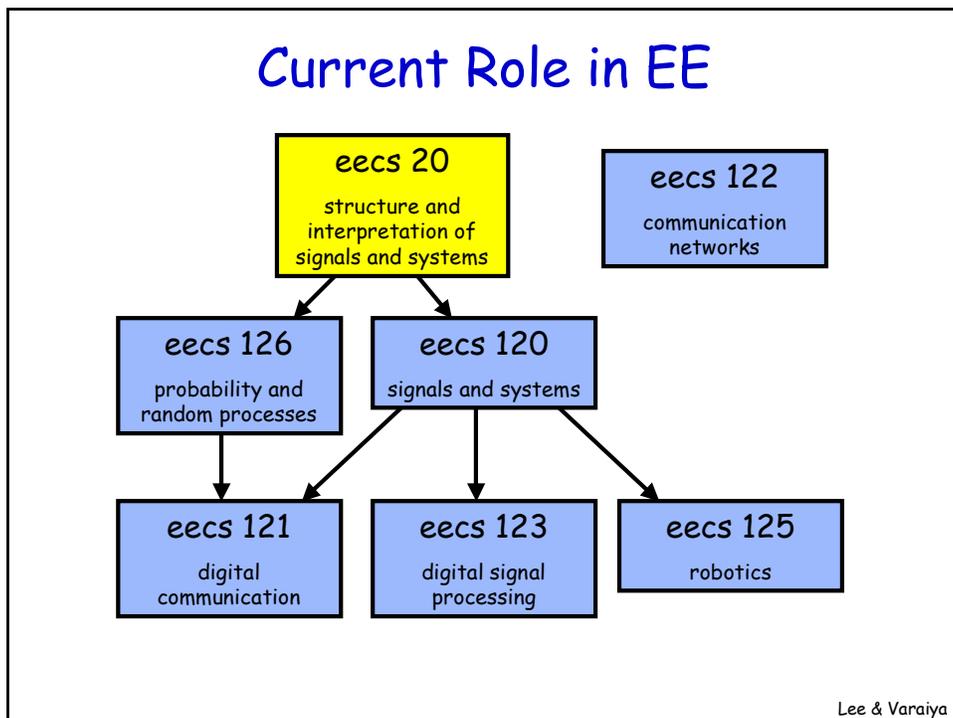
- The connection between *imperative* (Matlab) and *declarative* (Mathematical) descriptions of signals and systems.
- The use of *sets and functions* as a universal language for declarative descriptions of signals and systems.
- State machines and frequency domain analysis as complementary tools for designing and analyzing signals and systems.
- Early and often discussion of applications.

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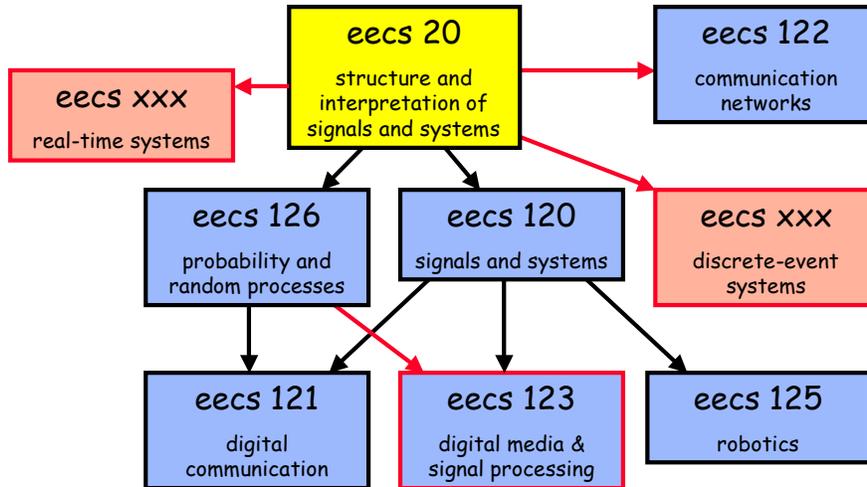
Role in the EECS Curriculum



Current Role in EE



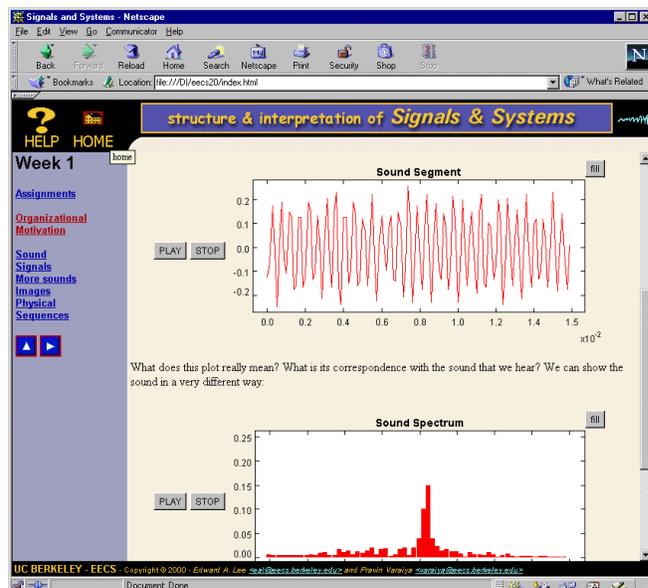
Future Role in EECS (speculative)



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Outline

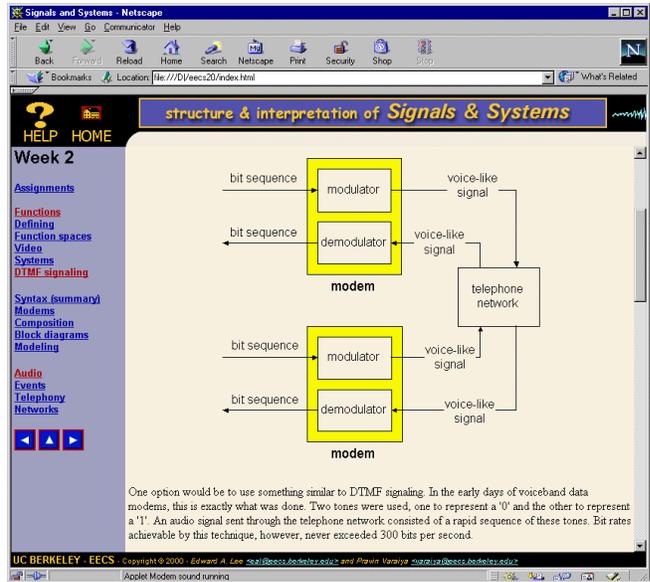
1. Signals
2. Systems
3. State
4. Determinism
5. Composition
6. Linearity
7. Freq Domain
8. Freq Response
9. LTI Systems
10. Filtering
11. Convolution
12. Transforms
13. Sampling
14. Design
15. Examples



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Notation

- Sets and functions
 - *Sound*: $Reals \rightarrow Reals$
 - *DigitalSound*: $Ints \rightarrow Reals$
 - *Sampler*: $[Reals \rightarrow Reals] \rightarrow [Ints \rightarrow Reals]$
- Our notation unifies
 - discrete and continuous time
 - event sequences
 - images and video, digital and analog
 - spatiotemporal models

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Problems with Standard Notation

- The form of the argument defines the domain

- $x(n)$ is discrete-time, $x(t)$ is continuous-time.
- $x(n) = x(nT)$? Yes, but...

- $X(j\omega) = X(s)$ when $j\omega = s$
- $X(e^{j\omega}) = X(z)$ when $z = e^{j\omega}$
- $X(e^{j\omega}) = X(j\omega)$ when $e^{j\omega} = j\omega$? No.

- $x(n)$ is a function

- $y(n) = x(n) * h(n)$
- $y(n-N) = x(n-N) * h(n-N)$? No.



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Using the New Notation

- Discrete-time Convolution :

$$\text{Convolution: } [Ints \rightarrow Reals] \times [Ints \rightarrow Reals] \\ \rightarrow [Ints \rightarrow Reals]$$

- Shorthand:

$$x * y = \text{Convolution}(x, y)$$

- Definition:

$$(x * y)(n) = \sum_{k=-\infty}^{\infty} x(k)y(n-k)$$

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structure & interpretation of *Signals & Systems*

Week 3

Example - Answering Machine

Verbal description:

- On the third ring, answer the phone, play a greeting, and record a message.
- After recording a message, hang up.
- If a telephone is taken off hook within three rings, do nothing.

State transition diagram:

```

    graph LR
      idle((idle)) -- "(ring)" --> count1((count1))
      count1 -- "(absent)" --> idle
      count1 -- "(ring)" --> count2((count2))
      count2 -- "(absent)" --> idle
      count2 -- "(ring)" --> play_greeting((play greeting))
      play_greeting -- "(absent)" --> idle
      play_greeting -- "(end greeting / record)" --> record_ing((record ing))
      record_ing -- "(end greeting / record)" --> idle
      idle -- "else" --> idle
      count1 -- "else" --> idle
      count2 -- "else" --> idle
      play_greeting -- "else" --> idle
      record_ing -- "else" --> idle
  
```

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Week 4

Simulation and Bisimulation

Consider deterministic state machine A :

```

    graph LR
      0and2((0 and 2)) -- "{1}/0" --> 1and3((1 and 3))
      1and3 -- "{1}/1" --> 0and2
  
```

and nondeterministic state machine B :

```

    graph LR
      0((0)) -- "{1}/0" --> 1((1))
      1 -- "{1}/1" --> 2((2))
      2 -- "{1}/0" --> 3((3))
      3 -- "{1}/1" --> 0
  
```

Consider a game, where each machine starts in its initial state. Then, given an input, A reacts, and B tries to react in such a way as to produce the same output (given the same input). For the above examples, B can always do this, so B is said to **simulate** A . Equivalently, we say that B **simulates** A .

The game can be turned around, where B makes a move and A tries to match it. In the above examples, this is again possible, so A simulates B . A and B are said to be **bisimilar**.

We can track this game by looking at the state responses of the two machines. They each start in their initial

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Week 5 **Feedback Composition**

Consider two state machines connected with a feedback loop:

Assumption:

- $Output_A = Input_B$
- $Output_B = Input_A$

Definition of the composition:

- $States = States_A \times States_B$
- $Inputs = Inputs_A$
- $Outputs = Outputs_B$

updates function is found by iteration to a fixed point:

- Start with *unknown* on the feedback arc.

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Week 6 **Infinite State Systems**

We study state machine with

$$States = \text{Reals}^N$$

$$Inputs = \text{Reals}^M$$

$$Outputs = \text{Reals}^K$$

The number of states and the sizes of the input and output alphabets are infinite, which tends to make things more complicated. However, these sets now have arithmetic properties, which opens up a huge new range of modeling and analysis possibilities.

A **block diagram** of such a system shows the input coming over M input ports and the output delivered over K output ports. (MIMO stands for multi-input, multi-output.)

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Week 7

Assignments	466
Linearity	494
Matrices	523
Vector, LTI state	554
Evaluation	587
SISO	622
Response	659
Convolution	698
Moving average	740
Frequencies	784
Musical scale	831
Phase	880
Timbre	
Images	

The psychophysical properties of the musical scale are fascinating. The musical scale is based on our perception of frequency, and harmonic relationships between frequencies. The choice of 12 evenly spaced notes is based on the so-called **circle of fifths**.

Frequencies that are harmonically related tend to sound good together. In the following applet, you can combine any set of frequencies in the scale.

Western Musical Scale

Time in seconds $\times 10^{-3}$

A
 B flat
 B
 C
 C sharp
 D
 D sharp
 E
 F sharp
 A flat
 F sharp
 Sum
 A flat
 A

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Week 8

Discrete Fourier series examples

The following applet illustrates how, unlike the continuous Fourier series, the discrete Fourier series converges to an exact representation of a periodic waveform in a finite number of steps.

Time Domain

Time in seconds $\times 10^{-3}$

Frequency Domain

Frequency in Hz $\times 10^3$

0
 1
 2
 3
 4
 5
 6
 7
 8
 9
 10
 11
 12
 13
 14
 15
 16

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Week 11

Assignments

Imparative
Impulses
Sums of Impulses
Impulse response

Convolution
FIR
Moving Average
Convolution
Causality

Implementation
Composition

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Week 12

Assignments

Freq. Response
Continuous time
Moving Average

Signal Spaces
Transforms
Symmetry
Inverse
Examples

Linearity
Using Linearity
Constants
Exponentials
Sinusoid
Discrete

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Week 13

Aliasing

In the following applet, you can change the frequency of a "continuous-time" sinusoid from 0 to 8,000 Hz. The sinusoid is sampled at 8 kHz and played through the computer audio system. Notice that as the frequency increases above 4 kHz, the Nyquist frequency, the sound you hear starts to decrease in frequency rather than increase. Frequencies above 4 kHz are indistinguishable from corresponding frequencies below 4 kHz. See the [text](#) for a mathematical presentation of this phenomenon.

start stop scale 7689

A sinusoid and its samples

Time in seconds $\times 10^{-3}$

If you sweep the frequency of a continuous-time sinusoid from 0 to 8 kHz, and this sinusoid is sampled at 8 kHz and transmitted through the telephone network, the sound you will hear at the other end has a perceived pitch that rises until you get to 4 kHz, but then it begins to fall, as shown in the following plot:

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Week 14

unless (it is the ratio of two magnitudes with the same units), we can express it in decibels as

$$G_{db} = 20 \log_{10}(G)$$

This has units of decibels, written dB. The same frequency response as above is given below in decibels:

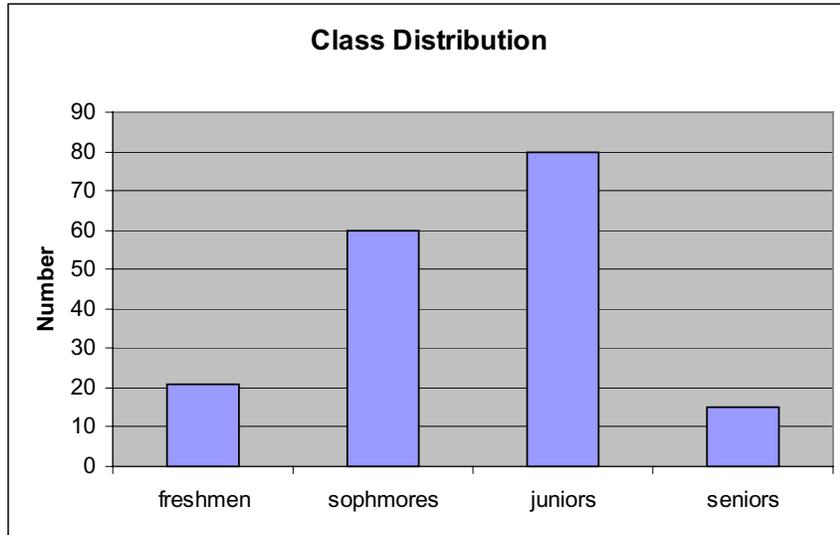
dB

frequency

Applet: frequency response (numm)

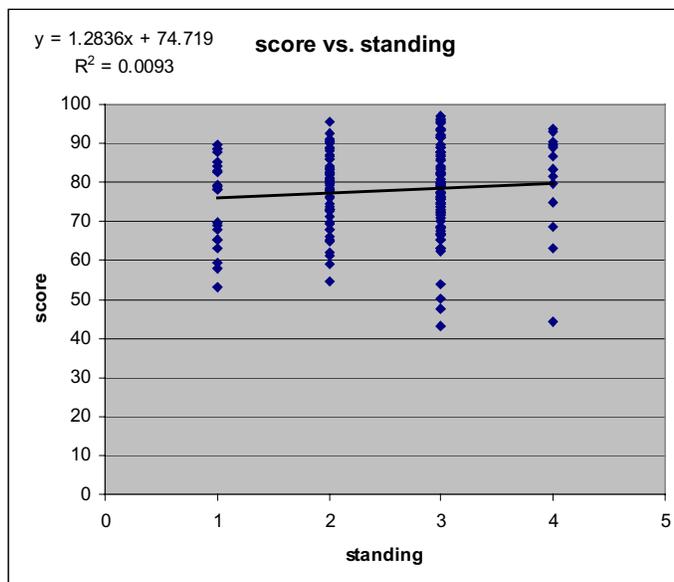
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Distribution by Class Standing



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Effect of Class Standing



80 and above:
A's
63 and above: B's
62 and below: C's

176 of the 227
students
responded (the
better ones).

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Effect of Showing Up

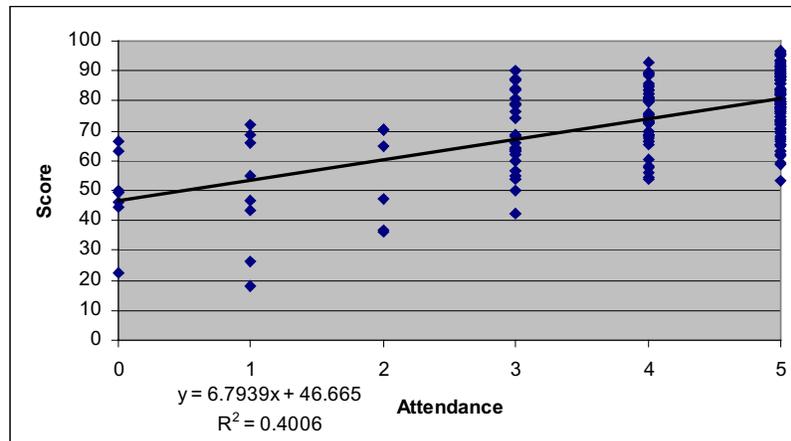
Students who answered the survey were those that showed up for the second to last lab.

- The mean for those who responded was 78, vs. 65 for those who did not respond (two grades, e.g. B to A-).
- The standard deviation is much higher for those who did not respond.
- A t-test on the means shows the data are statistically very significant.

We conclude that the respondents to the survey do not represent a random sample from the class, but rather represent the diligent subset.

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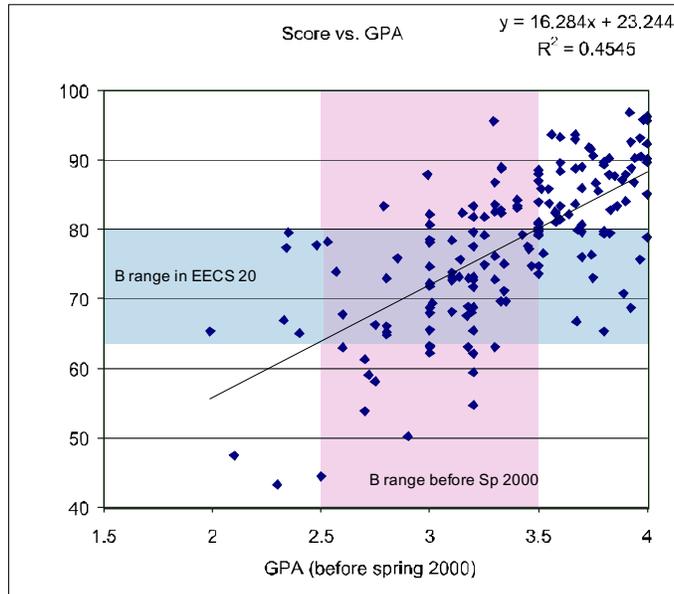
Attendance in Class vs. Score



Attendance is measured by presence for pop quizzes, of which there were five.

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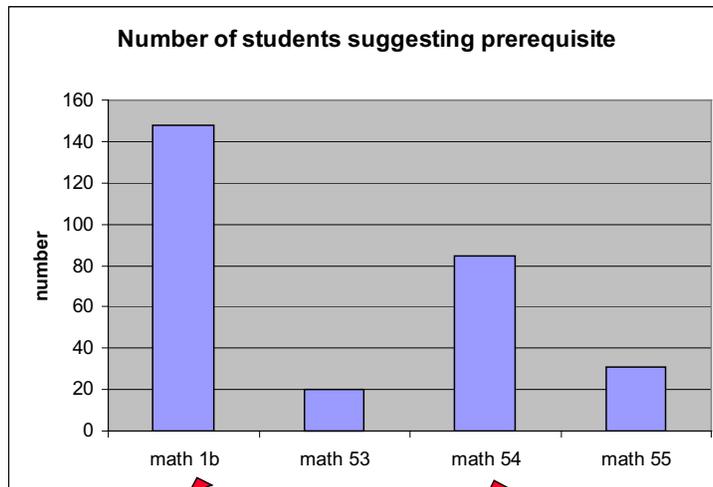
Effect on GPA



On average, students' GPA was not affected by this class.

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Student Opinion on Prerequisites



series

linear algebra

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Differences from Tradition

- No circuits
- More discrete-time, some continuous-time
- Broader than LTI systems
- Unifying sets-and-functions framework
- Emphasis on applications
- Many applets and demos
- Tightly integrated software lab

Text draft (Lee & Varaiya) and website available.

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Bottom-Up or Top-Down?



- Bottom-up:**
- foundations first
 - derive the applications

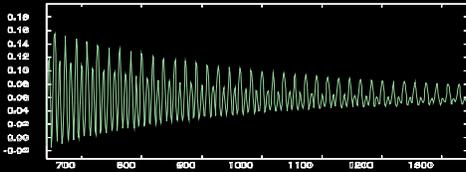
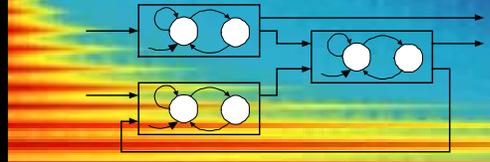


- Top-down:**
- applications first
 - derive the foundations

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Structure and Interpretation of Signals and Systems

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Textbook

Draft available on
the web.